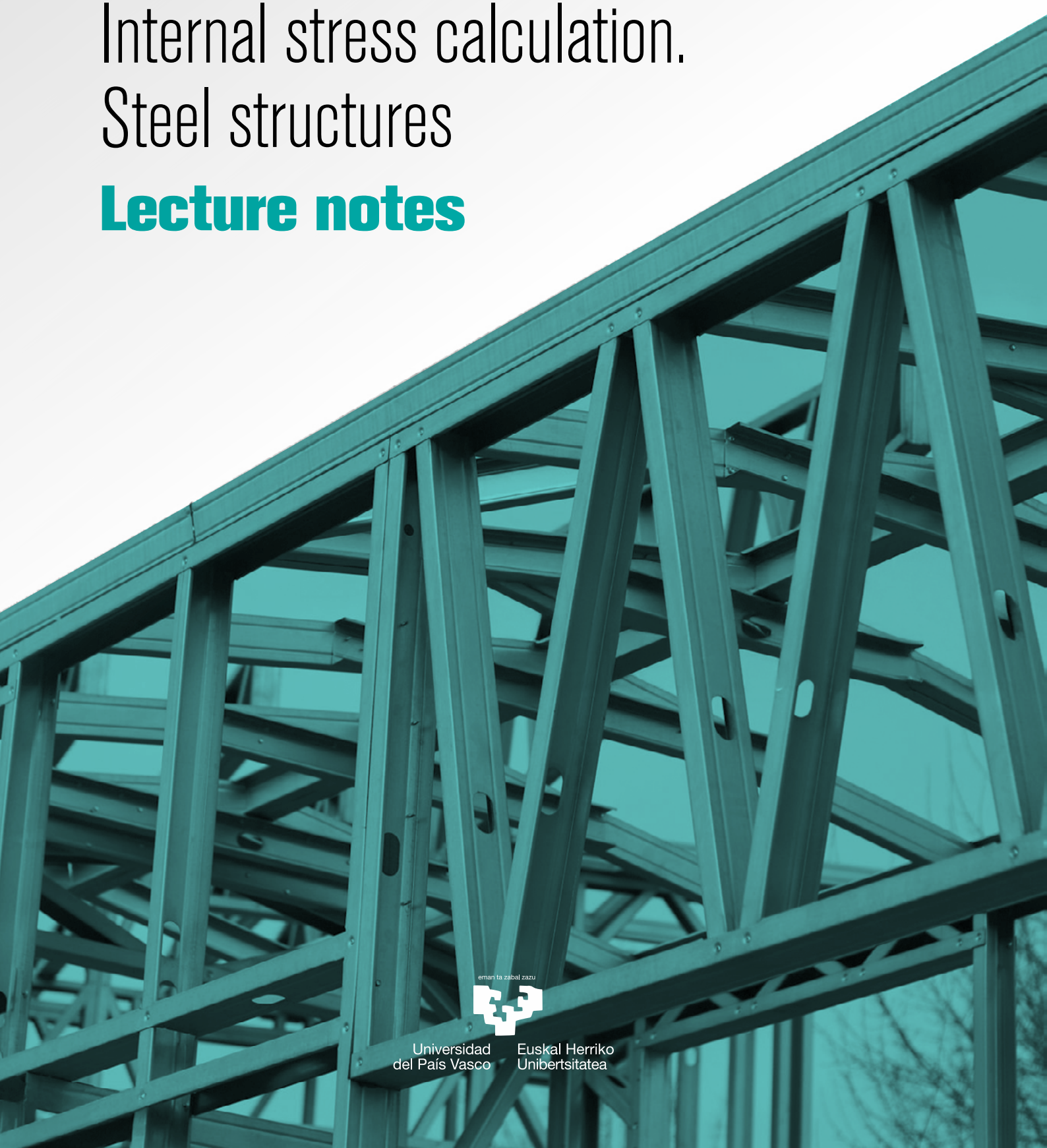
Gabriel Ruiz Múgica

Structures II

Internal stress calculation. Steel structures

Lecture notes



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Universidad del País Vasco
Euskal Herriko Unibertsitatea

Lecture notes

STRUCTURES II

INTERNAL STRESS CALCULATION
STEEL STRUCTURES

Carlos Gabriel Ruiz Múgica

ASSISTENT STUDENT
Adrian Mendoza Marcos
María de Artacho Sancho Arroyo

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1. INTRODUCTION

The course Structures II is the continuation of the course Structures I, taught in the first semester of this academic year. The combination of these two courses allows the student to have an overall and simplified idea of how to design and calculate a building structure.

In a summarized way, we can describe the structure design and calculation in these 4 steps:

1. The design of the building structure (frames)
2. The definition of the load state. (CTE – structural security)
3. The calculation of the frames (strengths diagrams)
4. The verification of the structure (Limit States method)

In the Structures I course the student received the basic notions to calculate a building frame (the 3rd step of the four above). However, the geometry and the load state of the frame were already given.

In the first part of this course, the student is going to learn basic criteria to design the structure of a building and to define the load state (the 1st and 2nd steps of the steps above). After this preliminary design, in the last part of the course, we will learn the procedure that we have to follow to verify a structure made of steel (4th step).

2. INTRODUCTION FOR A BUILDING STRUCTURE DESIGN

On this chapter we will learn the basic criteria to understand the behaviour of different structures. Even if these basic concepts are a kind of intuitive, they are absolutely necessary for the proper design of the structure.

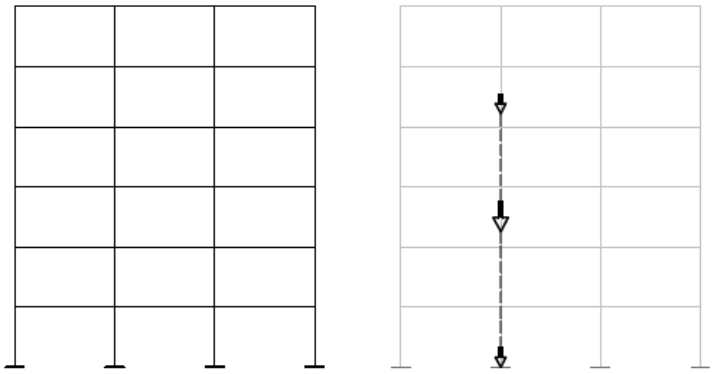
2.1. OVERALL DESIGN CRITERIA

This section has been separated in three smaller parts: vertical bearing of the loads, horizontal stability and types of joints. Each of these parts is going to provide a schematic solution, analysing the positive and negative aspects of each solution.

2.1.1. Vertical bearing of the loads

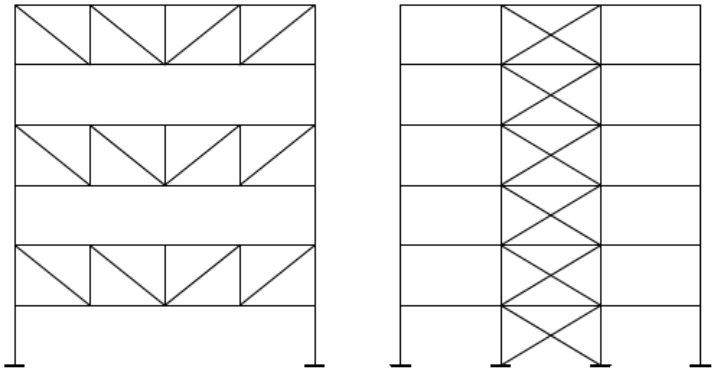
In these building structures there is the most rational tansmission of the loads: Directly from the floor slabs to the foundations, passing through the columns.

- Direct transmission

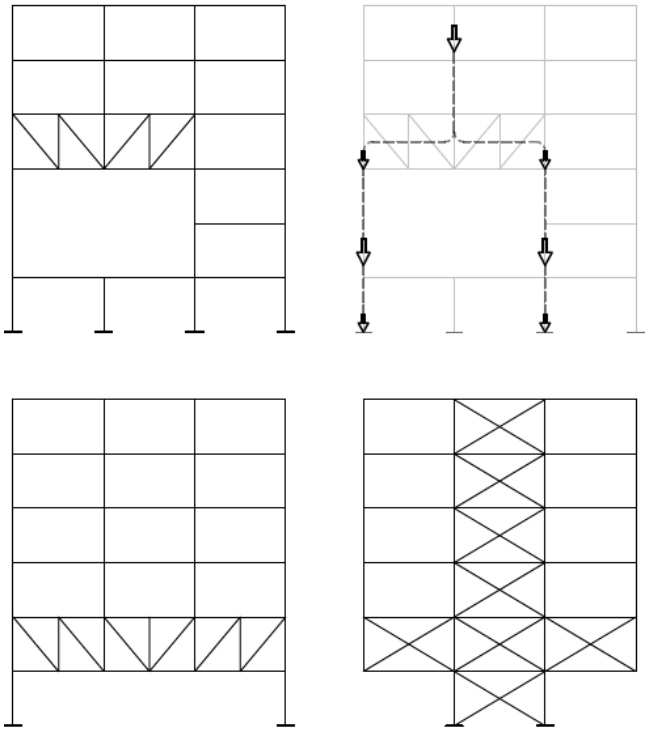


- Direct transmission: from the floor slabs to the columns and finally to the foundations.

- Is the most logical and cheapest structure system.



- Indirect transmission



- The need of a big space somewhere in the building.

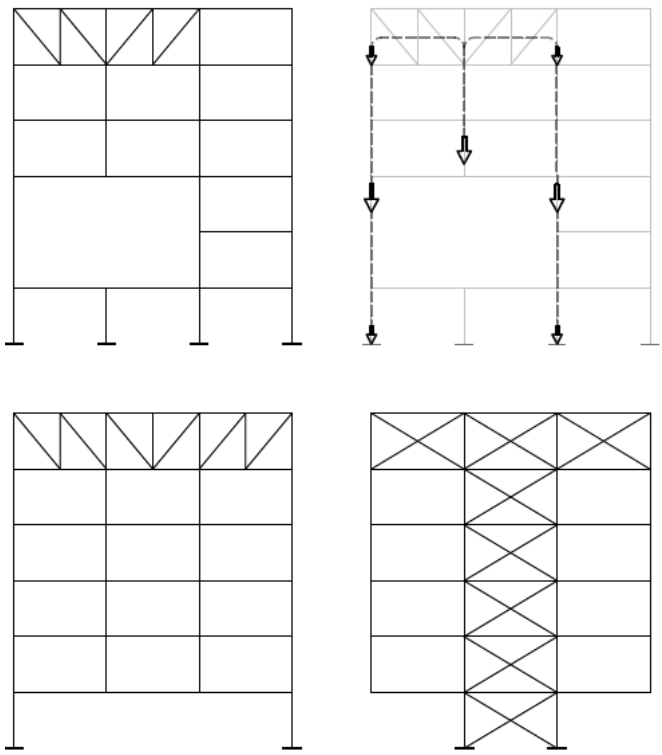
- Setbacks.

- The need of a different column organization in each floor.

- Special foundations.

- Special location: above a road, river or a train track.

- Suspended structures



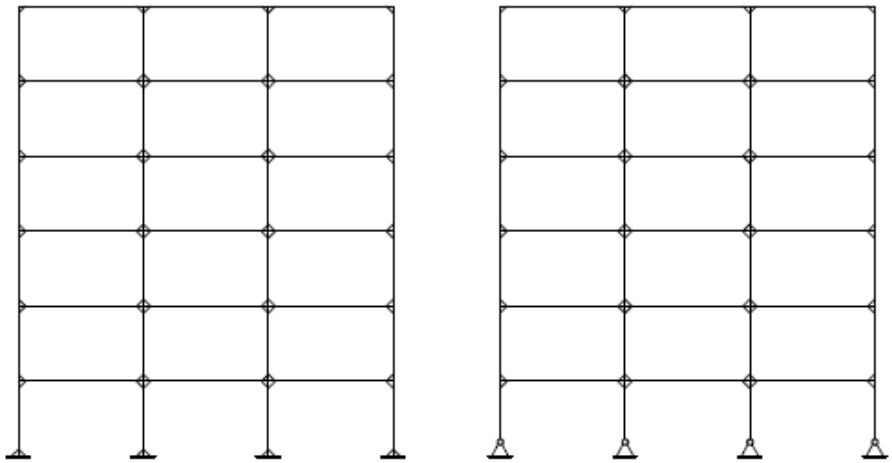
- The same situation as in the previous example. However, this time a part of the structure is hanging from the top, creating the same indirect transmission.

- This creates another problem that should be taken into consideration: The columns in the same floor are going to suffer different axial deformations.

2.1.2. Lateral stability

The capacity of a structure to hold horizontal stability is directly related to the nature of the joints between bars. That's why we are going to analyse structures with stiff and hinged joints, underlining the characteristics of each case.

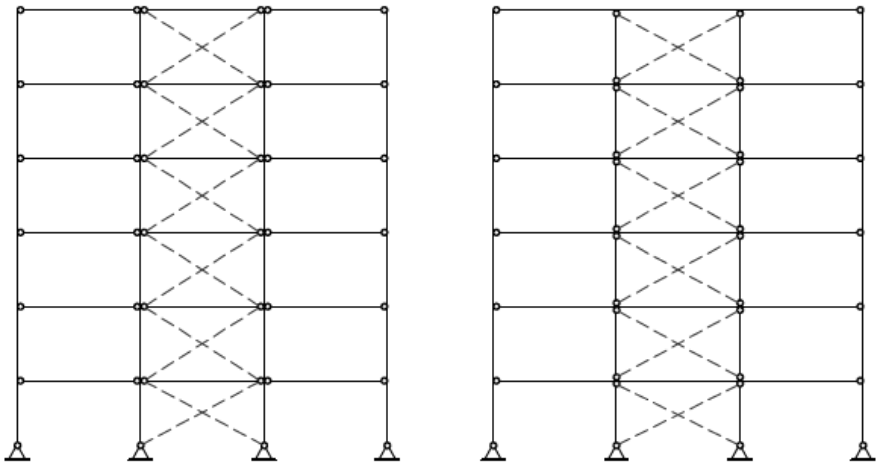
- Frames with stiff joints



The most common material used to create these rigid joints structures is concrete, but it could also be done using steel. These types of structures achieve the horizontal stability by transmitting the bending moment to the columns. These would be the main characteristics:

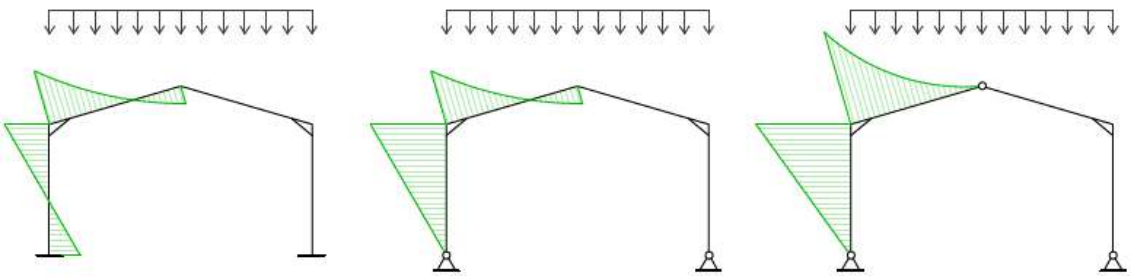
- The joint between column and beam must be stiff or semi-stiff.
- Depending on the case we may also need an additional bracing system.
- The connection between a column and the foundation can be stiff or hinged.
- Smaller efficiency. 10-15 floors (maximum 20). Better to create triangulations.

- Frames with hinged joints



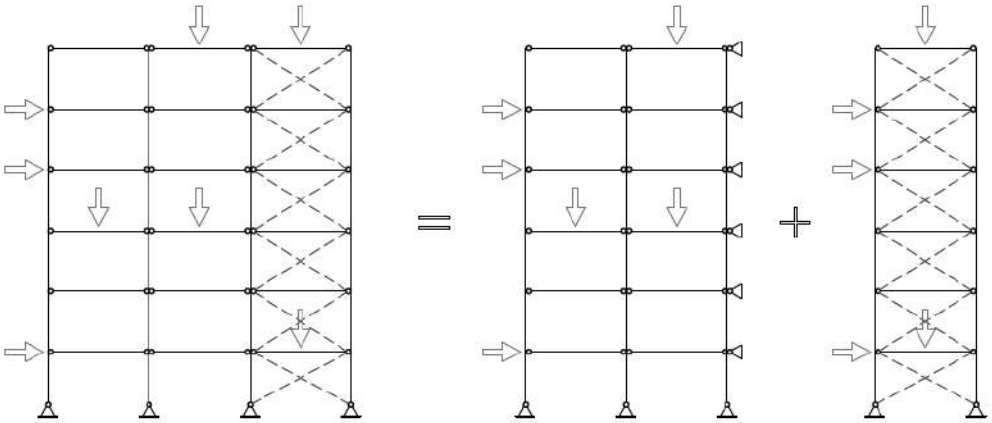
- Horizontal stability is achieved by an additional bracing system.
- Structures with continuous beams.

The comparison of the bending moment diagram in these basic frames can help us clarify the differences between hinged and stiff nodes.

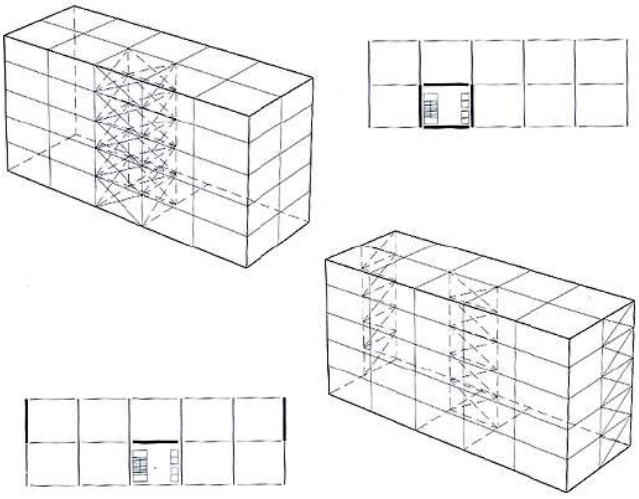


- Stiff frame → smaller deformations, bigger reactions.
- Two-hinged frame → smaller reactions.
- Three-hinged frame → biggest deformations, smallest reactions. Big bending moments in stiff nodes. There's no connection between cross-section inertia and strengths.

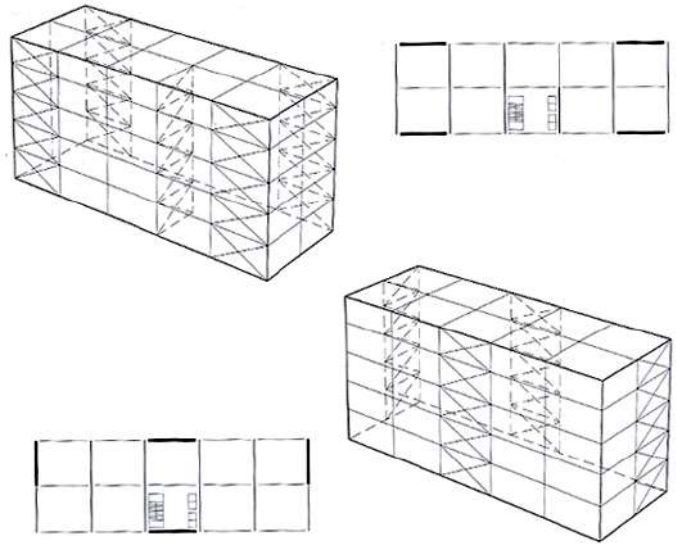
In the case of triangulations, we can simplify significantly the analysis of the frame by dividing it. We separate the bracing from the rest of the frame, and we analyse each part with their own load state.



The position of the different bracing inside the structure is also very important. Depending on this distribution we will get more homogeneous deformations. These are some distribution possibilities and their main characteristics.



The solution that works best is locating panSLS symmetrically (lower case) This way we avoid the torsion in the floor plan (upper case).

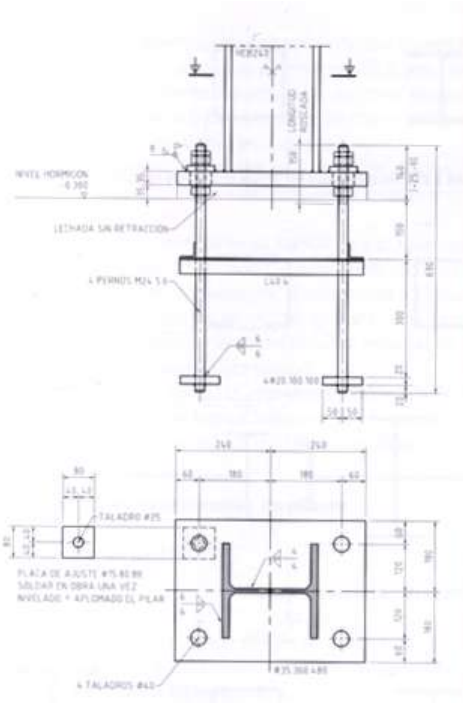


However the location of these planes can also be critical for the structure in case of thermal expansion. In the upper case the bracing panSLS don't allow the thermal expansion of the structure. Therefore we have to take into consideration the new stresses that this produces. In the lower case, the location of the panSLS allows for expansions.

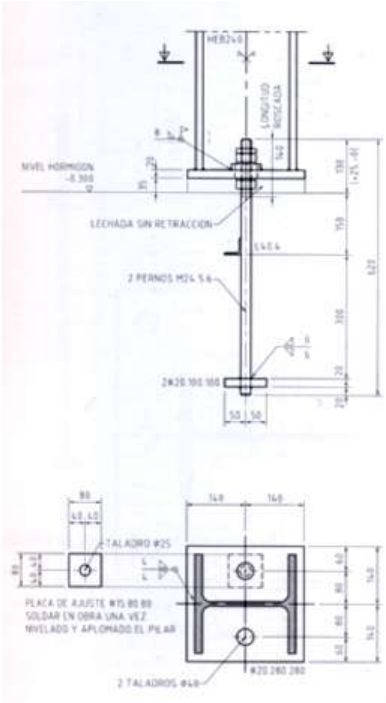
2.1.3. Types of joints

Before deciding if we use a stiff or hinged joint we need to take into account the difficulties to build this joint on site. We have to be sure that on the reality is going to work in the same way as in the project. These are some of the most common joints.

- Base plates



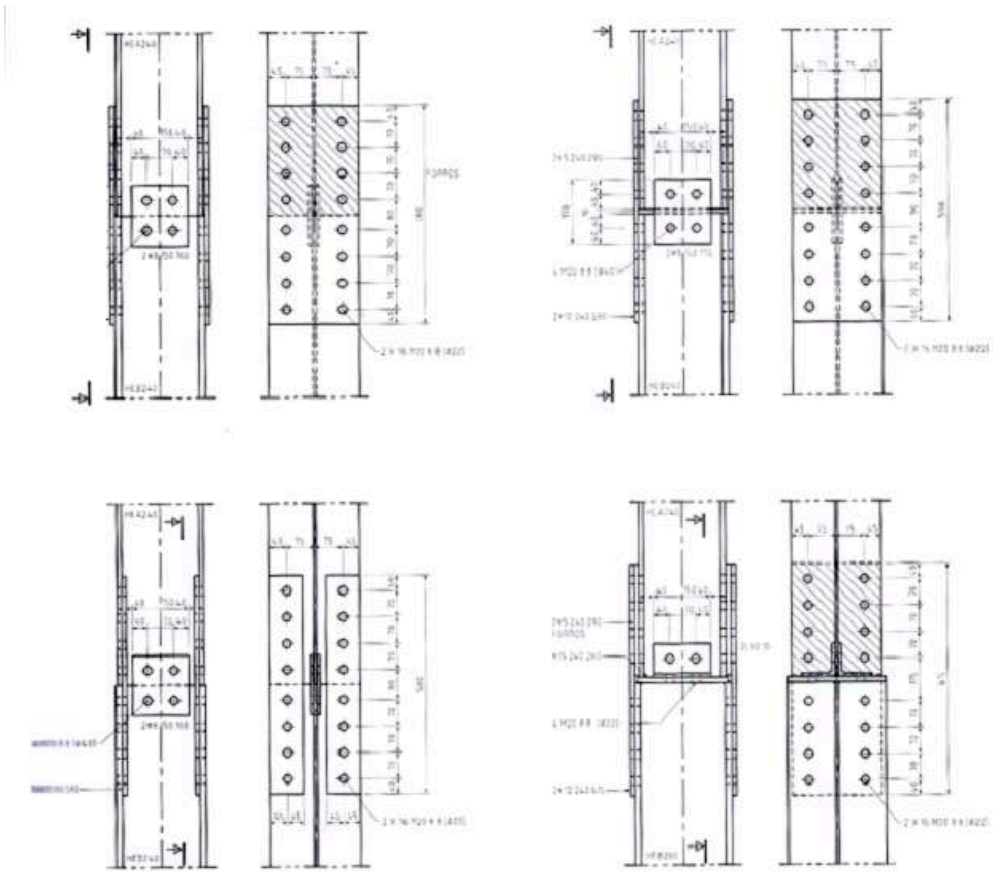
Stiff joint



Hinged joint

In the stiff joint (left side) there are four bolt connectors in two vertical planes parallel to the strong axis of inertia. It allows the union to create a couple of forces able to transmit the bending moment to the foundations. In the hinged joint (right side) there are just two bolt connectors located along the strong axis, making it unable to transmit the moment.

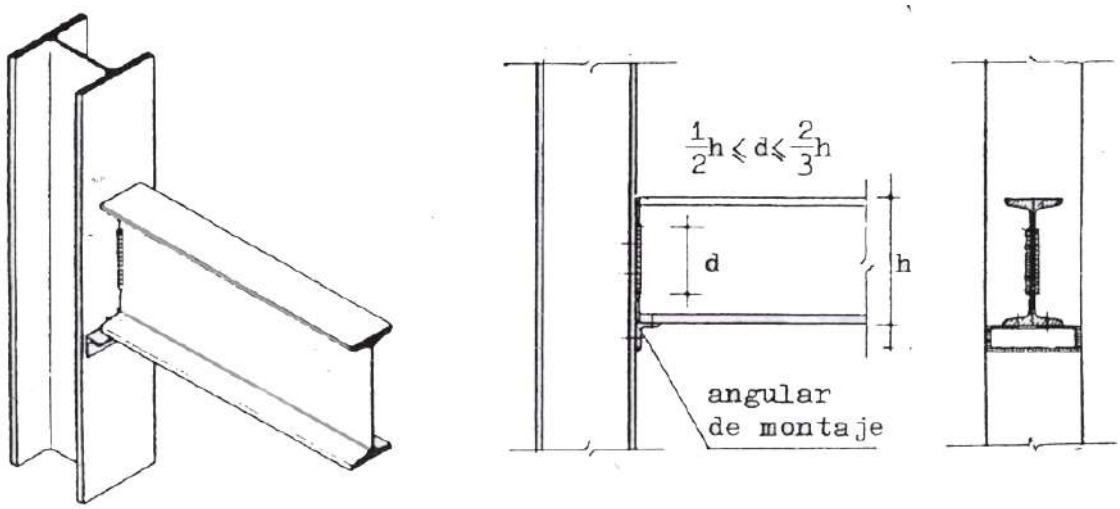
- Column's rigid continuity



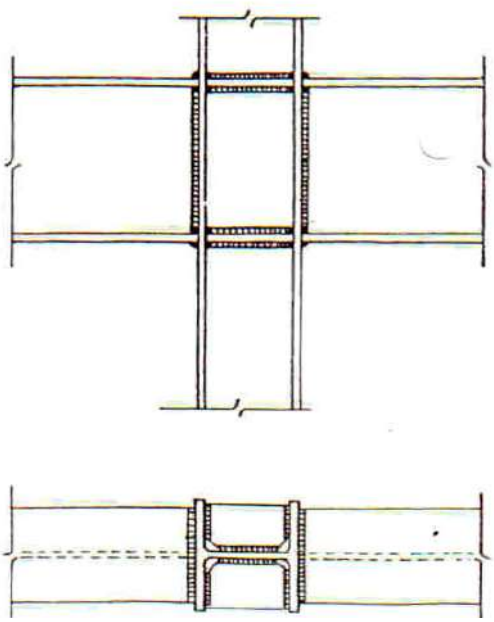
In any type of column is essential to guarantee its rigid continuity, otherwise the bending moment would not be transmitted. This can be a problem when we joint columns with different dimensions (a common situation in multi-story building).

- Beam-column joint

Basically, we can say that in a beam-column joint, the flanges transmit the bending moment and the web transmits the shear strength. Therefore, the joint is hinged when only the web of the beam is connected to the column; and a stiff joint when we connect the flanges of the beam as well.

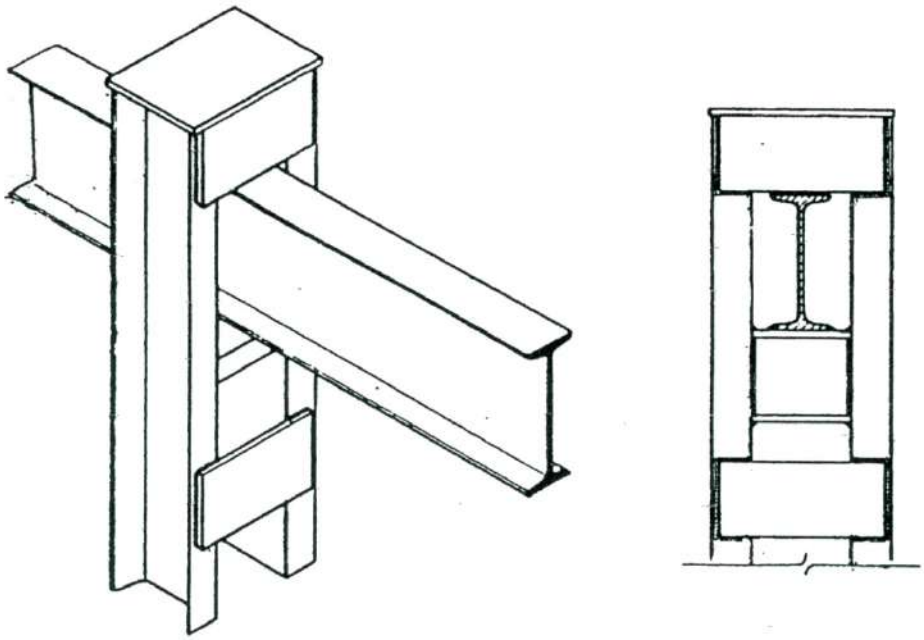


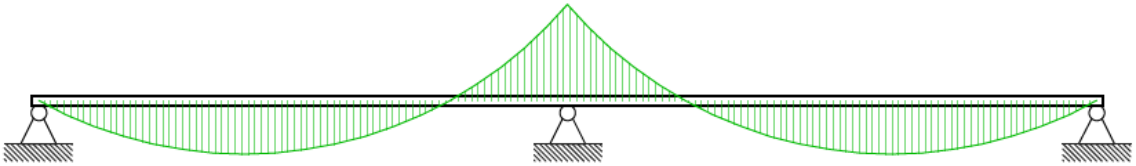
Hinged joint



Stiff joint

Finally, when the beam transmits the moment to the next part of the beam, but doesn't transmit bending moment to the column, we consider that we have a continuous beam:





2.2. FLOOR SLAB TIPOLOGIES

During the designing process we have to decide if the floor slab is going to transmit the loads in just one way (traditional structural frames) or in two directions (like a grid).

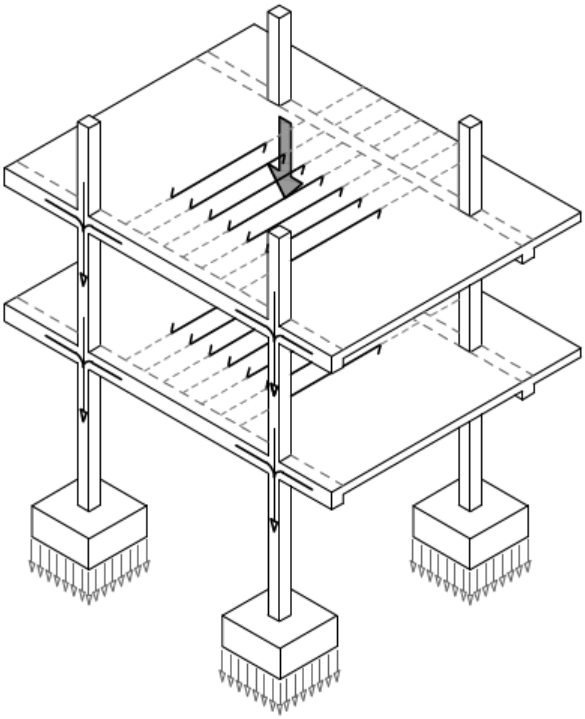
In this section we will see the most common typologies of floors, depending on the way they transmit loads. From all those types of floors we will have to choose the one that best fits our case. These are the main characteristics of each floor.

- Proportional relation between the depth and the span (acceptable span and cantilever)
- Own weight
- Economic cost

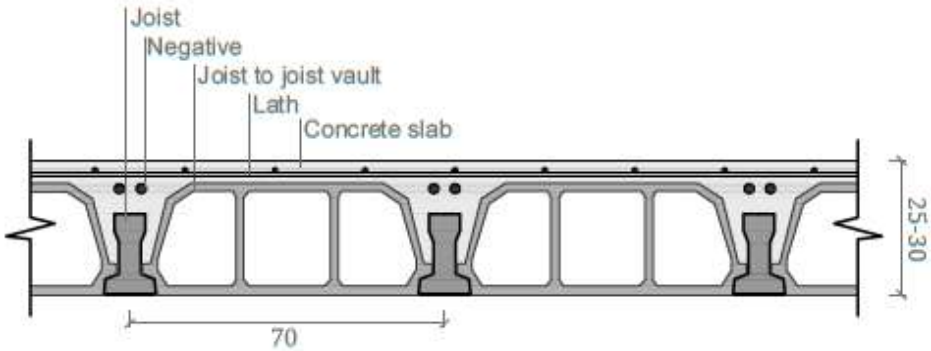
2.2.1. One way floors.

These are the most common floors that bear the load in one direction:

- Rib floor
- Steel rib floor
- On site cast rib floor
- Steel deck floor system
- Precast plank floor system
- Hollow core plank slab
- On site cast one-way lightened slab

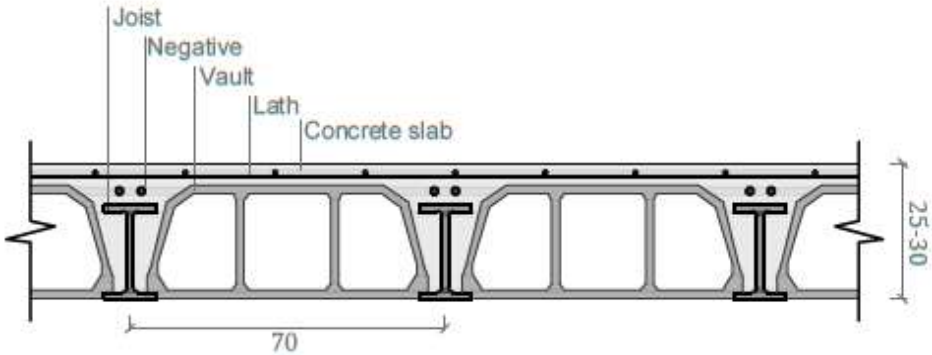


-Rib floor



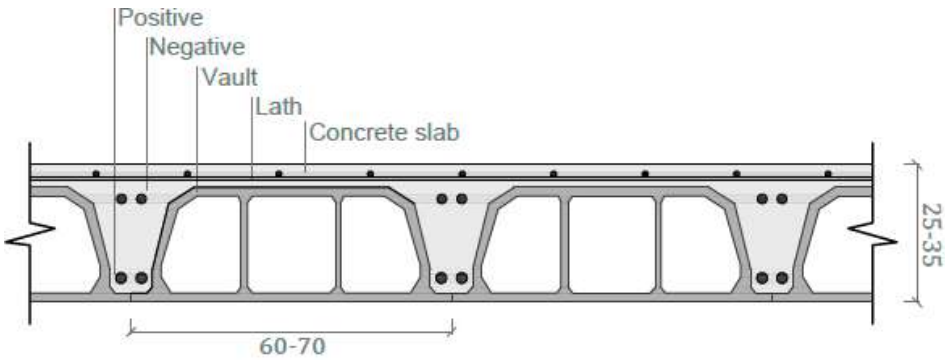
CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	0.50 – 0.80	< 6.50	0.20 – 0.35	2.75 – 4.75	30 - 50
Usual values (recommended)	0.70	4.50 – 6.00	0.25 – 0.30	3.25 – 3.75	35 - 45
Most common and economic floor type. Doubling the joists we can improve the behaviour in 10-20 %. The cantilever can be 6 or 8 times the depth of the floor. If the joists are not self-resistant, formwork is needed.			$H = \frac{L}{(18-22)}$	$H = P \cdot (11-14)$	$C = H \cdot (140-160)$

-Steel rib floor



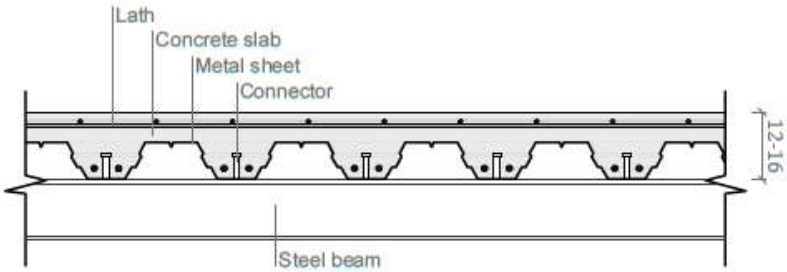
CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	0.50 – 0.80	< 7.50	0.20 – 0.35	2.75 – 4.75	35 - 60
Usual values (recommended)	0.70	4.50 – 6.50	0.25 – 0.30	3.25 – 3.75	40 - 50
This type of floor is not usually used because is not very resistant against fire. Doubling the joists we can improve the behaviour in 10-20 %. The cantilever can be 8 or 10 times the depth of the floor. There is no need of formwork.			$H = \frac{L}{(20-24)}$	$H = P \cdot (11-14)$	$C = H \cdot (150-170$

-On site cast rib floor



CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	0.50 – 0.80	< 10.00	0.20 – 0.40	2.50 – 4.00	50 - 90
Usual values (recommended)	0.60 - 0.70	6.00 – 9.00	0.25 – 0.35	3.00 – 3.50	60 - 70
Instead of using pre-cast joists we will cast them on site. This type of floor is used when we have a strange geometry. The cantilever can be 8 or 10 times the depth of the floor. We will always need formwork.			$H = \frac{L}{(23-27)}$	$H = P \cdot (10-12)$	$C = H \cdot (200-250)$

-Steel deck floor system



CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	-	< 5.00	0.10 – 0.20	1.75 – 3.50	40 - 70
Usual values (recommended)	-	3.00 – 4.00	0.12 – 0.16	2.25 – 3.75	45 - 55
Is the lightest floor slab. It has an easy and effective assembly. It can only be used with steel beams and joists. The continuation of the floor is achieved by the concrete slab on top of the steel sheet. The cantilevers vary between 0,5 – 1 meter.			$H = \frac{L}{(23-27)}$	$H = P \cdot (16-18)$	$C = H \cdot (300-40$

TABLAS DE RESISTENCIA

Apuntalamiento en el centro de vano

Canto del Forjado (mm)

Luces (m) | Sobrecarga de Uso (kp/m²)

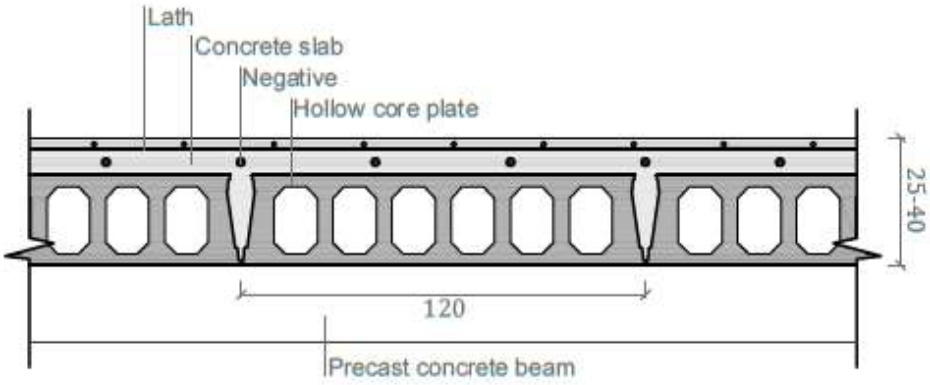
ESPESOR 0,75 mm

	12	14	16	18	20	21
2,0	1107	1382	1660	1939	2217	2352
2,2	905	1129	1356	1584	1812	1921
2,4	750	934	1123	1312	1500	1589
2,6	628	782	939	1097	1255	1329
2,8	530	659	792	925	1058	1120
3,0	450	559	672	785	898	950
3,2	384	477	573	670	766	809
3,4	329	408	490	573	655	691
3,6	262	349	420	491	561	592
3,8		299	360	421	481	506
4,0		256	308	360	412	433
4,2			263	308	352	369
4,4			224	262	300	313
4,6				221	253	264
4,8					212	221
5,0						

	12	14	16	18	20	21
2,0	1313	1669	1990	2232	2373	2437
2,2	1134	1415	1700	1986	2125	2180
2,4	946	1180	1417	1655	1893	1967
2,6	798	994	1195	1396	1596	1691
2,8	679	846	1017	1187	1358	1438
3,0	583	725	871	1018	1164	1232
3,2	503	625	751	877	1003	1061
3,4	436	541	650	760	869	919
3,6	379	470	565	660	755	798
3,8	331	410	492	575	658	695
4,0	289	357	430	502	574	606
4,2	253	312	375	439	502	528
4,4	221	272	328	383	438	461
4,6		237	286	334	382	401
4,8		206	248	290	332	348
5,0			215	252	288	301

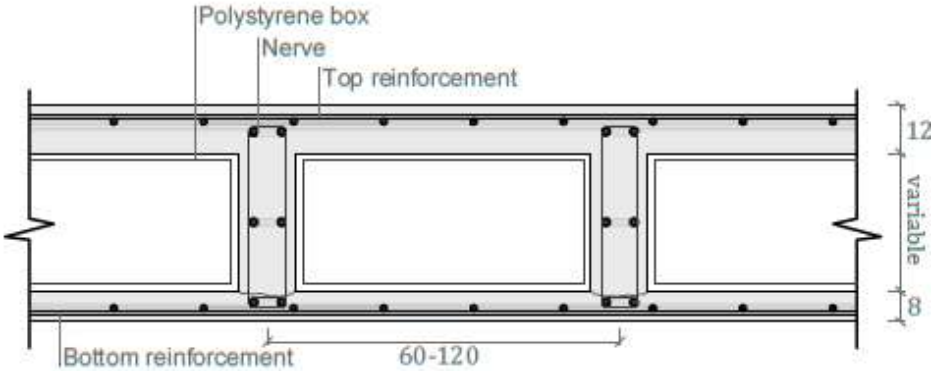
	12	14	16	18	20	21
2,0	1359	1727	2059	2310	2457	2523
2,2	1191	1487	1787	2072	2201	2259
2,4	993	1239	1489	1739	1988	2039
2,6	837	1044	1255	1465	1676	1776
2,8	713	888	1067	1246	1425	1510
3,0	611	761	914	1068	1222	1293
3,2	527	656	788	921	1053	1114
3,4	457	568	683	798	912	965
3,6	398	494	594	693	793	838
3,8	347	430	517	604	691	730
4,0	304	376	452	528	604	637
4,2	266	328	395	461	528	556
4,4	233	287	345	403	461	485
4,6	204	250	301	352	403	423
4,8		218	262	307	351	368
5,0			228	266	305	319

-Hollow core plank slab



CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	1.20	< 18.00	0.20 – 0.60	3.00 – 8.00	60 - 120
Usual values (recommended)	1.20	7.00 – 13.00	0.25 – 0.40	3.75 – 5.50	70 - 90
Is the floor slab with the most resistance against flection. We can use concrete or steel beams as a support. The planks don´t allow negative moments, therefore the cantilevers must be small.			$H = \frac{L}{(30-35)}$	$H = P \cdot (13-16)$	$C = H \cdot (250-300)$

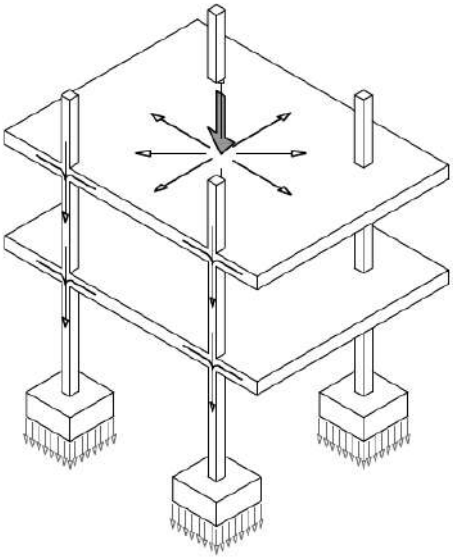
-On site cast one-way lightened slab

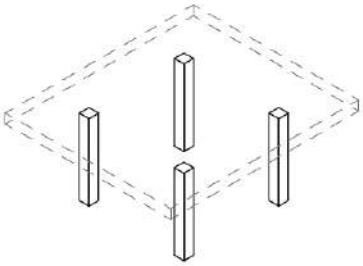


CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	1.20	< 18.00	0.20 – 0.60	3.00 – 8.00	60 - 120
Usual values (recommended)	1.20	7.00 – 13.00	0.25 – 0.40	3.75 – 5.50	70 - 90
This floor slab has the most resistance, so we can cover long spans and big cantilevers (6 or 8 times the depth of the slab). Therefore, because the inner part of the floor is empty, we should use a thick depth (0,5 m) to develop the ability of transmitting moments. We have to pour the concrete twice, so this increases the costs.			$H = \frac{L}{(30-35)}$	$H = P \cdot (13-16)$	$C = H \cdot (250-300)$

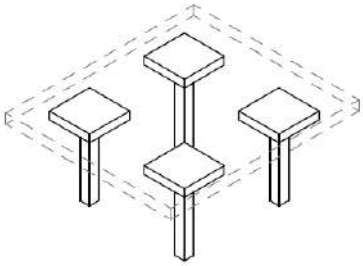
2.2.2. Two way slabs

- Concrete slabs
- Waffle slab
- On site cast two-way lightened slab
- Post-stressed concrete slab

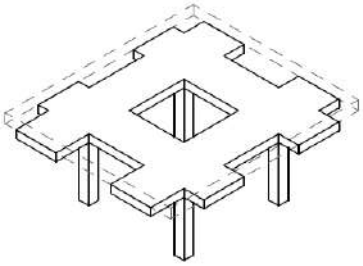




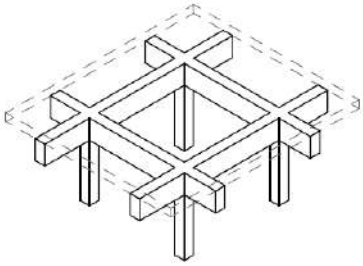
Concrete slab



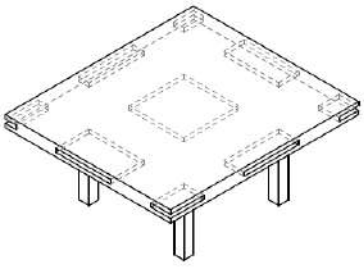
Slab with abacus



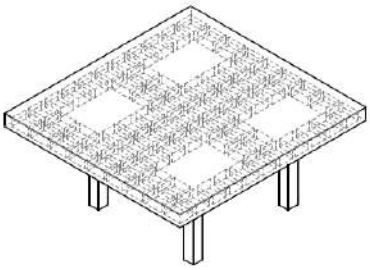
Slab with flat beams



Slab with beams on edge

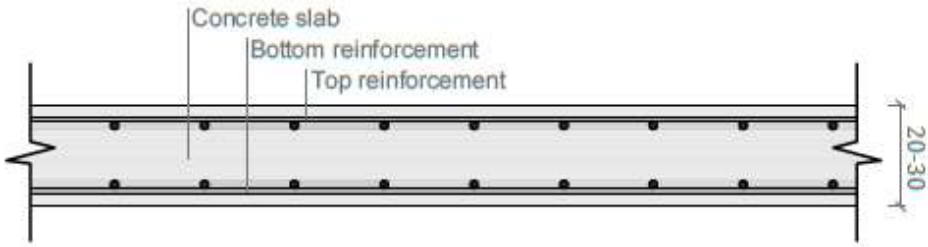


Lightened slab



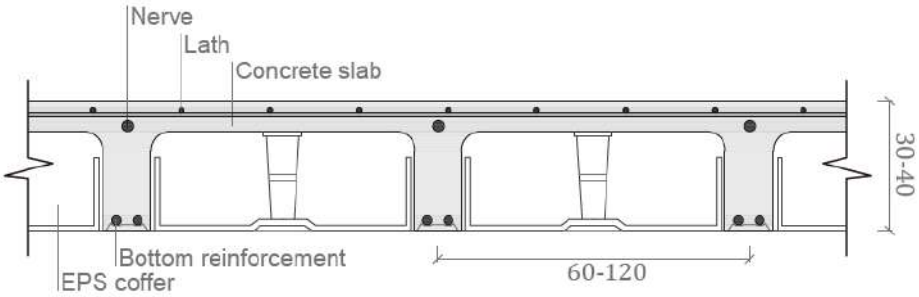
Waffle slab

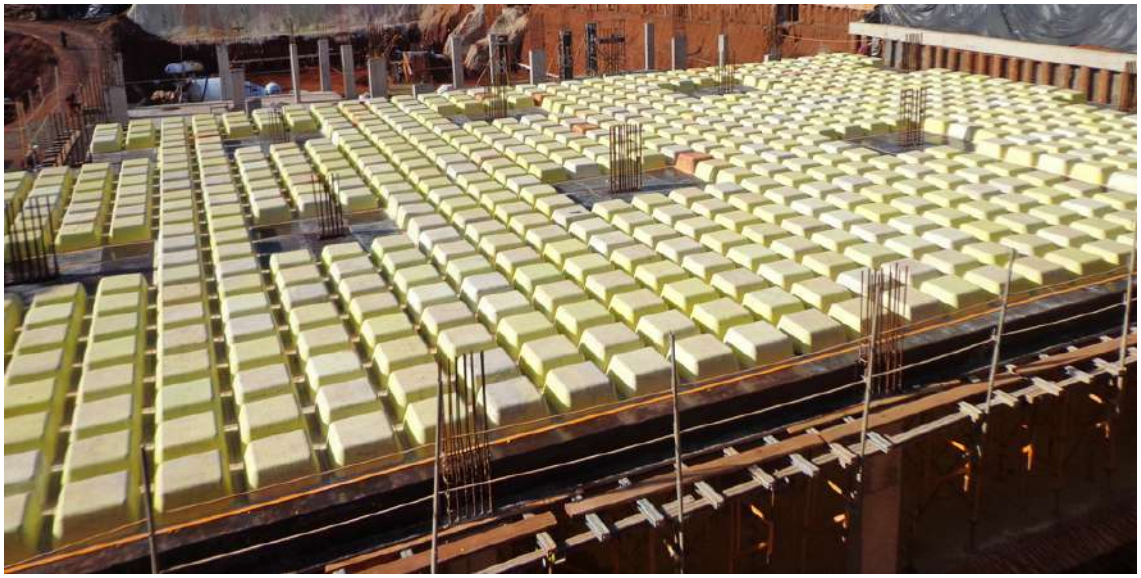
-Concrete slab



CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	-	< 10.00	0.15 – 0.40	2.25 – 10.00	30 - 100
Usual values (recommended)	-	3.00 – 8.00	0.20 – 0.30	5.00 – 7.50	50 - 80
Due to its great own weight, we only use it for small or medium spans. It is mainly used when the geometry of the floor or the location of the columns is irregular.			$H = \frac{L}{(24-30)}$	$H = P \cdot (25)$	$C = H \cdot (200-250)$

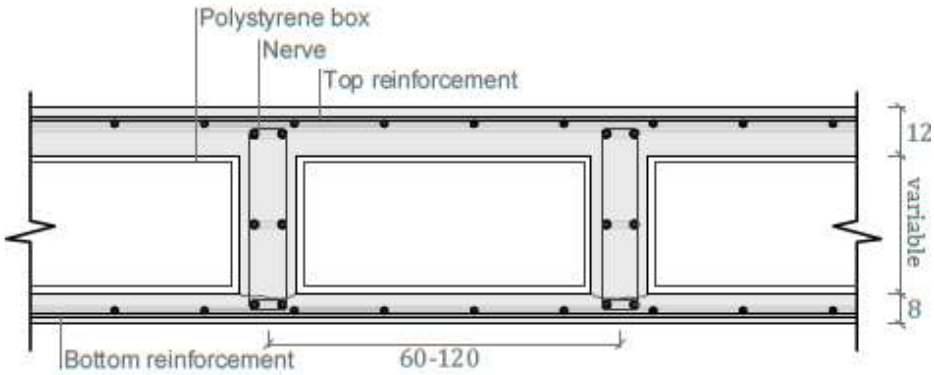
-Waffle slab





CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	0.60 – 1.00	< 14.00	0.25 – 0.60	3.50 – 6.00	60 - 120
Usual values (recommended)	0.70 – 0.80	6.00 – 12.00	0.30 – 0.40	4.00 – 5.00	70 - 90
The waffle slab is used for medium length spans. The formwork's location must be planned before construction. We usually don´t use beams, the forces are directly transferred to the columns.			$H = \frac{L}{(22-28)}$	$H = P \cdot (13-14)$	$C = H \cdot (200-250)$

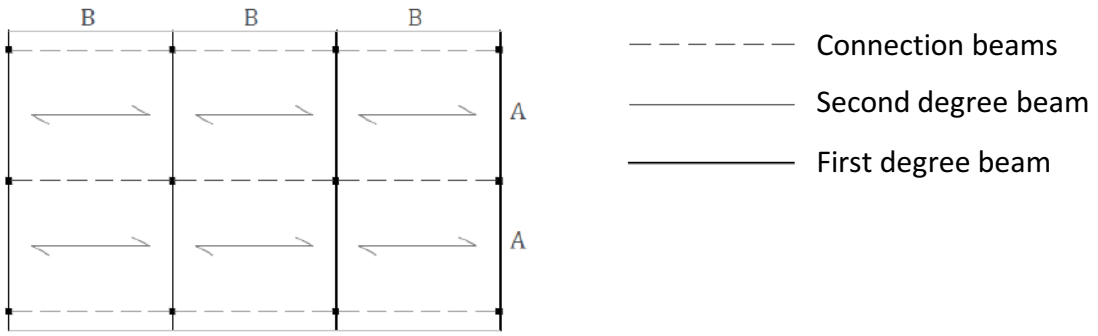
-In situ lightened concrete slab



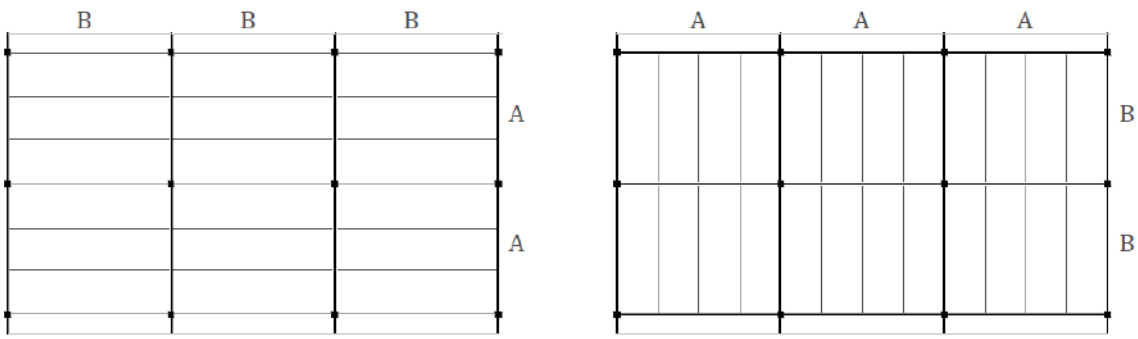
CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	0.50 – 2.00	< 22.00	0.40 – 1.20	5.50 – 16.50	110 - 300
Usual values (recommended)	0.60 – 1.20	12.00 – 18.00	0.50 – 0.80	7.50 – 12.00	140 - 180
This floor slab has the most resistance, so we can cover long spans and big cantilevers (6 or 8 times the depth of the slab). Therefore, because the inner part of the floor is empty, we should use a thick depth (0,5 m) to develop the ability of transmitting moments. We have to pour the concrete twice, so this increases the costs.			$H = \frac{L}{(20-24)}$	$H = P \cdot (13-15)$	$C = H \cdot (250-300)$

2.3. FLOOR SLABS WITH STEEL BEAMS

When using steel structures it is essential to decide if the beam grid is going to have on direction (sustaining the span between frames on their own) or if it is going to create a two direction system.



One way beams

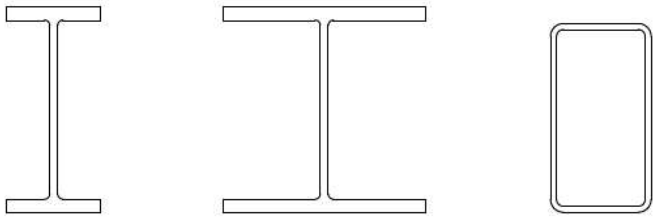


First and second degree beam system

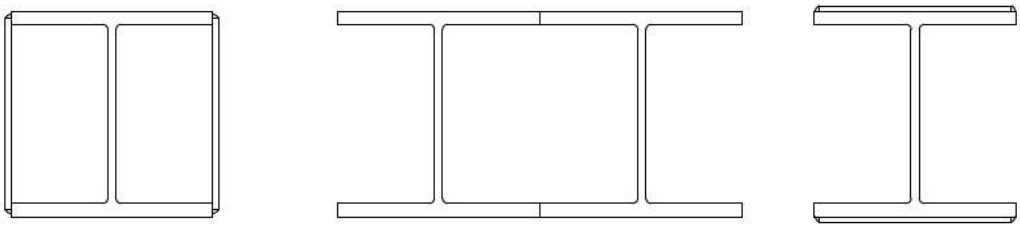
When A gap is bigger than B, the main beam increases its depth while the second degree beam becomes lighter (small span and bearing area). However, when B gap is bigger than A, the depth of main and second degree beams becomes similar. This way we obtain more homogeneous floor slabs but they are also heavier.

- Types of steel beams

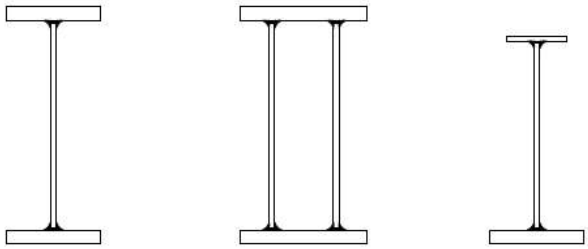
Common profiles



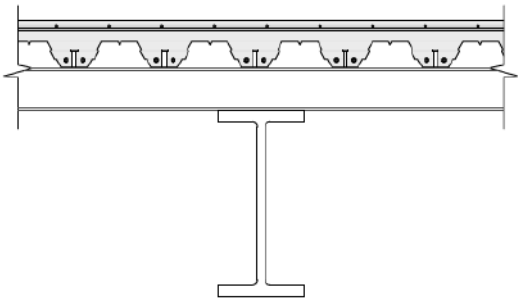
Common profiles with reinforcement

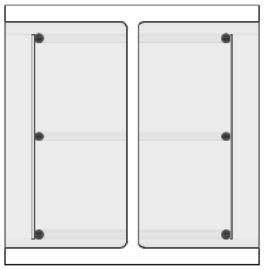


Composed beams



Mixed structures

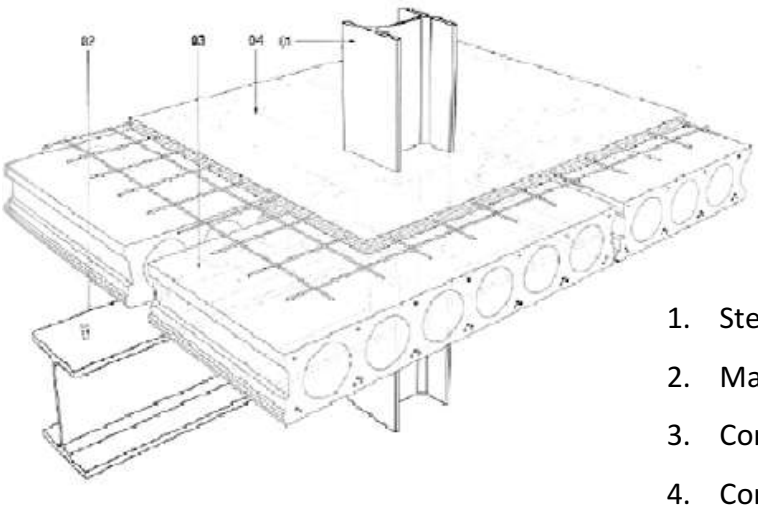




Relation between beam's depth and span:

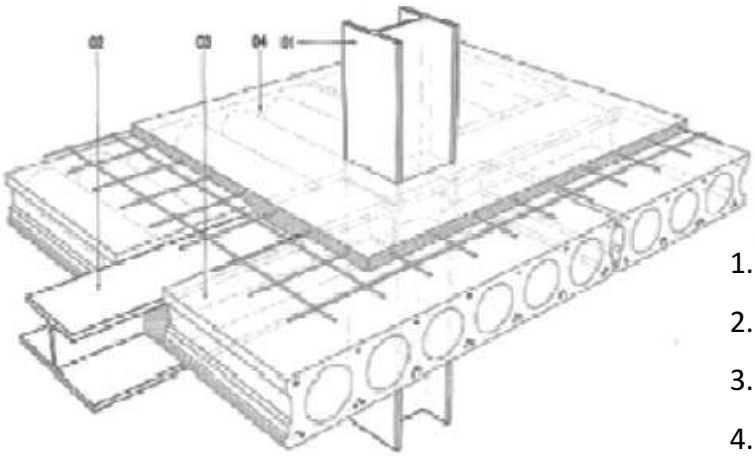
- Main beams with regular sections
  - Span A 4,5 – 9 m
  - Span / depth A / D 15 – 20
- Main beams with mixed sections
  - Span A 6 – 12 m
  - Span / depth A / D 20 – 25
- Main beams with composed section
  - Span A 7,5 – 20 m
  - Span / depth A / D 10 – 30
- Second degree beams with regular sections
  - Span A 0 – 7,5 m
  - Span / depth A / D 15 – 20
- Second degree beams with mixed sections
  - Span A 0 – 10,5 m
  - Span / depth A / D 20 – 25

<u>Beams located underneath the slab</u>



1. Steel column
2. Main steel beam
3. Concrete hollow core plates
4. Concrete slab

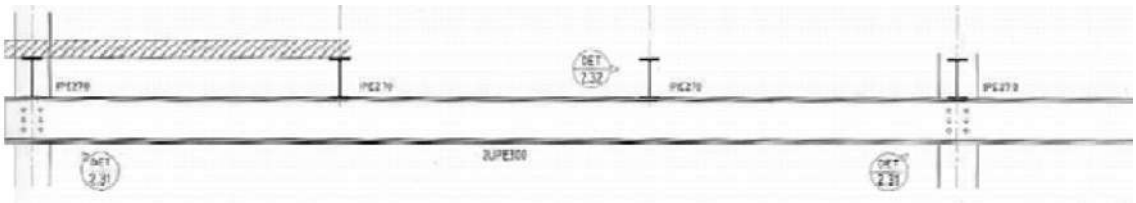
- Beams located inside the slab



1. Steel column
2. Main steel beam
3. Concrete hollow core plates
4. Concrete slab

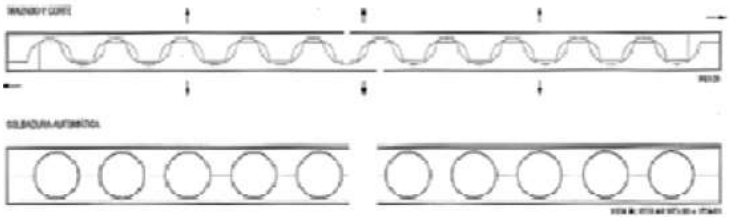
- Slabs with steel continuous beams

  Continious beam floors



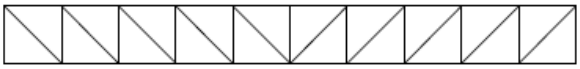
- Primary continious beams
  - Span A 7,5 – 9 m
  - Span / depth A / D 15 – 20
- Secondary continuous beams
  - Span A 7,5 – 12 m
  - Span / depth A / D 20 – 25

- Slabs with Boyd beams



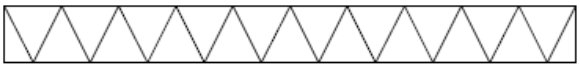
○ Span	A	7,5 – 15 m
○ Span / depth	A / D	15 – 20

- Truss

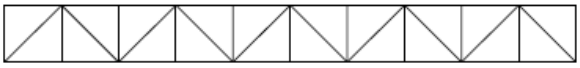


Diagonals under traction

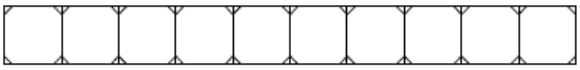
Diagonals under compression



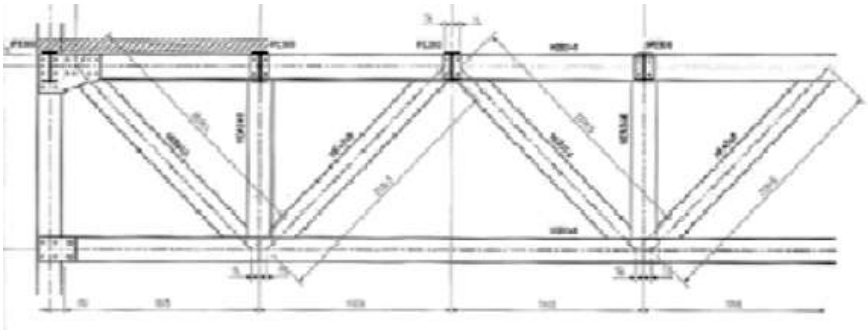
Without vertical bars



Triangulation and vertical bars combination



Vierendeel beam (stiff nodes)



- Primary truss
    - Span A 9 – 30 m
    - Span / depth A / D 10 – 15
- Secondary truss
    - Span A 7,5 – 15 m
    - Span / depth A / D 15 – 20

3. ACTIONS IN CONSTRUCTION. RESEARCH AND CALCULATION (CTE)

3.1. INTRODUCTION

As said at the beginning of the course, to develop the design and the calculation of a building structure we have to follow these four basic steps:

1. Criteria for designing a structure (frame)
2. Loading definition. (CTE-DB-SE-AE)
3. Frame calculation (strengths)
4. Verify the structure

To develop the first step (criteria for designing a structure) we will have to follow the explanation in this chapter.

Continuing with this scheme, in order to define the loading of the structure, we have to follow the procedures defined by the regulations. In this chapter, we will get an overall view of this procedure.

3.2. ACTIONS: CLASSES, CHARACTERISTIC VALUES AND CALCULATION VALUES

As it is mentioned in the **CTE-DB-SE-AE** (Código Técnico de la Edificación-Documento Básico-Seguridad Estructural-Acciones en la Edificación), defines the following load types: Permanent actions, variable actions and accidental actions.

- Permanent actions
  - Own weight
    - Structural elements
    - Façade
    - Partition walls
    - Cladding
  - Ground actions, foundations (DB-SE-Cimientos)

- Variable actions
  - Use overload
  - Wind
  - Snow
  - Thermal actions
- Accidental actions
  - Seismic actions
  - Actions due to fire
  - Collision actions

3.2.1. Permanent actions

Own weight

The CTE gives us some general charts to define the own weight values. However, on most of those technical solutions the architect will be the one defining the specific values.

Anejo C. Prontuario de pesos y coeficientes de rozamiento interno

Tabla C.1 Peso específico aparente de materiales de construcción

Materiales y elementos	Peso específico aparente kN/m³
Materiales de albañilería	
Arenisca	21,0 a 27,0
Basalto	27,0 a 31,0
Calizas compactas, mármoles	28,0
Diorita, gneis	30,0
Granito	27,0 a 30,0
Sienita, diorita, pórfido	28,0
Terracota compacta	21,0 a 27,0
Fábricas	
Bloque hueco de cemento	13,o a 16,0
Bloque hueco de yeso	10,0
Ladrillo cerámico macizo	18,0
Ladrillo cerámico perforado	15,0
Ladrillo cerámico hueco	12,0
Ladrillo silicocalcáreo	20,0
Mampostería con mortero	
de arenisca	24,0
de basalto	27,0
de caliza compacta	26,0
de granito	26,0
Sillería	
de arenisca	26,0
de arenisca o caliza porosas	24,0
de basalto	30,0
de caliza compacta o mármol	28,0
de granito	28,0
Hormigones y morteros	
Hormigón ligero	9,0 a 20,0
Hormigón normal [1]	24,0
Hormigón pesado	> 28,0
Mortero de cemento	19,0 a 23,0
Mortero de yeso	12,0 a 28,0
Mortero de cemento y cal	18,0 a 20,0
Mortero de cal	12,0 a 18,0

Materiales y elementos	Peso específico aparente kN/m³
Madera	
Aserrada, tipos C14 a C40	3,5 a 5,0
Laminada encolada	3,7 a 4,4
Tablero contrachapado	5,0
Tablero cartón gris	8,0
Aglomerado con cemento	12,0
Tablero de fibras	8,0 a 10,0
Tablero ligero	4,0
Metales	
Acero	77,0 a 78,5
Aluminio	27,0
Bronce	83,0 a 85,0
Cobre	87,0 a 89,0
Estaño	74,0
Hierro colado	71,0 a 72,5
Hierro forjado	76,0
Latón	83,0 a 85,0
Plomo	112,0 a 114,0
Zinc	71,0 a 72,0
Plásticos y orgánicos	
Caucho en plancha	17,0
Lámina acrílica	12,0
Linóleo en plancha	12,0
Mástico en plancha	21,0
Poliestireno expandido	0,3
Otros	
Adobe	16,0
Asfalto	24,0
Baldosa cerámica	18,0
Baldosa de gres	19,0
Papel	11,0
Pizarra	29,0
Vidrio	25,0

[1] En hormigón armado con armados usuales o fresco aumenta 1 kN/m³

Tabla C.2 Peso por unidad de superficie de elementos de cobertura

Materiales y elementos	Peso kN/m²	Materiales y elementos	Peso kN/m²
Aislante (lana de vidrio o roca)		Tablero de madera, 25 mm espesor	0,15
por cada 10 mm de espesor	0,02	Tablero de rasilla, una hoja	
Chapas grecadas, canto 80 mm,		una hoja sin revestir	0,40
Acero 0,8 mm espesor	0,12	una hoja más tendido de yeso	0,50
Aluminio, 0 8 mm espesor	0,04	Tejas planas (sin enlistonado)	
Plomo, 1,5 mm espesor	0,18	ligeras (24 kg/pieza)	0,30
Zinc, 1,2 mm espesor	0,10	corrientes (3,0 kg/pieza)	0,40
Cartón embreado, por capa	0,05	pesadas (3,6 kg/pieza)	0,50
Enlistonado	0,05	Tejas curvas (sin enlistonado)	
Hoja de plástico armada, 1,2 mm	0,02	ligeras (1,6 kg/pieza)	0,40
Pizarra, sin enlistonado		corrientes (2,0 kg/pieza)	0,50
solape simple	0,20	pesadas (2,4 kg/pieza)	0,60
solape doble	0,30	Vidriera (incluida la carpintería)	
Placas de fibrocemento, 6 mm espesor	0,18	vidrio normal, 5 mm espesor	0,25
		vidrio armado, 6 mm espesor	0,35

Tabla C.3 Peso por unidad de superficie de elementos de pavimentación

Materiales y elementos	Peso kN/m²	Materiales y elementos	Peso kN/m²
Baldosa hidráulica o cerámica (incluyendo material de agarre)		Linóleo o loseta de goma y mortero	
0,03 m de espesor total	0,50	20 mm de espesor total	0,50
0,05 m de espesor total	0,80	Parque y tarima de 20 mm de espesor	
0,07 m de espesor total	1,10	sobre rastreles	0,40
Corcho aglomerado		Tarima de 20 mm de espesor	
tarima de 20 mm y rastrel	0,40	rastreles recibidos con yeso	0,30
		Terrazo sobre mortero, 50 mm espesor	0,80

Tabla C.4 Peso por unidad de superficie de tabiques

Tabiques (sin revestir)	Peso kN/m²	Revestimientos (por cara)	Peso kN/m²
Rasilla, 30 mm de espesor	0,40	Enfoscado o revoco de cemento	0,20
Ladrillo hueco, 45 mm de espesor	0,60	Revoco de cal, estuco	0,15
de 90 mm de espesor	1,00	Guarnecido y enlucido de yeso	0,15

Tabla C.5 Peso propio de elementos constructivos

Elemento		Peso
Forjados		kN / m²
	Chapa grecada con capa de hormigón; grueso total < 0,12 m	2
	Forjado unidireccional, luces de hasta 5 m; grueso total < 0,28 m	3
	Forjado uni o bidireccional; grueso total < 0,30 m	4
	Forjado bidireccional, grueso total < 0,35 m	5
	Losa maciza de hormigón, grueso total 0,20 m	5
Cerramientos y particiones (para una altura libre del orden de 3,0 m) incluso enlucido		kN / m
	Tablero o tabique simple; grueso total< 0,09 m	3
	Tabicón u hoja simple de albañilería; grueso total < 0,14 m	5
	Hoja de albañilería exterior y tabique interior; grueso total < 0,25 m	7
Solados (incluyendo material de agarre)		kN / m²
	Lámina pegada o moqueta; grueso total < 0,03 m	0,5
	Pavimento de madera, cerámico o hidráulico sobre plastón; grueso total < 0,08 m	1,0
	Placas de piedra, o peldañeado; grueso total < 0,15 m	1,5
Cubierta, sobre forjado (peso en proyección horizontal)		kN / m²
	Faldones de chapa, tablero o paneles ligeros	1,0
	Faldones de placas, teja o pizarra	2,0
	Faldones de teja sobre tableros y tabiques palomeros	3,0
	Cubierta plana, recrecido, con impermeabilización vista protegida	1.5
	Cubierta plana, a la catalana o invertida con acabado de grava	2,5
Rellenos		kN / m³
	Agua en aljibes o piscinas	10
	Terreno , como en jardineras, incluyendo material de drenaje (1)	20

(1) El peso total debe tener en cuenta la posible desviación de grueso respecto a lo indicado en planos.

Ground actions

With the “DB-SE-cimientos” document we will identify and calculate the actions the ground creates in our building structure.

3.2.2. Variable Actions

Use overload

According to the use of our building we will have a specific overload value. In the case that we have different uses along our building, we will have to combine different values according to each use. Uniform loads will be homogenously distributed on the area they refer to. Punctual loads, on the other hand, will fall upon one exact point.

Tabla 3.1. Valores característicos de las sobrecargas de uso

Categoría de uso		Subcategorías de uso		Carga uniforme [kN/m^2]	Carga concentrada [kN]
A	Zonas residenciales	A1	Viviendas y zonas de habitaciones en, hospitales y hoteles	2	2
		A2	Trasteros	3	2
B	Zonas administrativas			2	2
C	Zonas de acceso al público (con la excepción de las superficies pertenecientes a las categorías A, B, y D)	C1	Zonas con mesas y sillas	3	4
		C2	Zonas con asientos fijos	4	4
		C3	Zonas sin obstáculos que impidan el libre movimiento de las personas como vestíbulos de edificios públicos, administrativos, hoteles; salas de exposición en museos; etc.	5	4
		C4	Zonas destinadas a gimnasio u actividades físicas	5	7
		C5	Zonas de aglomeración (salas de conciertos, estadios, etc)	5	4
D	Zonas comerciales	D1	Locales comerciales	5	4
		D2	Supermercados, hipermercados o grandes superficies	5	7
E	Zonas de tráfico y de aparcamiento para vehículos ligeros (peso total < 30 kN)			2	20 (1)
F	Cubiertas transitables accesibles sólo privadamente (2)			1	2
G	Cubiertas accesibles únicamente para conservación (3)	G1(7)	Cubiertas con inclinación inferior a 20º	1(4) (6)	2
			Cubiertas ligeras sobre correas (sin forjado) (5)	0,4(4)	1
		G2	Cubiertas con inclinación superior a 40º	0	2

In the case of bigger buildings, we will have a reduction upon the use overload value:

Tabla 3.2. Coeficiente de reducción de sobrecargas

Elementos verticales			Elementos horizontales			
Número de plantas del mismo uso			Superficie tributaria (m^2)			
1 ó 2	3 ó 4	5 ó más	16	25	50	100
1,0	0,9	0,8	1,0	0,9	0,8	0,7

Wind

$$q_e = q_b \cdot C_e \cdot C_p$$

q_b Wind dynamic pressure. In a simplify way we can use $0{,}5\ kN/m^2$ if our building is in Spain. Depending on our geographical location we can take more specific values in the D annex of the DB-SE-AE.

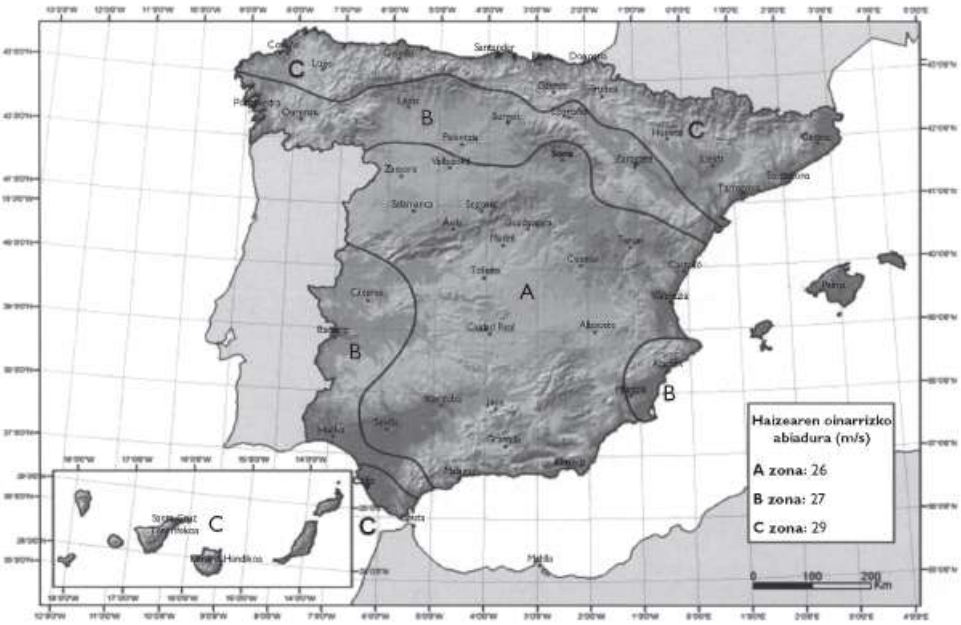
C_e Exposure coefficient. It depends on the height of the point and the roughness degree of the location we are analysing. For buildings located inside the city and with less than 8 floors we can use $C_e = 2{,}0$.

C_p Pressure or wind coefficient. It depends on the surface form and orientation against the wind.

- Dynamic pressure (q_b)

$$q_b = 0{,}5 \cdot \delta \cdot {v_b}^2$$

2.5 irudia
Zona eolikoak



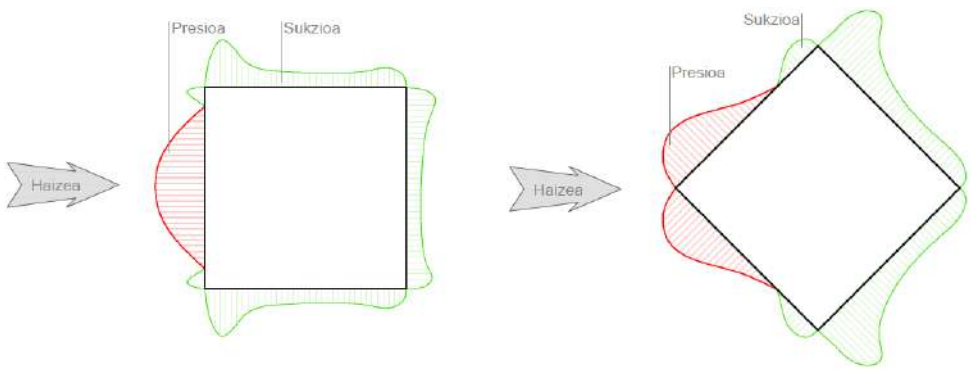
- Exposure coefficient (C_e)

Tabla 3.4. Valores del coeficiente de exposición c_e

	Grado de aspereza del entorno	Altura del punto considerado (m)							
		3	6	9	12	15	18	24	30
I	Borde del mar o de un lago, con una superficie de agua en la dirección del viento de al menos 5 km de longitud	2,4	2,7	3,0	3,1	3,3	3,4	3,5	3,7
II	Terreno rural llano sin obstáculos ni arbolado de importancia	2,1	2,5	2,7	2,9	3,0	3,1	3,3	3,5
III	Zona rural accidentada o llana con algunos obstáculos aislados, como árboles o construcciones pequeñas	1,6	2,0	2,3	2,5	2,6	2,7	2,9	3,1
IV	Zona urbana en general, industrial o forestal	1,3	1,4	1,7	1,9	2,1	2,2	2,4	2,6
V	Centro de negocio de grandes ciudades, con profusión de edificios en altura	1,2	1,2	1,2	1,4	1,5	1,6	1,9	2,0

- Wind coefficient (C_p, C_s)

Due to the wind´s impact we will suffer pressures and suctions on our building's façades. Surfaces exposed to the wind will work under pressure (positive coefficients), however, the rest of the façade surfaces will work under suction (negative coefficients).



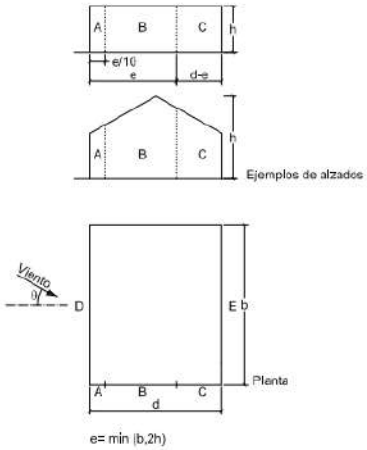
On a conventional residential building with many floors, small windows and inner partitions we can use the following coefficients.

Tabla 3.5. Coeficiente eólico en edificios de pisos

	Esbeltez en el plano paralelo al viento					
	< 0,25	0,50	0,75	1,00	1,25	≥ 5,00
Coeficiente eólico de presión, c_p	0,7	0,7	0,8	0,8	0,8	0,8
Coeficiente eólico de succión, c_s	-0,3	-0,4	-0,4	-0,5	-0,6	-0,7

If the building we are designing doesn´t meet these requirements, we will have to find our own wind coefficients in the D annex (DB-SE-AE). This annex gathers different situations so we will have to find the one that best suits our case.

Tabla D.3 Paramentos verticales



A (m^2)	h/d	Zona (según figura), $-45° < \theta < 45°$ A	B	C	D	E
≥ 10	5	-1,2	-0,8	-0,5	0,8	-0,7
	1	"	"	"	"	-0,5
	≤ 0,25	"	"		0,7	-0,3
5	5	-1,3	-0,9	-0,5	0,9	-0,7
	1	"	"	"	"	-0,5
	≤ 0,25	"	"	"	0,8	-0,3
2	5	-1,3	-1,0	-0,5	0,9	-0,7
	1	"	"	"	"	-0,5
	≤ 0,25	"	"	"	0,7	-0,3
≤ 1	5	-1,4	-1,1	-0,5	1,0	-0,7
	1	"	"	"	"	-0,5
	≤ 0,25	"	"	"	"	-0,3

Moreover, the impact of the wind will create some airstreams inside our building, therefore, we will also suffer from wind pressure and suction inside it. Even though these values will be higher in an empty building, residential ones will also have to take this into account.

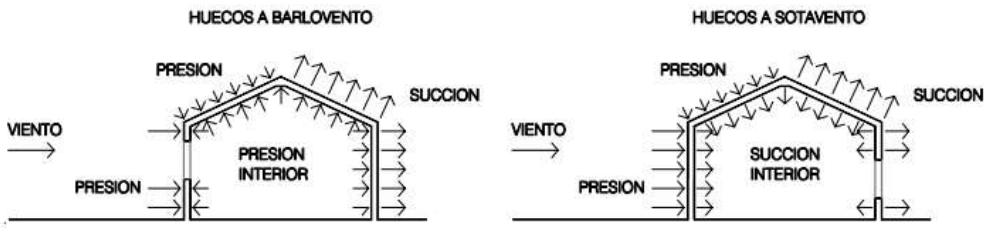


Fig. 3.1 Presiones ejercidas por el viento en una construcción diáfana

Snow

$$q_n = \mu \cdot S_k$$

$$\mu = Coefficient\ according\ to\ the\ shape\ of\ the\ deck$$

$$S_k = Snow\ overload\ value$$

If the building is located in the capital city of a province or big municipality we can take the snow overload value S_k from the following chart:

Tabla 3.8 Sobrecarga de nieve en capitales de provincia y ciudades autónomas

Capital	Altitud m	s_k kN/m^2	Capital	Altitud m	s_k kN/m^2	Capital	Altitud m	s_k kN/m^2
Albacete	690	0,6	Guadalajara	680	0,6	Pontevedra	0	0,3
Alicante / *Alacant*	0	0,2	Huelva	0	0,2	Salamanca	780	0,5
Almería	0	0,2	Huesca	470	0,7	SanSebas-	0	0,3
Ávila	1.130	1,0	Jaén	570	0,4	tián/*Donostia*	0	0,3
Badajoz	180	0,2	León	820	1,2	Santander	1.000	0,7
Barcelona	0	0,4	Lérida / *Lleida*	150	0,5	Segovia	10	0,2
Bilbao / *Bilbo*	0	0,3	Logroño	380	0,6	Sevilla	1.090	0,9
Burgos	860	0,6	Lugo	470	0,7	Soria	0	0,4
Cáceres	440	0,4	Madrid	660	0,6	Tarragona	0	0,2
Cádiz	0	0,2	Málaga	0	0,2	Tenerife	950	0,9
Castellón	0	0,2	Murcia	40	0,2	Teruel	550	0,5
Ciudad Real	640	0,6	Orense / *Ourense*	130	0,4	Toledo	0	0,2
Córdoba	100	0,2	Oviedo	230	0,5	Valencia/*València*	690	0,4
Coruña / *A Coruña*	0	0,3	Palencia	740	0,4	Valladolid	520	0,7
Cuenca	1.010	1,0	Palma de Mallorca	0	0,2	Vitoria / *Gasteiz*	650	0,4
Gerona / *Girona*	70	0,4	Palmas, Las	0	0,2	Zamora	210	0,5
Granada	690	0,5	Pamplona/*Iruña*	450	0,7	Zaragoza	0	0,2
						Ceuta y Melilla		

Nevertheless, if the construction is located anywhere SLSe, the snow overload value is going to be taken from the E annex, which relates the location and its altitude creating a custom value. For inclined roofs we will have to reduce this value using a specific equation.

Form coefficient is 1 in flat decks, over than 60º of inclination the coefficient would be 0, and if the shape of the deck tends to accumulate snow, we will have to take it into account.

Thermal actions

To measure the impact of the thermal actions first we have to define which the maximum possible temperature changes are that our building can suffer in its lifetime. This way we can calculate the expansions produced by those temperature changes.

$$\Delta l = \Delta t \cdot l \cdot \alpha$$

$$\alpha = Linear\ dilatation\ coefficient$$

$$\Delta t = Temperature\ change$$

$$l = Bar's\ length$$

Once we have quantified those expansions, we are going to calculate the dimensions of the expansion joints and how many of them we have to use (if because of the special design of the building we cannot use these expansion joints, we will have to take into consideration the inner tensions produced by those thermal actions).

According to the regulations we have to use the following expression in order to measure the thermal changes:

$$\Delta t = (t_{max} - t); (t - t_{min})$$

$$t_{max} = maximum\ temperature$$

$$t_{min} = minimum\ temperature$$

$$t = average\ temperature$$

The regulation accepts and average temperature of 10 degrees for Spain.

DB-SE-AE document explains the effect of thermal actions in construction. The E annex of this document explains the consequences of temperatures changes in buildings. These are the highest and the lowest temperatures we can find in our country:

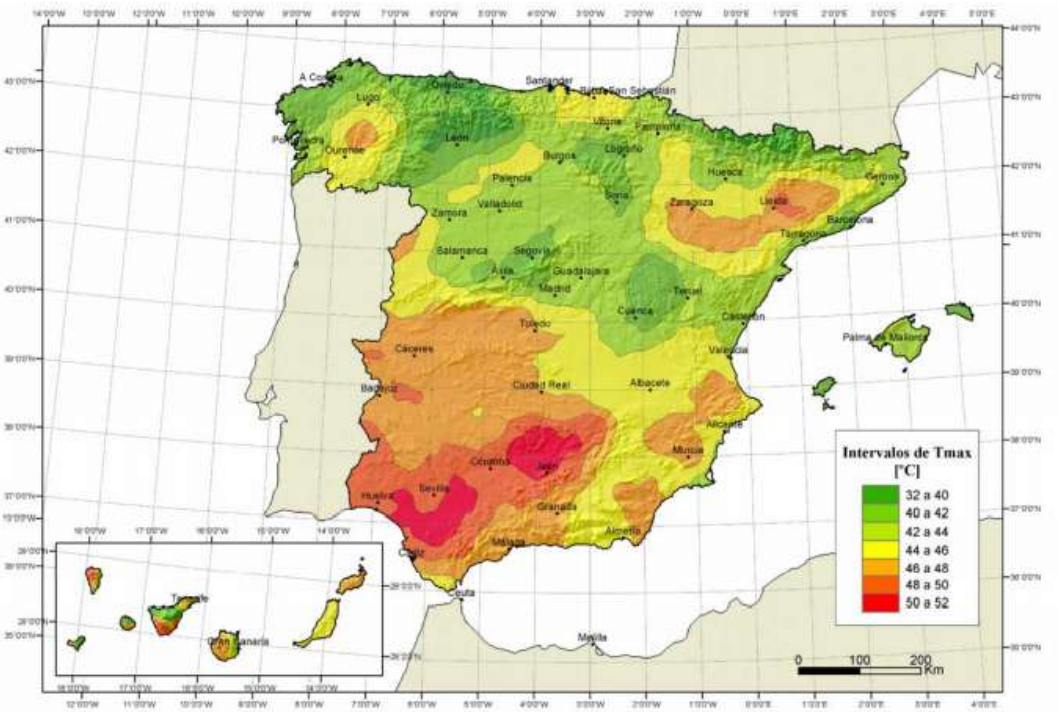


Figura E.1 Isotermas de la temperatura anual máxima del aire (T_{max} en ºC)

Although these are the highest temperatures of the environment, the façades of our building can overcome those temperatures due to the radiant power of the sun. In fact, sun radiation can increase the temperature of some surface in 42 degrees.

Tabla 3.7 Incremento de temperatura debido a la radiación solar

Orientación de la superficie	Color de la superficie		
	Muy claro	Claro	Oscuro
Norte y Este	0 ºC	2 ºC	4 ºC
Sur y Oeste	18 ºC	30 ºC	42 ºC

The minimum temperature will depend on the geographic location in the map under, and the altitude of the building.

Tabla E.1 Temperatura mínima del aire exterior (ºC)

Altitud (m)	**Zona de clima invernal**, (según figura E.2)						
	1	**2**	**3**	**4**	**5**	**6**	**7**
0	-7	-11	-11	-6	-5	-6	6
200	-10	-13	-12	-8	-8	-8	5
400	-12	-15	-14	-10	-11	-9	3
600	-15	-16	-15	-12	-14	-11	2
800	-18	-18	-17	-14	-17	-13	0
1.000	-20	-20	-19	-16	-20	-14	-2
1.200	-23	-21	-20	-18	-23	-16	-3
1.400	-26	-23	-22	-20	-26	-17	-5
1.600	-28	-25	-23	-22	-29	-19	-7
1.800	-31	-26	-25	-24	-32	-21	-8
2.000	-33	-28	-27	-26	-35	-22	-10

Figura E.2 Zonas climáticas de invierno

3.2.3. Accidental Actions

Collision actions

In case of an accidental car collision against our building, the CTE foresees a 5 tons punctual load 60 cm high on a frontal impact; and a 2,5 ton 60 cm high on a side impact.

Fire

In case of a fire we have to provide a space of 5 meters around the building so that the firetruck can pass through. The load of this truck is 20 tons, which will be represented as a punctual load.

Seismic action

Seismic actions are regulated by the NSCE (Norma de construcción sismorresistente) code, not by the CTE.

In this map we can see the impact of earthquakes through the accelerations they create. Taking into account that the impact in our territory is quite small, we can solve this problem with simple constructive solutions (stiff joints…).

3.3. LOAD HYPOTHESIS. REGULATIONS

The CTE uses the State Limit method to check and verify the building structures. Later in this notes we will learn how to calculate the different State Limit methods, but first, we will learn how to define the load state for each method.

a) Serviceability Limit State (SLS)

These are the load hypothesis calculations to measure and restrict the deformation of the structure. These are the two main deformations that we will have to verify:

- Vertical deflection
- Horizontal deflection

In these kinds of verifications we don't apply the magnification factors. Load values with no magnification factors are "Characteristic values". They carry sub K.

b) Ultimate limit state (ULS)

These are the load hypothesis calculations to measure and restrict the internal stress and the instability phenomenon. These are ultimate limit states that we will have to verify:

- Cross section Resistance
  - Axial strength
  - Bending moment
  - Shear strength
- Stability
  - Buckling
  - Lateral torsional buckling
  - Bending-compression stress
  - Dent

In these kinds of verifications we apply the magnification factors. Load values with magnification factors are “Calculation values”. They carry sub D.

3.3.1. Serviceability Limit State (SLS)

The serviceability limit state aim is to find or maintain the wellbeing of the daily users. Therefore, the deformations, vibrations and in general the damage of the building can´t be too big, so that the users don´t perceive them. Because these kinds of verifications are far from the structural collapse, we don’t apply the magnification factors.

$$\sum_{j\geq 1} G_{k,j} + P + Q_{k,1} + \sum_{i>1} \Psi_{0,i} \cdot Q_{k,i}$$

G_k: Any permanent action, in characteristic values.

P: Value of pre-stress in concrete.

Q_k: Main variable action of the hypothesis, in characteristic value. With all the variable actions that we have, we have to write a hypothesis where this action is the main one. Therefore, the amount of hypothesis combinations and variable actions will be the same.

$\Psi_0 \cdot Q_k$: The rest of variable actions, applying a simultaneity factor (4.2 table in CTE).

Tabla 4.2 Coeficientes de simultaneidad (ψ)

	ψ_0	ψ_1	ψ_2
Sobrecarga superficial de uso (Categorías según DB-SE-AE)			
• Zonas residenciales (Categoría A)	0,7	0,5	0,3
• Zonas administrativas(Categoría B)	0,7	0,5	0,3
• Zonas destinadas al público (Categoría C)	0,7	0,7	0,6
• Zonas comerciales (Categoría D)	0,7	0,7	0,6
• Zonas de tráfico y de aparcamiento de vehículos ligeros con un peso total inferior a 30 kN (Categoría E)	0,7	0,7	0,6
• Cubiertas transitables (Categoría F)		(1)	
• Cubiertas accesibles únicamente para mantenimiento (Categoría G)	0	0	0
Nieve			
• para altitudes > 1000 m	0,7	0,5	0,2
• para altitudes ≤ 1000 m	0,5	0,2	0
Viento	0,6	0,5	0
Temperatura	0,6	0,5	0
Acciones variables del terreno	0,7	0,7	0,7

(1) En las cubiertas transitables, se adoptarán los valores correspondientes al uso desde el que se accede.

3.3.2. Ultimate Limit State (ULS)

The ultimate limit state method verifies the resistance of the structure against the building collapse. Therefore, this time we have to apply a magnification factor and we have to verify the worst one.

$$\sum_{j\geq 1} \gamma_{G,j} \cdot G_{k,j} + \gamma_P \cdot P + \gamma_{Q,1} \cdot Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \cdot \Psi_{0,i} \cdot Q_{k,i}$$

$\gamma_G \cdot G_k$: Any permanent action, in calculation values.

$\gamma_p \cdot P_k$: Value of pre-stress in concrete, in calculation values.

$\gamma_Q \cdot Q_k$: Main variable action of the hypothesis, in calculation value. With all the variable actions that we have, we have to write a hypothesis where this action is the main one. Therefore, the amount of hypothesis combinations and variable actions will be the same

$\gamma_Q \cdot \Psi_0 \cdot Q_k$: The rest of variable actions, applying a simultaneity factor (4.2 table in theCTE).

γ: Magnification factors (4.1 table in the CTE)

Tabla 4.1 Coeficientes parciales de seguridad (γ) para las acciones

Tipo de verificación [1]	Tipo de acción	Situación persistente o transitoria	
		desfavorable	favorable
Resistencia	Permanente		
	Peso propio, peso del terreno	1,35	0,80
	Empuje del terreno	1,35	0,70
	Presión del agua	1,20	0,90
	Variable	1,50	0
		desestabilizadora	estabilizadora
Estabilidad	Permanente		
	Peso propio, peso del terreno	1,10	0,90
	Empuje del terreno	1,35	0,80
	Presión del agua	1,05	0,95
	Variable	1,50	0

[1] Los coeficientes correspondientes a la verificación de la resistencia del terreno se establecen en el DB-SE-C

The previous load hypothesis formula takes into account ordinary variable loads. However, when we have an accidental load we will have to use the following load hypothesis formula:

$$\sum_{j\geq 1} \gamma_{G,j} \cdot G_{k,j} + \gamma_P \cdot P + A_d + \gamma_{1,1} \cdot Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \cdot \Psi_{2,i} \cdot Q_{k,i}$$

$\gamma_G \cdot G_k$Any permanent action, in calculation values.

A_d:Any accidental action, in calculation value.

$\gamma_Q \cdot \Psi_1 \cdot Q_k$: Main variable action of the hypothesis, in calculation value. With all the variable actions that we have, we have to write a hypothesis where this action is the main one. Therefore, with each accidental action, the amount of hypothesis combinations and variable actions will be the same.

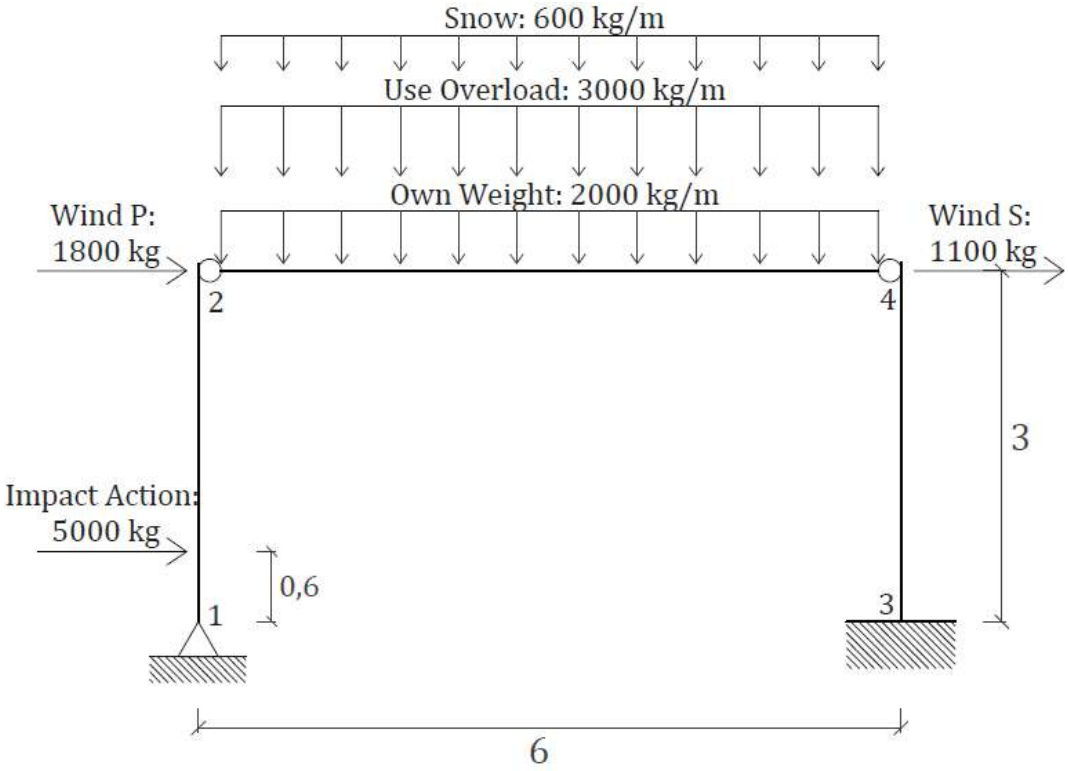
$\gamma_Q \cdot \Psi_2 \cdot Q_k$: The rest of variable actions, applying a simultaneity factor (4.2 table in theCTE).

γ: Magnification factors (4.1 table in the CTE)

3.3.3. Examples

Example. Load combination

Define the load combinations of the following frame.



Serviceability limit state (SLS):

$$\sum_{j\geq 1} G_{k,j} + Q_{k,1} + \sum_{i>1} \Psi_{0,i} \cdot Q_{k,i}$$

- Vertical deflection:

1. Hypothesis: Use overload is the main variable action

$G: Own\ weight = 2000\ kg/m$

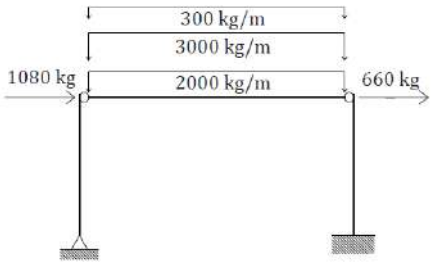
$Q_1: Use\ overload = 3000\ kg/m$

$Q_2: Snow = 600\ kg/m$

$Q_3: Wind = 1800\ kg - 1100kg$

$\Psi_{0\ Snow} = 0{,}5$

$\Psi_{0\ Wind} = 0{,}6$



2. Hypothesis: Snow is the main variable action

$G: Own\ weight = 2000\ kg/m$

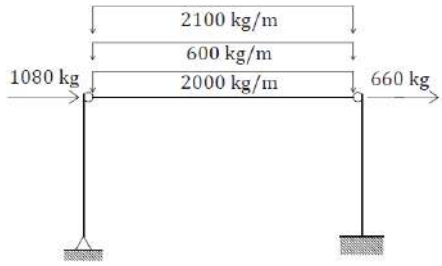
$Q_1: Snow = 600\ kg/m$

$Q_2: Use\ overload = 3000\ kg/m$

$Q_3: Wind = 1800\ kg - 1100kg$

$\Psi_{0\ U.O.} = 0{,}7\ \rightarrow Public\ use$

$\Psi_{0\ Wind} = 0{,}6$



3. Hypothesis: Wind is the main variable action

$G: Own\ weight = 2000\ kg/m$

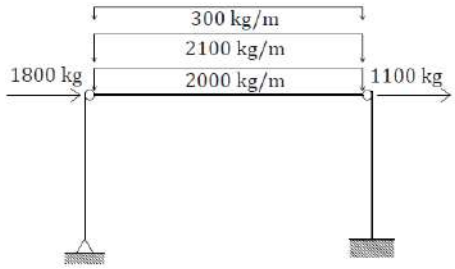
$Q_1: Wind = 1800\ kg - 1100kg$

$Q_2: Use\ overload = 3000\ kg/m$

$Q_3: Snow = 600\ kg/m$

$\Psi_{0\ U.O.} = 0{,}7\ \rightarrow Public\ use$

$\Psi_{0\ Snow} = 0{,}5$



	O.W.	U.O.	Snow	Wind
SLS – U.O.	1	1	1 x 0,5	1 x 0,6
SLS - Snow	1	1 x 0,7	1	1 x 0,6
SLS - Wind	1	1 x 0,7	1 x 0,5	1

Ultimate limit state (ULS):

$$\sum_{j\geq 1} \gamma_{G,j} \cdot G_{k,j} + \gamma_{Q,1} \cdot Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \cdot \Psi_{0,i} \cdot Q_{k,i}$$

1. Hypothesis: Use overload is the main variable action

$G: Own\ weight = 2000\ kg/m$

$Q_1: Use\ overload = 3000\ kg/m$

$Q_2: Snow = 600\ kg/m$

$Q_3: Wind, pressure = 1800\ kg/m$

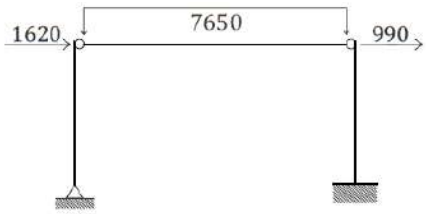
$Wind, suction = 1100\ kg/m$

$\gamma_G = 1{,}35$

$\gamma_Q = 1{,}5$

$\Psi_{0\ Snow} = 0{,}5$

$\Psi_{0\ Wind} = 0{,}6$



$$\overbrace{1{,}35 \cdot 2000}^{\text{O.W.}} + \overbrace{1{,}5 \cdot 3000}^{\text{U.O.}} + \overbrace{1{,}5 \cdot 0{,}5 \cdot 600}^{\text{Snow}} + \overbrace{1{,}5 \cdot 0{,}6 \cdot 1800 + 1{,}5 \cdot 0{,}6 \cdot 1100}^{\text{Wind}} = \overbrace{7650}^{\text{Vert.}} + \overbrace{1620 + 990}^{\text{Horiz.}}$$

2. Hypothesis: Snow is the main variable action

$G: Own\ weight = 2000\ kg/m$

$Q_1: Snow = 600\ kg/m$

$Q_2: Use\ overload = 3000\ kg/m$

$Q_3: Wind, pressure = 1800\ kg/m$

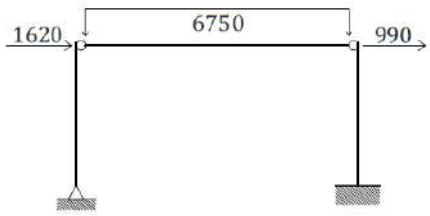
$Wind, suction = 1100\ kg/m$

$\gamma_G = 1{,}35$

$\gamma_Q = 1{,}5$

$\Psi_{0\ Wind} = 0{,}6$

$\Psi_{0\ U.O.} = 0{,}7 \rightarrow Public\ use$



$$\overbrace{1{,}35 \cdot 2000}^{O.W:} + \overbrace{1{,}5 \cdot 600}^{\text{Snow}} + \overbrace{1{,}5 \cdot 0{,}7 \cdot 3000}^{\text{U.O..}} + \overbrace{1{,}5 \cdot 0{,}6 \cdot 1800 + 1{,}5 \cdot 0{,}6 \cdot 1100}^{\text{Wind}} = \overbrace{6750}^{\text{Vert.}} + \overbrace{1620 + 990}^{\text{Horiz.}}$$

3. Hypothesis: Wind is the main variable action

$G: Own\ weight = 2000\ kg/m$

$Q_1: Wind, pressure = 1800\ kg/m$

$\quad Wind, suction = 1100\ kg/m$

$Q_2: Use\ overload = 3000\ kg/m$

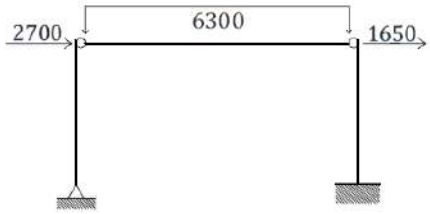
$Q_3: Snow = 600\ kg/m$

$\gamma_G = 1{,}35$

$\gamma_Q = 1{,}5$

$\Psi_{0\ Snow} = 0{,}5$

$\Psi_{0\ U.O.} = 0{,}7\ \rightarrow Public\ use$



$$\overbrace{1{,}35 \cdot 2000}^{O.W.} + \overbrace{1{,}5 \cdot 1800 + 1{,}5 \cdot 1100}^{\textbf{Wind}} + \overbrace{1{,}5 \cdot 0{,}7 \cdot 3000}^{\textbf{U.O.}} + \overbrace{1{,}5 \cdot 0{,}5 \cdot 600}^{\textbf{Snow}} = \overbrace{6300}^{\textbf{Vert.}} + \overbrace{2700 + 1650}^{\textbf{Horiz.}}$$

	O.W.	U.O.	Snow	Wind
ULS – U.O.	1,35	1,5	1,5 x 0,5	1,5 x 0,6
ULS - Snow	1,35	1,5 x 0,7	1,5	1,5 x 0,6
ULS - Wind	1,35	1,5 x 0,7	1,5 x 0,5	1,5

Ultimate Limit State (ULS) – Accidental actions:

$$\sum_{j\geq 1} \gamma_{G,j} \cdot G_{k,j} + A_d + \gamma_{1,1} \cdot Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \cdot \Psi_{2,i} \cdot Q_{k,i}$$

1. Hypothesis: Use overload is the main variable action

$G: Own\ weight = 2000\ kg/m\ ;\ A_d: Accidental\ action = 5000\ kg$

$Q_1: Use\ overload = 3000\ kg/m$

$Q_2: Snow = 600\ kg/m$

$Q_3: Wind, pressure = 1800\ kg/m$

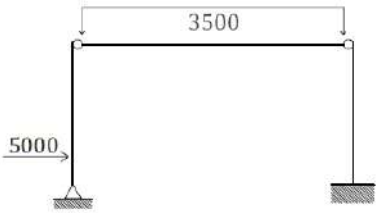
$\quad Wind, suction = 1100\ kg/m$

$\gamma_G = \gamma_Q = 1$

$\Psi_{0\ U.O.} = 0{,}5\ \rightarrow Public\ use$

$\Psi_{0\ Snow} = 0$

$\Psi_{0\ Wind} = 0$



$$\overbrace{1 \cdot 2000}^{\textbf{O.W.}} + \overbrace{5000}^{\textbf{A.A.}} + \overbrace{1 \cdot 0{,}5 \cdot 3000}^{\textbf{U.O.}} + \overbrace{0 \cdot 600}^{\textbf{Snow}} + \overbrace{0 \cdot 1800 + 0 \cdot 1100}^{\textbf{Wind}} = \overbrace{3500}^{\textbf{Vert.}} + \overbrace{5000}^{\textbf{Horiz.}}$$

2. Hypothesis: Snow is the main variable action

$G: Own\ weight = 2000\ kg/m\ ;\ A_d: Accidental\ action = 5000\ kg$

$Q_1: Snow = 600\ kg/m$

$Q_2: Use\ overload = 3000\ kg/m$

$Q_3: Wind, pressure = 1800\ kg/m$

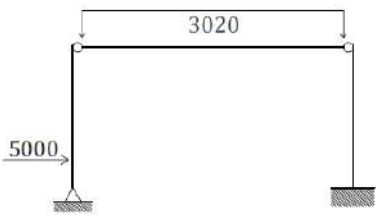
$\quad Wind, suction = 1100\ kg/m$

$\gamma_G = \gamma_Q = 1$

$\Psi_{0\ U.O.} = 0{,}3\ \rightarrow Public\ use$

$\Psi_{0\ Snow} = 0{,}2$

$\Psi_{0\ Wind} = 0$



$$\overbrace{1 \cdot 2000}^{\textbf{O.W.}} + \overbrace{5000}^{\textbf{A.A.}} + \overbrace{1 \cdot 0{,}2 \cdot 600}^{\textbf{Snow}} + \overbrace{1 \cdot 0{,}3 \cdot 3000}^{\textbf{U.O.}} + \overbrace{0 \cdot 1800 + 0 \cdot 1100}^{\textbf{Wind}} = \overbrace{3020}^{\textbf{Vert.}} + \overbrace{5000}^{\textbf{Horiz.}}$$

3. Hypothesis: Wind is the main variable action

$G: Own\ weight = 2000\ kg/m$; $A_d: Accidental\ actions = 5000\ kg$

$Q_1: Wind, pressure = 1800\ kg/m$

$Wind, suction = 1100\ kg/m$

$Q_2: Use\ overload = 3000\ kg/m$

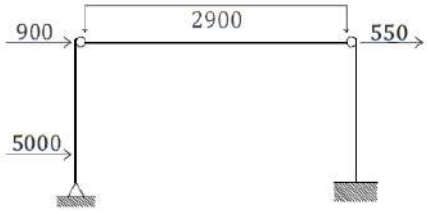
$Q_3: Snow = 600\ kg/m$

$\gamma_G = \gamma_Q = 1$

$\Psi_{0\ U.O.} = 0{,}3\ \rightarrow Public\ use$

$\Psi_{0\ Snow} = 0$

$\Psi_{0\ Wind} = 0{,}5$



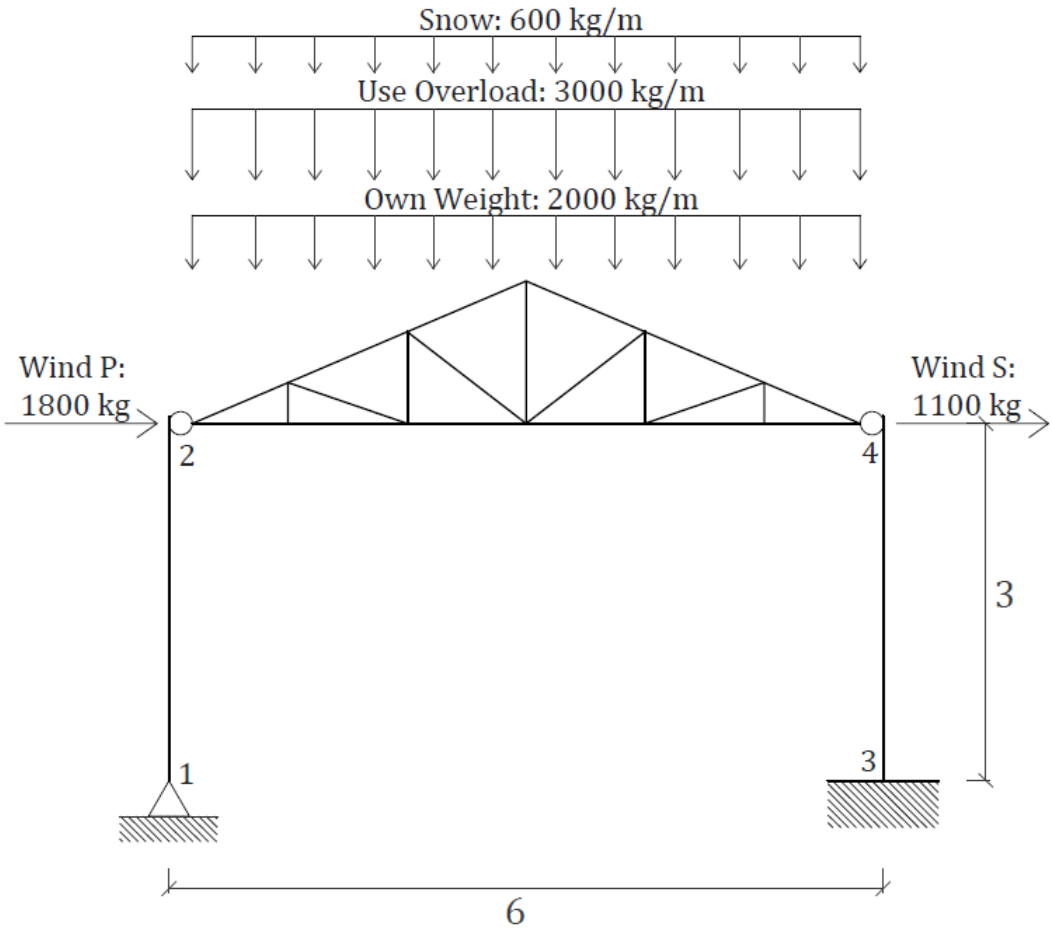
$$\overbrace{1 \cdot 2000}^{\textbf{O.W.}} + \overbrace{5000}^{\textbf{A.A.}} + \overbrace{1 \cdot 0{,}5 \cdot 1800 + 1 \cdot 0{,}5 \cdot 1100}^{\textbf{Wind}} + \overbrace{1 \cdot 0{,}3 \cdot 3000}^{\textbf{U.O.}} + \overbrace{0 \cdot 600}^{\textbf{Snow}} = \overbrace{2900}^{\textbf{Vert.}} + \overbrace{5000 + 900 + 550}^{\textbf{Horiz.}}$$

	O.W.	U.O.	Snow	Wind	Impact
ULS – U.O.	1	1 x 0,5	1 x 0	1 x 0	1
ULS - Snow	1	1 x 0,3	1 x 0,2	1 x 0	1
ULS - Wind	1	1 x 0,3	1 x 0	1 x 0,5	1

<u>All the hypothesis combinations:</u>

Hypo.		O.W.	U.O.	Snow	Wind	Impact
1	SLS – U.O.	1	1	1 x 0,6	0	0
2	SLS - Snow	1	1 x 0,7	1	0	0
3	SLS - Wind	1	0	0	1	0
4	ULS – U.O.	1,35	1,5	1,5 x 0,5	1,5 x 0,6	0
5	ULS - Snow	1,35	1,5 x 0,7	1,5	1,5 x 0,6	0
6	ULS- Wind	1,35	1,5 x 0,7	1,5 x 0,5	1,5	0
7	ULS - Impact + U.O.	1	1 x 0,5	1 x 0	1 x 0	1
8	ULS - Impact + Snow	1	1 x 0,3	1 x 0,2	1 x 0	1
9	ULS - Impact + Wind	1	1 x 0,3	1 x 0	1 x 0,5	1

Exercise 1. For the following frame, define a hypothesis to analyse the risk of bearing compression in the bottom beam of the truss.



In this case, as we are analysing the bottom beam of the truss, vertical loads work in our favour (as they are bearing traction to this bottom beam). Nevertheless, horizontal loads are working against us, as they bear compression to the bottom beam. Therefore, in addition to the previous exercise hypothesis we have to add this one to:

<u>Ultimate limit state (ULS) – wind/truss:</u>

$G: Own\ weight = 2000\ kg/m$

$Q_1: Wind, pressure = 1800\ kg/m$

$\quad Wind, sukzioa = 1100\ kg/m$

$Q_2: Use\ overload = 3000\ kg/m$

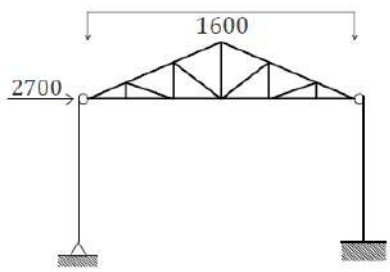
$Q_3: Snow = 600\ kg/m$

$\gamma_G = 0{,}8$

$\gamma_{Q\ Wind} = 1{,}5$

$\gamma_{Q\ Snow} = 0$

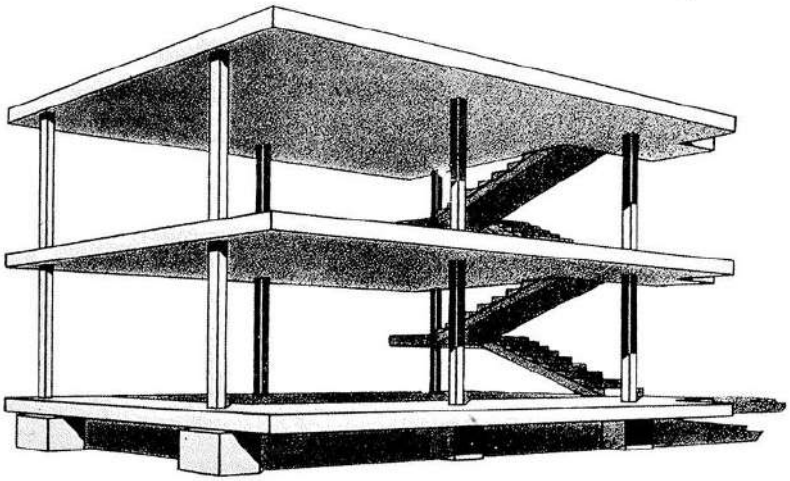
$\gamma_{0\ U.O.} = 0 \rightarrow Public\ use$

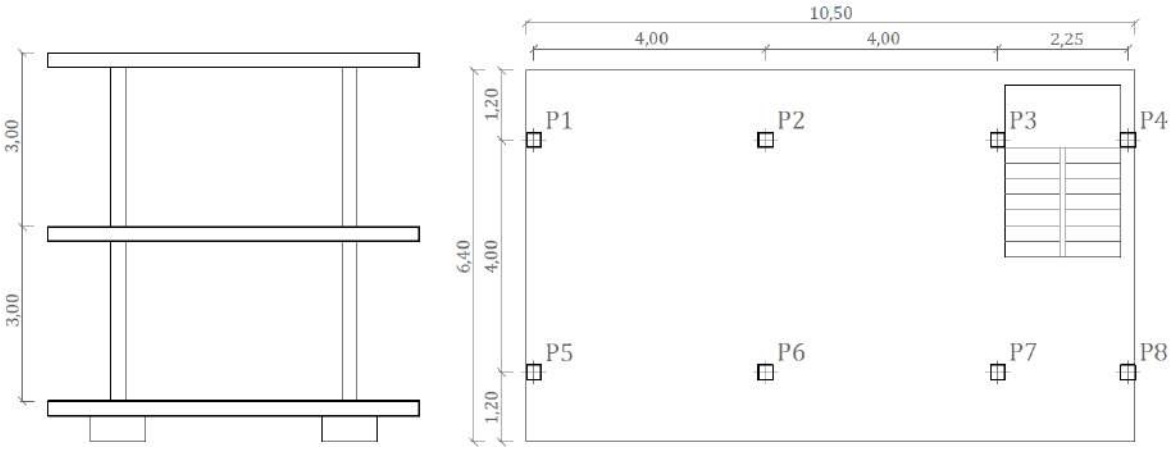


$$\overbrace{0{,}8 \cdot 2000}^{\textbf{O.W.}} + \overbrace{1{,}5 \cdot 1800 + 1{,}5 \cdot 0}^{\textbf{Wind}} + \overbrace{0 \cdot 3000}^{\textbf{U.O.}} + \overbrace{0 \cdot 600}^{\textbf{Snow}} = \overbrace{1600}^{\textbf{Vert.}} + \overbrace{2700}^{\textbf{Horiz.}}$$

Exercise 2

Dimension the structural elements of Le Corbusier´s “Maison Domino” building.



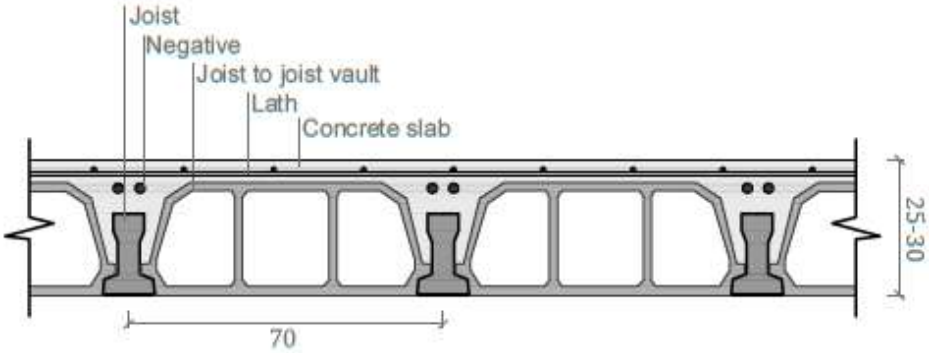


Data:

- Building´s use: Residential
- Façade´s weight: 800 kg/ml
- Floor´s and inner partition´s weight: 150 kg/m^2
- False ceiling’s weight: 50 kg/m^2
- Flat roof´s weight: 150 kg/m^2

Floor type:

-Rib floor

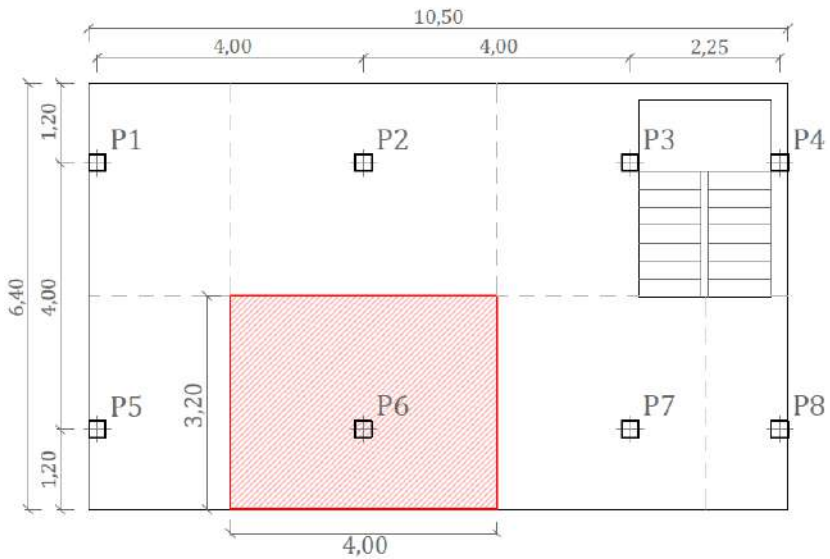


CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Possible values	0.50 – 0.80	< 6.50	0.20 – 0.35	2.75 – 4.75	30 - 50
Usual values (recommended)	0.70	4.50 – 6.00	0.25 – 0.30	3.25 – 3.75	35 - 45
Most common and economic floor type. Doubling the joists we can improve the behaviour in 10-20 %. The cantilever can be 6 or 8 times the depth of the floor. If the joists are not self-resistant, formwork is needed.			$H = \frac{L}{(18-22)}$	$H = P \cdot (11-14)$	$C = H \cdot (140-160)$

Column´s dimensioning:

- Eccentricity increases axial strength in 20%
- Material´s yield strength. $^{250}/_{1,5}\ kg/m^2$
- Slenderness < 35
- Minimum dimensions of the column: $25\ x\ 25$

<u>P6 is bearing the greatest load, then we are going to analyse this column:</u>



Bearing load area:

$A = 4 \cdot 3{,}2 = 12{,}8\ m^2$

• Each floor´s weight:

- <u>Permanent loads (own weight):</u>

$G_1 : Floor\ slab\ \rightarrow 25\ cm\ thickness = 325\ kg/m^2$

$G_2 : Floor\ and\ inner\ partitions = 150\ kg/m^2$

$G_3 : False\ ceiling = 50\ kg/m^2$

$G_4 : Flat\ roof = 150\ kg/m^2$

$G_5 : Façade = 800\ kg/ml$

- <u>Variable loads:</u>

$Q_1 : Use\ overload = 200\ kg/m^2$

$Q_2 : Snow = 50\ kg/m^2$

- Floor´s weight:

$$\underline{Ground\ floor}: G_1 + G_2 + G_5 + Q_1 = 675\ kg/m^2 + 800\ kg/ml$$

$$\underline{First\ floor}: G_1 + G_2 + G_3 + G_5 + Q_1 = 725\ kg/m^2 + 800\ kg/ml$$

$$\underline{Roof}: G_1 + G_3 + G_4 + Q_1 + Q_2 = 775\ kg/m^2$$

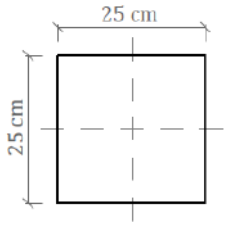
• Column´s increased weight:

$$N_{P6} = [(675 \cdot 12{,}8 + 800 \cdot 4) + (725 \cdot 12{,}8 + 800 \cdot 4) + (775 \cdot 12{,}8)] \cdot 1{,}5$$
$$= \boxed{51360\ kg}$$

• P6 column dimensioning:

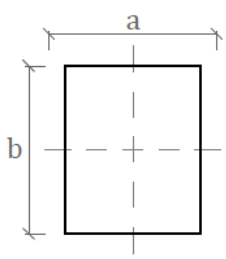
$$\vartheta = \frac{N \cdot 1{,}2}{A} \leq \delta_{CD} \ \rightarrow A \geq \frac{N \cdot 1{,}2}{\delta_{CD}} = \frac{61632}{166,\hat{6}} = 370cm^2 \ \rightarrow 19\ x\ 19 \ < \underbrace{25\ x\ 25}_{\substack{minimum \\ dimensions}}$$

$$\delta_{CD} = \frac{250}{1{,}5} = 166,\hat{6}\ kg/cm^2$$



Beam´s dimensioning:

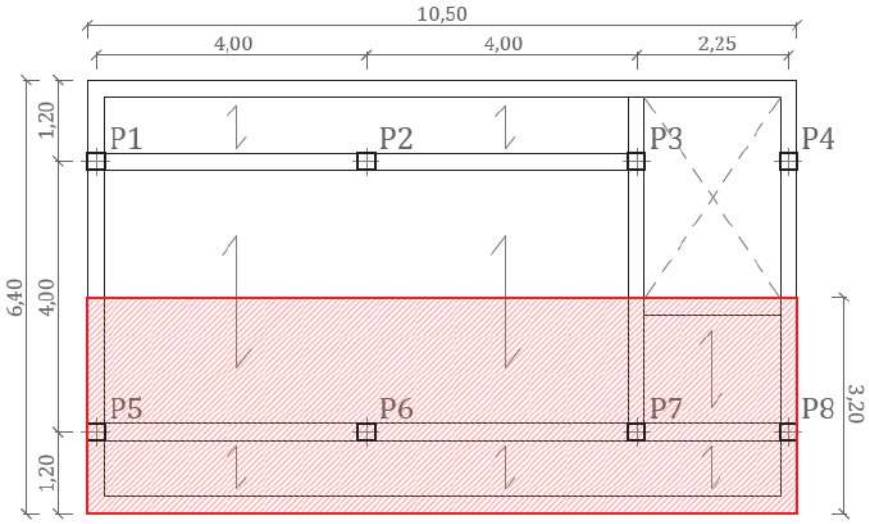
- Span-depth ratio:
  - One gap: $b = {}^{L}/_{15} - {}^{L}/_{20}$
  - More than one gap: $b = {}^{L}/_{20} - {}^{L}/_{24}$
- Cross section width-height ratio:
  - $a = {}^{b}/_{2}\ ; a = 0{,}2m - 0{,}3m$



$$b = \frac{L}{20} = 20 \ \rightarrow$$

To improve the connection between the beam and the column we will use a beam with the size of 25 x 25 cm.

Primary frame



Bearing load area:

$$A = 10{,}5 \cdot 3{,}2 = 33{,}6\ m^2$$

<u>Loads:</u>

• Own weight:

$$\underline{Ground\ floor}: (G_1 \cdot 3{,}2) + (G_2 \cdot 3{,}2) + \left(G_5 \cdot \frac{10{,}5+3{,}2+3{,}2}{10{,}5}\right) = 2840\ kg/ml$$

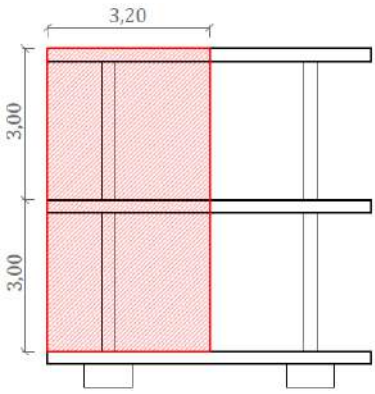
$$\underline{First\ floor}: (G_1 \cdot 3{,}2) + (G_2 \cdot 3{,}2) + (G_3 \cdot 3.2) + \left(G_5 \cdot \frac{16{,}9}{10{,}5}\right) = 3000\ kg/ml$$

$$\underline{Roof}: (G_1 \cdot 3{,}2) + (G_3 \cdot 3{,}2) + (G_4 \cdot 3{,}2) = 1680\ kg/ml$$

• Use overload: $Q_1 \cdot 3{,}2 = 640\ kg/ml$

• Snow: $Q_2 \cdot 3{,}2 = 160\ kg/ml$

• Wind:



Bearing load area:

$$A = 3 \cdot 3{,}2 = 9{,}6\ m^2$$

$q_e = q_b \cdot C_e \cdot C_p$

q_b :	Wind´s dynamic pressure	$= 0{,}5$
C_e :	Exposure coefficient	$= 2$
C_p :	Wind coefficient	$= 0{,}8/-0{,}4$

Tabla 3.5. Coeficiente eólico en edificios de pisos

	Esbeltez en el plano paralelo al viento					
	< 0,25	0,50	0,75	1,00	1,25	≥ 5,00
Coeficiente eólico de presión, c_p	0,7	0,7	0,8	0,8	0,8	0,8
Coeficiente eólico de succión, c_s	-0,3	-0,4	-0,4	-0,5	-0,6	-0,7

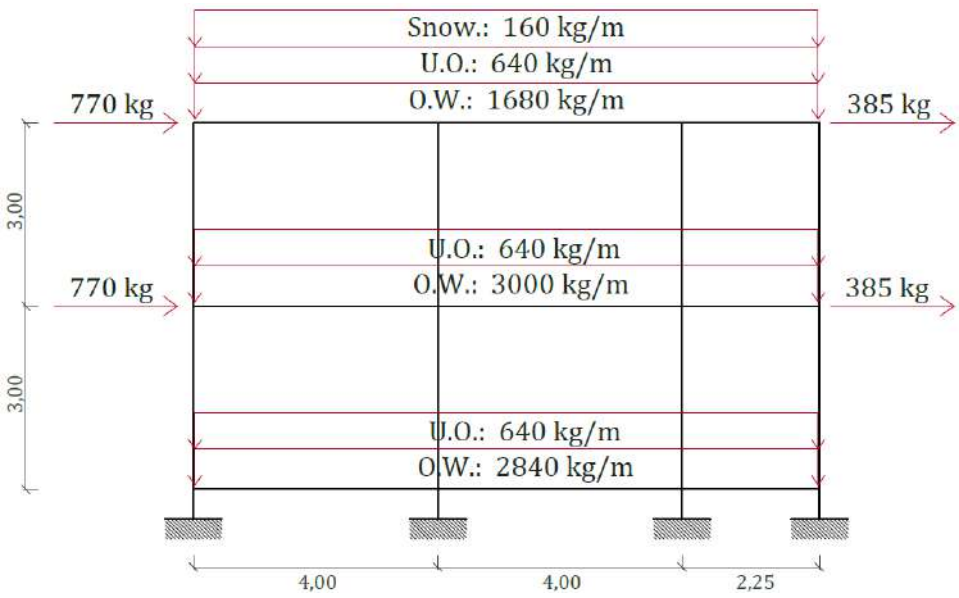
Pressure side: $q_e = q_b \cdot C_e \cdot C_p = 0{,}5 \cdot 2 \cdot 0{,}8 = 0{,}8\ kN/m^2 = 80\ kg/m^2$

Suction side: $q_e = q_b \cdot C_e \cdot C_p = 0{,}5 \cdot 2 \cdot -0{,}4 = -0{,}4\ kN/m^2 = -40\ kg/m^2$

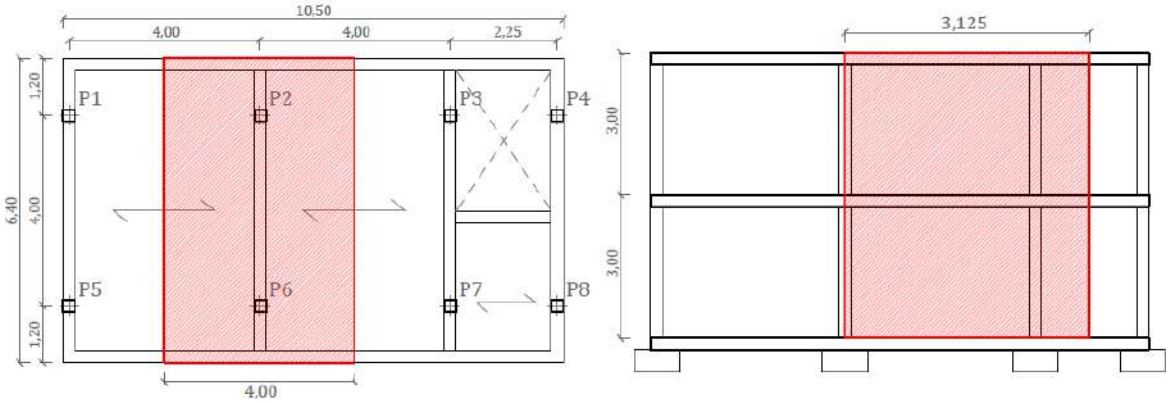
$Q_{3\ pressure} = 80 \cdot (3 \cdot 3{,}2) \approx 770\ kg$

$Q_{3\ suction} = -40 \cdot (3 \cdot 3{,}2) \approx 385\ kg$

• Summary:



Secondary frame:



$$Q_{4\ pressure} = 80 \cdot (3 \cdot 5{,}125) \approx 1230\ kg$$

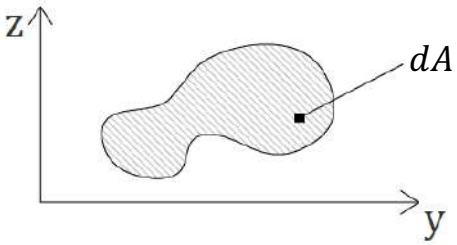
$$Q_{4\ suction} = -40 \cdot (3 \cdot 5{,}125) \approx 615\ kg$$

4. GEOMETRIC PROPERTIES OF FLAT SECTIONS

If we want to verify if the strengths created by the actions in our structure are acceptable or not, we have to calculate the inner deformations and stress resulted from those previous actions (SLS and ULS). To be able to calculate the stress, we need to know some geometric properties of the cross section. In this chapter then, we will learn some formulas to calculate the geometric properties of flat sections. After seeing this, in the following chapter, we will learn how to use them to calculate the stress and deflections.

4.1. STATIC MOMENT AND CENTROID

To calculate the static moment we are going to use a generic flat area A.



$$A = \int_A dA$$

When we have defined the area, by means of an integer mathematical operation in the Y and Z axis, depending on dA differential element, these are going to be S_y and S_z static moments:

$$S_y = \int_A z \cdot dA$$

$$S_z = \int_A y \cdot dA$$

The static moments can be positive or negative, depending on the location of the axis (if the area is under the axis, the static moment is going to be negative).

Using the static moment we can now define the centroid, with the following expression:

$$Y_{G.C.} = \frac{S_z}{A} = \frac{\int_A y \cdot dA}{A}$$

$$Z_{G.C.} = \frac{S_y}{A} = \frac{\int_A z \cdot dA}{A}$$

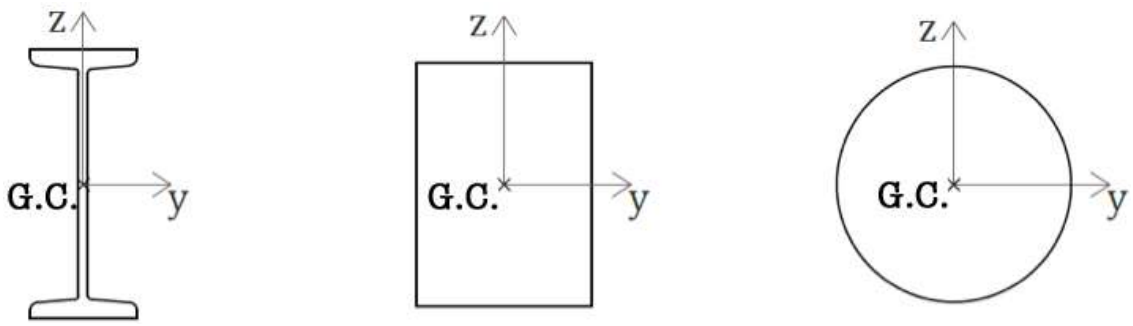
If the density of the materials in the section is uniform, then we can say that the centroid and the centre of gravity are located in the same point.

By definition, the static moment will be null in every axis passing through the centre of gravity (G.C.):

$$Y_{G.C.} = \frac{S_z}{A};\ Y_{G.C.} = 0\ (axis\ G.C.)\ \rightarrow\ S_z = 0 \cdot A = 0$$

Then, we can define the centre of gravity in a different way: the centre of gravity is the point where the static moment is zero about every axis passing through that point.

If an area is symmetric, its centre of gravity is going to be located in the symmetry axis. Moreover, if this section has two symmetry axis, the centre of gravity is going to be located in these axis intersection:



If our section is composed by the combination of different small pieces and we now the centre of gravity of those pieces we can define the overall centre of gravity of the area using this mathematical expression:

$$Y_{G.C.} = \frac{\sum_{i=1}^{n} Y_{G.C._i} \cdot A_i}{\sum_{i=1}^{n} A_i}$$

$$Z_{G.C.} = \frac{\sum_{i=1}^{n} Z_{G.C._i} \cdot A_i}{\sum_{i=1}^{n} A_i}$$

When the geometry has a hole we can consider that hole as a negative area, and therefore, introduce it in the mathematical expression with the negative sign.

4.2. MOMENT OF INERTIA

The Moment of inertias I_y and I_z of a specific area related to Y and Z axis are the following ones:

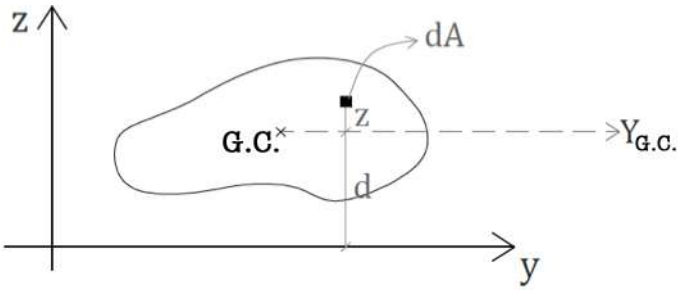
$$I_y = \int_A z^2 \cdot dA$$

$$I_z = \int_A y^2 \cdot dA$$

The Moment of inertia of a composed section about an axis, is the addition of all the inertias of the subareas composing the cross section about the same axis.

4.3. STEINER´S THEOREM

Using the following method we can also calculate a section´s Moment of inertia about any axis:



$$I_y = I_{y_{G.C.}} + A \cdot d^2$$

Where:

- I_y : Moment of inertia about Y axis
- $I_{G.C.}$: Moment of inertia about $Y_{G.C.}$ axis,
  where Y and $Y_{G.C.}$ are parallel
- A : Area
- d : distance between Y and $Y_{G.C.}$

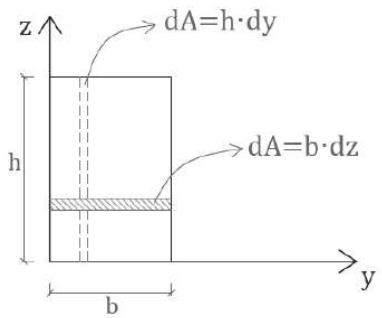
We can easily justify this theorem by developing Moment of inertia´s expression:

$$I_y = \int_A (d+z)^2 dA = \int_A (z^2 + 2dz + d^2) dA = \int_A z^2 dA + 2d \int_A z\, dz + d^2 \int_A dA$$

$$\left.\begin{array}{l} \int_A z^2 dA = I_{y_{G.C.}} \\ \int_A z\, dA = S_y \quad about\ \mathrm{G.C.} \rightarrow 0 \\ \int_A dA = A \end{array}\right\} \text{Then: } I_y = I_{y_{G.C.}} + d^2 \cdot A$$

Examples:

<u>- Area of a rectangle:</u>



$$A = \int_A dA = \int_0^h b \cdot dz = b \cdot z \Big|_0^h = b \cdot h$$

$$\boxed{A = b \cdot h}$$

$$S_y = \int_A z \cdot dA = \int_0^h z \cdot b \cdot dz = b \frac{z^2}{2} \Big|_0^h = \frac{b \cdot h^2}{2}$$

$$S_z = \int_A y \cdot dA = \int_0^b y \cdot h \cdot dy = h \frac{y^2}{2} \Big|_0^b = \frac{h \cdot b^2}{2}$$

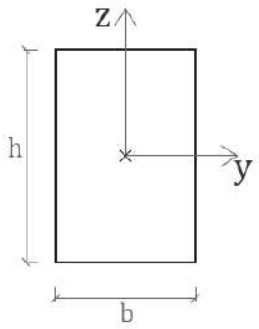
Centre of gravity location:

$$Y_{G.C.} = \frac{S_z}{A} = \frac{(h \cdot b^2)/2}{h \cdot b} = \frac{b}{2}$$

$$Z_{G.C.} = \frac{S_y}{A} = \frac{(b \cdot h^2)/2}{h \cdot b} = \frac{h}{2}$$

$$G.C.\left(\frac{b}{2}, \frac{h}{2}\right) \text{ because of symmetry}$$

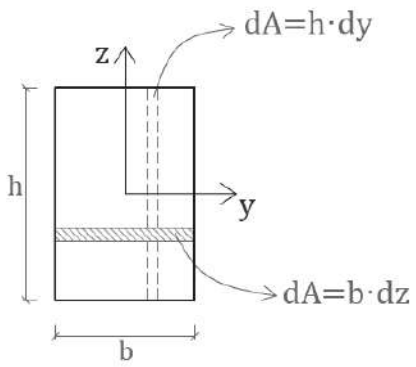
Even though we already know that the static moment of any area to its centre of gravity is null, we can easily prove it:



$$S_y = \int_A z \cdot dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} z \cdot b \cdot dz = b\frac{z^2}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}} = b\left(\frac{h^2}{8} - \frac{h^2}{8}\right) = 0$$

$$S_z = \int_A y \cdot dA = \int_{-\frac{b}{2}}^{\frac{b}{2}} y \cdot h \cdot dy = h\frac{y^2}{2}\int_{-\frac{b}{2}}^{\frac{b}{2}} = h\left(\frac{b^2}{8} - \frac{b^2}{8}\right) = 0$$

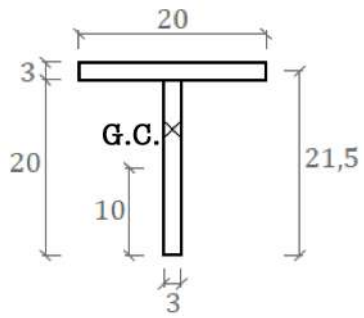
Rectangle´s Moment of inertia about the main axis:



$$I_y = \int_A z^2 \cdot dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 \cdot b \cdot dz = b\frac{z^3}{3}\int_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{b}{3}\left(\frac{h^3}{8} - \frac{-h^3}{8}\right) = \frac{bh^3}{12}$$

$$I_z = \int_A y^2 \cdot dA = \int_{-\frac{b}{2}}^{\frac{b}{2}} y^2 \cdot h \cdot dy = h\frac{y^3}{3}\int_{-\frac{b}{2}}^{\frac{b}{2}} = \frac{h}{3}\left(\frac{b^3}{8} - \frac{-b^3}{8}\right) = \frac{hb^3}{12}$$

- Composed section:



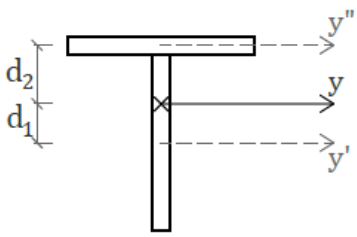
We already know that because of the symmetry of the profile the centre of gravity is going to be located in this symmetry axis. Therefore we just need to calculate the height of this point.

$$Z_{G.C.} = \frac{\sum_i^n Z_{G.C._i} \cdot A_i}{\sum_i^n A_i} = \frac{Z_{G.C._1} \cdot A_1 + Z_{G.C._2} \cdot A_2}{A_1 + A_2} = \frac{60 \cdot 10 + 60 \cdot 21{,}5}{120} = 15{,}75\ cm$$

$Z_{G.C._1} = 10\ ;\ A_1 = 20 \cdot 3 = 60$

$Z_{G.C._2} = 21{,}5\ ;\ A_2 = 30 \cdot 2 = 60$

To calculate this Moment of inertia, first we need to know what the subarea's Moment of inertias are. Then we can just add them.



$$I_y = \left(I_{y'} + A_1 \cdot {d_1}^2\right) + \left(I_{y''} + A_2 \cdot {d_2}^2\right)$$

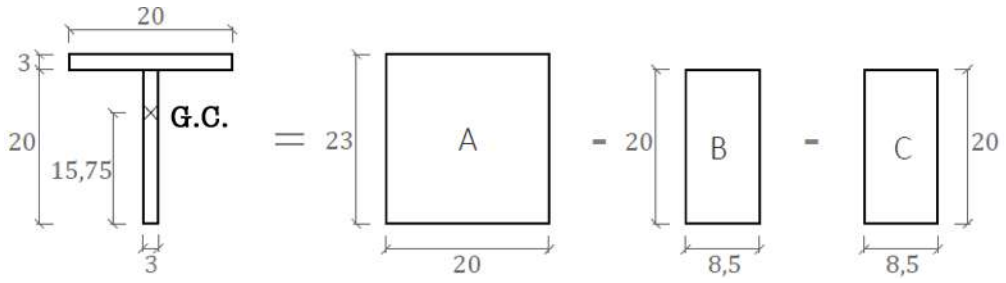
$$I_y = \frac{b' \cdot h'^3}{12} + b' \cdot h' \cdot {d_1}^2 + \frac{h'' \cdot b''^3}{12} + b'' \cdot h'' \cdot {d_2}^2$$

$d_1 = 5{,}75\ cm$

$d_2 = 5{,}75\ cm$

$$I_y = \frac{3 \cdot 20^3}{12} + 3 \cdot 20 \cdot 5{,}75^2 + \frac{20 \cdot 3^3}{12} + 20 \cdot 3 \cdot 5{,}75^2 = 6012{,}5\ cm^4$$

- Same Composed profile calculated in a different way:



Therefore, we can apply Steiner´s theorem in this other way:

$$I_y = \left(I_{y1} \cdot A_1 \cdot {d_1}^2\right) - \left(I_{y2} \cdot A_2 \cdot {d_2}^2\right) \cdot 2$$

$$I_y = \frac{20 \cdot 23^2}{12} + 20 \cdot 23 \cdot 4{,}25^2 - 2 \cdot \left(\frac{8{,}5 \cdot 20^3}{12} + 20 \cdot 8{,}5 \cdot 5{,}75^2\right)$$

$$I_y = 6012{,}5\ cm^4$$

5. STRESS IN STRUCTURES

As we saw in the previous chapter, to verify our frame's cross sections, we need to first calculate what the internal strengths are, and later on, what the inner stress that they create are. So, summarizing, that's the calculation scheme we follow:

$$Actions \rightarrow Strengths \rightarrow Stress$$

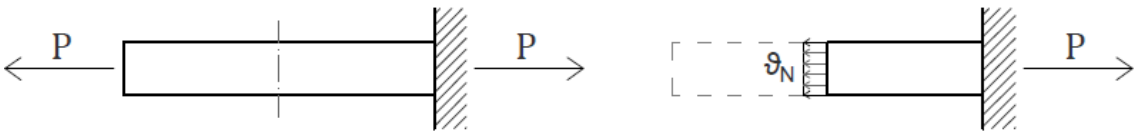
First we have to calculate the strengths on our structure produced by the actions. After this we are going to quantify the inner stress generated by the previous strengths. Finally, to check that the material resists those stress, we have to verify that materials yield strength is higher than our inner stress. This last verification is the most important step of the whole process.

5.1. STRESS TYPES. NORMAL AND TANGENTIAL STRESS

The different srtess happening in a specific section are directly related to the strengths happening in the structure:

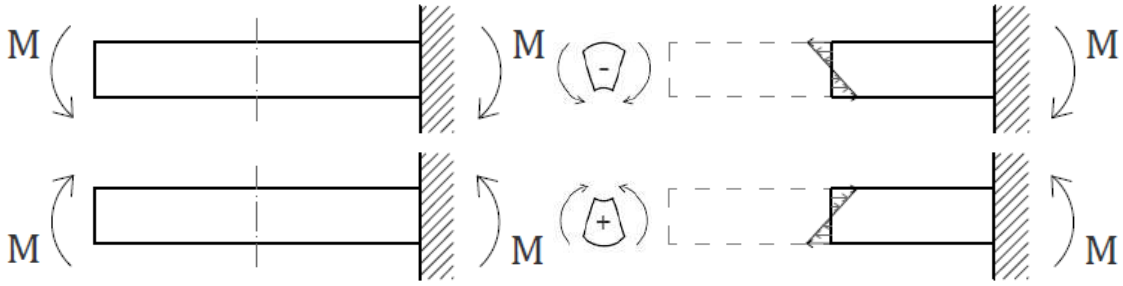
- Compression strengths: → constant **normal** stress

- Traction strengths: → constant **normal** stress



- Shear strength : → variable **tangential stress**

- Bending moment: → variable **normal** stress



- Torsion strength: → variable **tangential stress**

As we have seen in the previous schemes, we can have two different kinds of stress in our cross sections:

- Normal stress (σ) → related to traction-compression and flection
- Tangential stress (τ) → related to cutting and torsion strengths

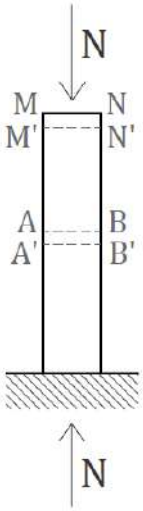
And now, by using differential equations, we can calculate what the stress created by each strength is, because in each section, the strength and the stress are in equilibrium:

$$N = \int_A \vartheta_N \cdot dA \ ; \ M_y = \int_A \vartheta_N \cdot z \cdot dA \ ; M_z = \int_A \sigma_N \cdot y \cdot dA \ ; V_y = \int_A \tau_y \cdot dA$$

$$V_z = \int_A \tau_z \cdot dA \ ; \ M_T = \int_A (z \cdot \tau_y - y \cdot \tau_z) \cdot dA$$

In the following chapter we will develop these differential equations.

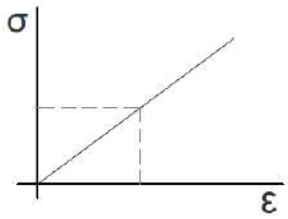
5.1.1. Normal stress due to axial strength



If the axial strength is applied in the centre of gravity and towards the length axis, we verify empirically that the sections we have determined stay parallel to one another.

This is: $AA' = BB'$

Hook's law implies that if all the points of a section are equally deformed, each point's normal stress (σ_N) is the same.



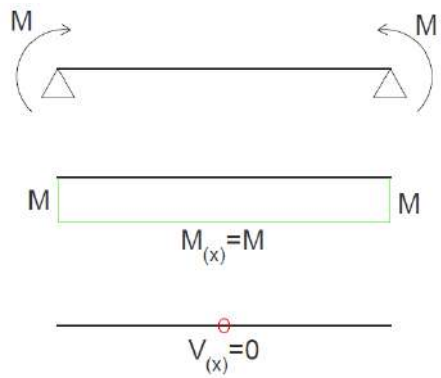
$$\varepsilon = const. \rightarrow \ \sigma = const.$$

In the same way, we know that all the stress are in equilibrium with the strengths received in the section. On the case of only axial strength:

$$N = \int_A \sigma_N \cdot dA \ \rightarrow \ \sigma_N = kte \ \rightarrow N = \sigma_N \cdot \int_A dA = \sigma_N \cdot A \ ; \ \boxed{\sigma_N = \frac{N}{A}}$$

5.1.2. Normal stress due to bending moment

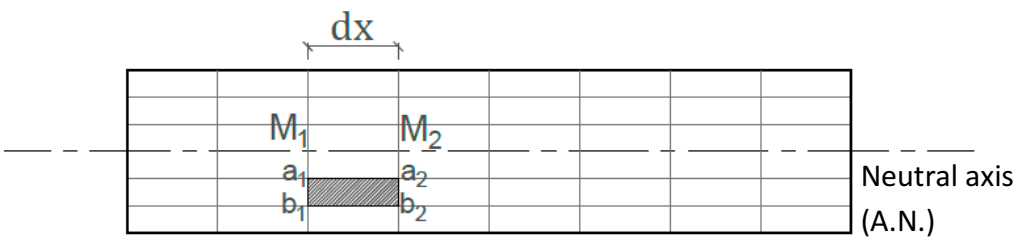
Pure bending moment:



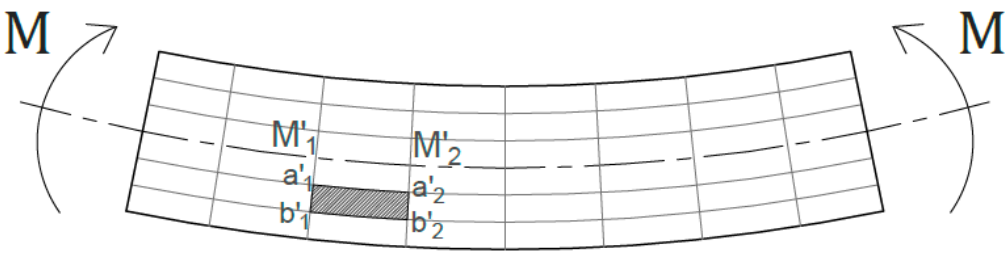
To analyse the tensional state produced by the bending moment in any section of the bar, we have to follow some rules and principles:

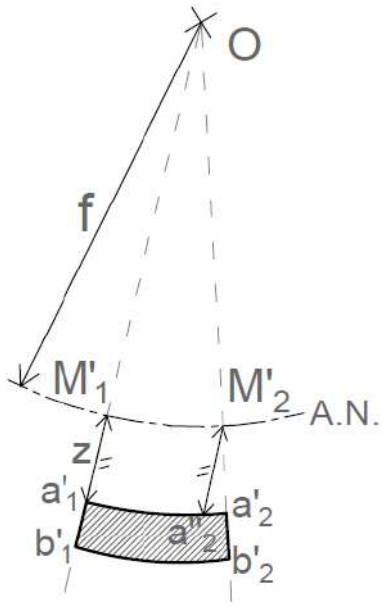
- The bar must be made up by an homogeneous material.
- If before the deformation the cross sections are flat, they have to remain flat after the deformation too.
- We will move inside Hook's law.

Before the deformation:



After the deformation:





-As $M'_1M'_2$ is in the neutral axis, then the M_1M_2 distance, and dx have to be the same.

-We are going to define an a''_2 point that is going to be distance Z away from M'_2, drawing a parallel from M'_2 point to the line connecting $M'_1a'_1$.

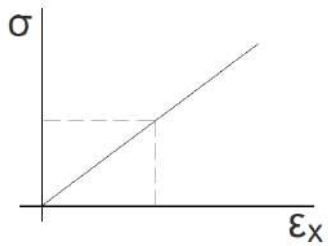
Therefore, this is going to be the unitary elongation of a_1a_2:

$$\varepsilon_x = \frac{\Delta l}{l} = \frac{a''_2a'_2}{dx} = \frac{a''_2a'_2}{M'_1M'_2}$$

What´s more, taking into account the proportionality between the triangles:

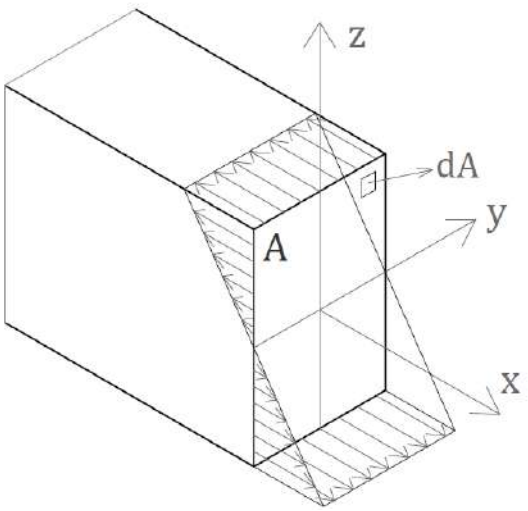
$$\frac{M'_1M'_2}{f} = \frac{a''_2a'_2}{z} \; ; \; \frac{z}{f} = \frac{a''_2a'_2}{M'_1M'_2} \rightarrow \boxed{\frac{z}{f} = \varepsilon_x}$$

This means that once we know "f" radius of curvature, we can calculate any "z" fibre "ε_x" unitary elongation. Through this, we will be able to know how much stress is received by each fibre, applying Hook's law:



$$E = \frac{\sigma}{\varepsilon_x} \; ; \; \sigma = \varepsilon_x \cdot E \; ; \; \sigma = \frac{z}{f} \cdot E$$

However, it is not easy to calculate the "f" radius of curvature. Therefore, we are going to try to simplify this expression using the first static equation:



$$\Sigma F_x = 0$$

$$\int_a \sigma_x \cdot dA = 0\,;\ \int_A E \cdot \varepsilon_x \cdot dA = 0\,;$$

$$\int_A E \cdot \frac{z}{f} \cdot dA = 0\,;\ \frac{E}{f}\int_A z \cdot dA = 0$$

$$\frac{E}{f} \neq 0 \ \rightarrow\ \int_A z \cdot dA = 0 \ \rightarrow\ S_y = 0 \text{ (Static moment zbout neutral axis)}$$

Therefore, this means that in the case of pure bending moment the neutral axis passes through the centre of gravity, because the static moment about the neutral axis is null.

Let's apply now the third static equation:

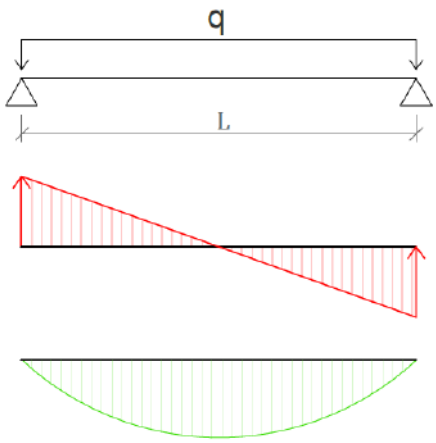
$$\Sigma M = 0\,;\ M_y = \int_A z \cdot \sigma_x \cdot dA$$

$$M_y = \int_A z \cdot \sigma_x \cdot dA\ ; M = \int_A z \cdot E \cdot \varepsilon_x \cdot dA\ ; M = \frac{E}{f}\int_A z^2 \cdot dA\ ;\ \boxed{M_y = \frac{E}{f} \cdot I_y}$$

And now comparing it with the stress expression:

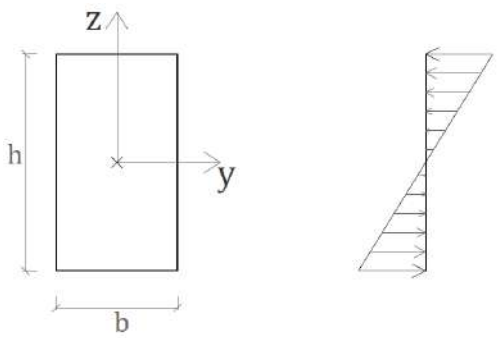
$$\left.\begin{aligned} \sigma_x &= \frac{E \cdot z}{f} \\ M_y &= \frac{E}{f} \cdot I_y \end{aligned}\right\} \qquad \frac{E \cdot z}{\sigma_x} = \frac{E \cdot I_y}{M_y} \rightarrow \boxed{\sigma_x = \frac{M_y \cdot z}{I_y}}$$

We say that a structural element is working under simple bending moment if in addition to the bending moment it is also working under shear strength.



To calculate the strengths suffered by any section we are going to use the superposition principle. This way, first we are going to identify and quantify the stress caused by the bending moment, and then the stress caused by the shear strength.

As we have analysed in the previous part, this would be the strength caused by the bending moment:

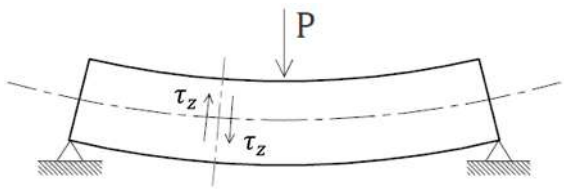


$$\sigma_x = \frac{M_y \cdot z}{I_y}$$

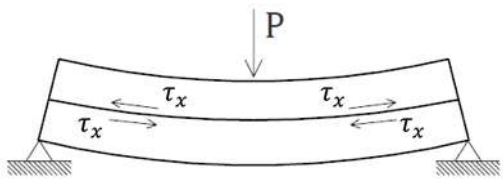
5.1.3. Tangential stress due to shear strength

The shear strength causes two different types of tangential stress:

- Cross tangential stress τ_z, perpendicular to the length axis of the bar.
- Longitudinal tangential stress τ_x, parallel to the length axis.

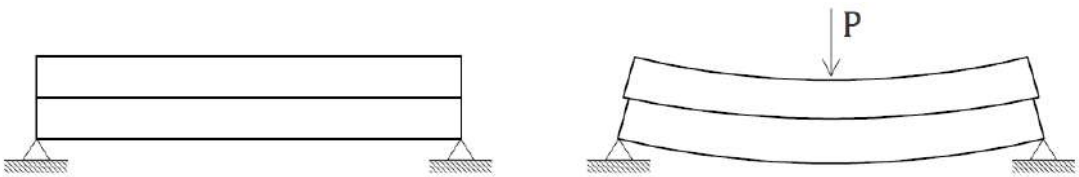


Cross tangential stress

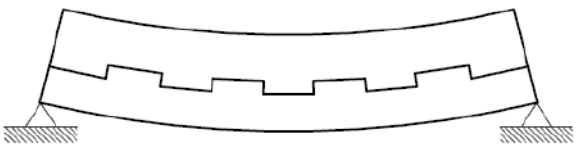


Longitudinal tangential stress

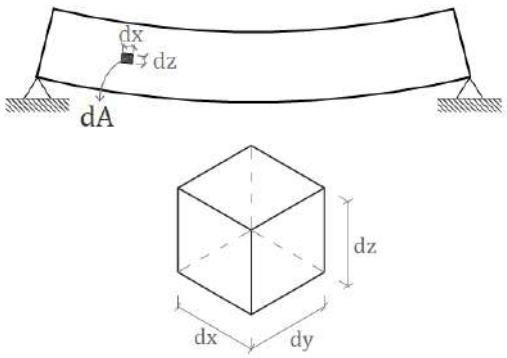
To understand the impact of the longitudinal tangential stress we will analyse these two bars, one on top of the other.

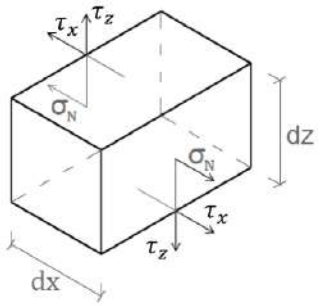


Taking into account the longitudinal deformation, we can see that there is a displacement between those two bars. If we could attach the bars we would prevent the displacement. Therefore, the aim of the longitudinal tension is to obstruct that displacement:



First of all we will prove that the longitudinal and cross stresses are equal (Cauchy's expression):





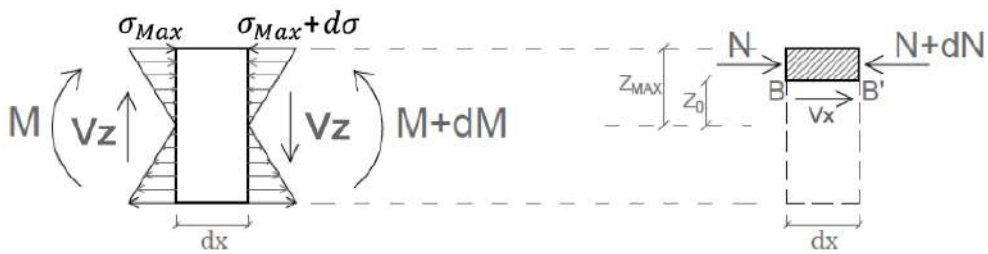
Cubic differential element:

$$dx = dy = dz$$

$$\Sigma M_0 = 0 \; ; \; \tau_z \cdot dA \cdot dx + \sigma_N \cdot dA \cdot \frac{dz}{2} = \tau_x \cdot dA \cdot dz + \sigma_N \cdot dA \cdot \frac{dz}{2} \; ; \; \boxed{\tau_z = \tau_x}$$

This means that a differential element working under simple flection has equal values for longitudinal and cross stresses.

To define the relation between the shear strength and the tangential stress, we are going to analyse a differential element of a beam working under simple flection. The cross section will be rectangular. We have to consider that the bending moment is variable (if it was constant, it would be working under pure bending moment, and the shear strength would be null).



As the bending moments are different, the tensions they produce are going to be different too:

$$M < M + dM \; ; \; \sigma < \sigma + d\sigma$$

Then, when isolating the B-B' differential element, in order to be balanced we have to take into account the strengths produced by the longitudinal sections:

$$V_x + N = N + dN$$

Having these strengths:

$$N = \int_{Z_0}^{Z_{MAX}} \sigma \cdot dz$$

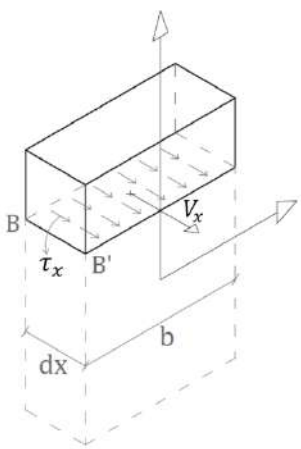
$$N + dN = \int_{Z_0}^{Z_{MAX}} (\sigma + d\sigma) \cdot dz$$

$$V_x = \int_A \tau_x \cdot dA$$

V_x longitudinal shear strength is going to change along the beam's length, and is also going to change in each fibre of the section with the normal stress. To calculate its value we are going to solve the equilibrium equation of $B \cdot B'$ differential element:

$$V_x + N = N + dN \; ; V_x = dN \; ; V_x = \int_{Z_0}^{Z_{MAX}} d\sigma \cdot dz \; ; V_x = \int_{Z_0}^{Z_{MAX}} \frac{dM \cdot z}{I_y} \cdot dz \; ;$$

$$V_x = \frac{dM}{I_y} \cdot \int_{Z_0}^{Z_{MAX}} z \cdot dz \rightarrow \int_{Z_0}^{Z_{MAX}} z \cdot dz = S_y^{(z_0 - z_{\max})} \rightarrow \boxed{V_x = \frac{dM \cdot S_y^{(z_0 - z_{\max})}}{I_y}}$$



Now we are going to link V_x longitudinal strength and τ_x longitudinal tangential stress:

$$V_x = \int_A \tau_x \cdot dA = \tau_x \cdot (d_x \cdot b); \boxed{\tau_x = \frac{V_x}{d_x \cdot b}}$$

And putting these two expressions together:

$$\left.\begin{aligned} V_x &= \frac{dM \cdot S_y^{(z_0 - z_{\max})}}{I_y} \\ \tau_x &= \frac{V_x}{d_x \cdot b} \end{aligned}\right\} \tau_x = \frac{dM}{d_x} \cdot \frac{S_y^{(z_0 - z_{\max})}}{I_y \cdot b}$$

And as we already know:

$$V_z = \frac{dM}{d_x} \rightarrow V_z : Shear\ strength$$

Then:

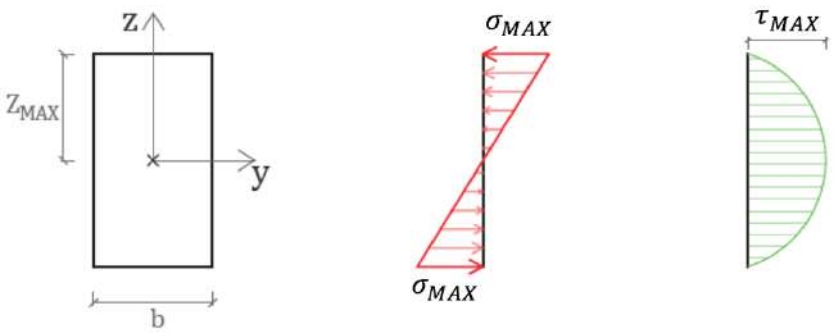
$$\tau_x = \tau_z = \frac{dM}{d_x} \cdot \frac{S_y^{(z_0 - z_{\max})}}{I_y \cdot b} = \frac{V_z \cdot S_y^{(z_0 - z_{\max})}}{I_y \cdot b}$$

- $\tau_x : Longitudinal\ tangential\ stress$
- $\tau_z : Cross\ tangential\ stress$
- $V_z : Shear\ strength$
- $S_y^{(z_0 - z_{\max})} : Analysing\ fibre´s\ above\ area´s\ Static\ Moment$
- $I_y : Analysing\ section´s\ Inerti\ Moment$
- $b : Analysing\ section´s\ thickness$

In this expression, we normally don't use the z subindex in the cross tangential stress and the shear strentgh, because it is suposed to be known. In the same way, we don't use supraindex in the static moment S_y. Therefore, we can write this expression in a simplified way:

$$\boxed{\tau = \frac{V \cdot S_y}{I_y \cdot b}} \text{ Collignon} - \text{en formula}$$

Summarizing the information we've learnt, these are the stress diagrams for a rectangular cross section working under bending moement:



$$\sigma_{MAX} = \frac{M_y \cdot Z_{MAX}}{I_y} = \frac{M_y}{W_y} \qquad \tau_{MAX} = \frac{V \cdot S_y}{b \cdot I_y}$$

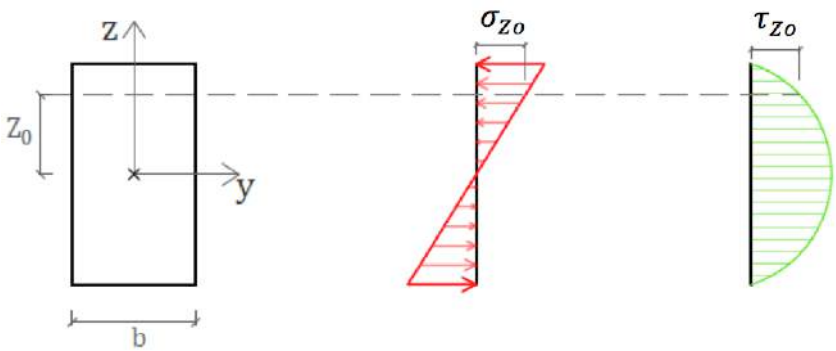
5.1.4. Stress combination (Von Mises)

In sections working under simple flection, when we have σ_{MAX} , τ is null. And viceversa, when we have τ_{MAX}, σ is null.

However, on some of the fibres we have both types of tensions. To quantify the impact of the combination of these two tensions we can use Von Mises' expression:

$$\sigma_{CO} = \sqrt{{\sigma_{Zo}}^2 + 3{\tau_{Zo}}^2}$$

This would be the graphical expression of Von Mises' formula



σ_{co} is a value of Normal stress, and therefore, we have to verify with the material's normal yield strength:

$$\sigma_{CO} \leq f_{yD} \left(f_{yD} : \text{Material´s yield strength}\right)$$

When the Normal stress is null, we will use this expression to define the tangential yield strength:

$$\sigma_{CO} \leq f_{yD}\ ;\ \sqrt{\sigma^2 + 3\tau^2} \leq f_{yD}\ ;\ \tau \cdot \sqrt{3} \leq f_{yD}\ ;\ \boxed{\tau \leq \frac{f_{yD}}{\sqrt{3}}}$$ Tangential yield strength

6. STEEL STRUCTURES

6.1. INTRODUCTION. ROLLED STEEL

The steel comes from the alloy between iron and carbon, which is achieved in a blast furnace. This iron and carbon alloy has a lot of advantages, that's why is used in construction:

- High elastic limit.
- It's a homogeneous material.
- Allows a fast assembly.
- It offers a high range of variety in profiles and pieces.
- It can be easily reused (unlike concrete).
- It doesn´t suffer a lot from rheological actions (except from thermal actions).
- Is a good material for rehabilitation and reinforcement works.
- Is pliable and ductile.

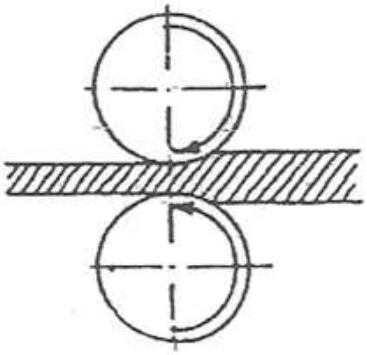
However, it also has some disadvantages against concrete and other different materials:

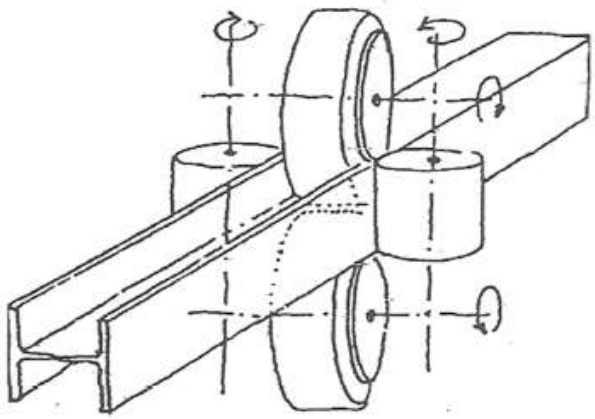
- It's an expensive material.
- It has an unstable behaviour against fire.
- Stability problems (buckling, dent…).
- It's corrosive.
- To unite different steel pieces we have to weld them, which can´t be well controlled on site.

In spite of being the same steel alloy, we can find three different ways of producing the pieces: glued, forged and rolled. These days we just work with the **rolled steel**, because it offers better properties and characteristics.

6.2. ROLLED STEEL'S MECHANIC PROPERTIES

This type of steel is the most used in construction. To produce it, they first heat it in a blast furnace and while is still warm they give the shape they want by passing it through some rolls. This way the steel receives the form of a determined profile.

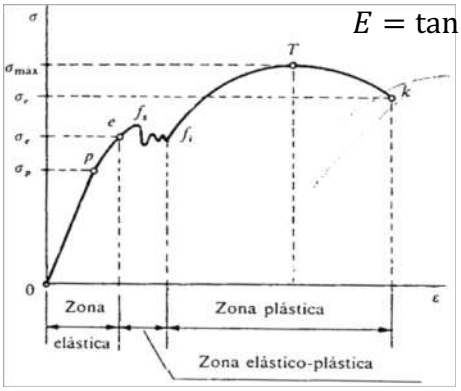




Among the physical and mechanical properties of the rolled steel these are the most important ones:

- Proportionality
- Elastic limit
- Plastic behaviour
- Spilled
- Rupture

$$E = \tan\beta = \frac{\delta_\vartheta}{\delta_\varepsilon}$$

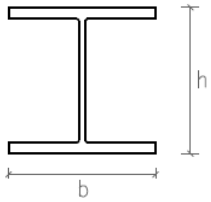


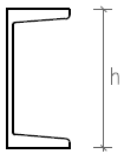
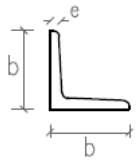
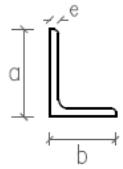
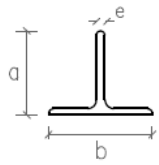
Inside the same steel alloy we can also make small changes to modify some of its properties.

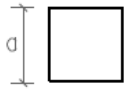
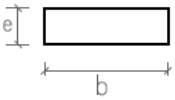
DESIGNACIÓN	Espesor nominal t (mm)				Temperatura del ensayo Charpy °C
	Tensión de límite elástico f_y (N/mm^2)			Tensión de rotura f_u (N/mm^2)	
	$t \leq 16$	$16 < t \leq 40$	$40 < t \leq 63$	$3 \leq t \leq 100$	
S235JR					20
S235J0	235	225	215	360	0
S235J2					-20
S275JR					20
S275J0	275	265	255	410	0
S275J2					-20
S355JR					20
S355J0	355	345	335	470	0
S355J2					-20
S355K2					-20 (1)
S450J0	450	430	410	550	0

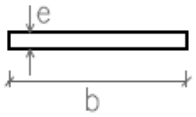
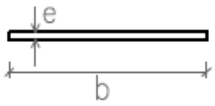
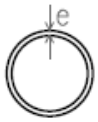
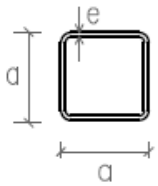
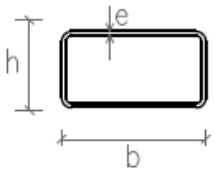
(1) Se le exige una energía mínima de 40J.

6.3. PROFILE TYPES

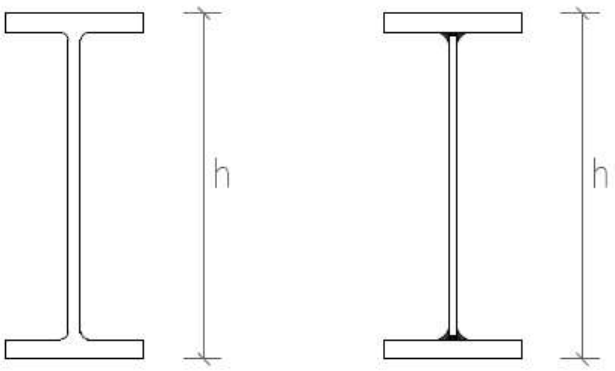
PROFILE TYPE	NOMENCLATURE	DEFINITION	DIMENSIONS
Flection (beams and joists)			
	IPN	h	$80 \leq h \leq 600$
	IPE	h	$80 \leq h \leq 600$
Compression and bending-compression (columns)			
	IPN h=b HEB b>h HEM b<h	h	$100 \leq h \leq 300$
flection (rarely) and Composed columns			

	UPN	h	$80 \leq h \leq 300$
Helping and reinforcing profiles			
	IPN	L b.e	$20 \leq b \leq 200$ $3 \leq e \leq 13$
	LD	L a.b.e	$80 \leq a \leq 150$ $50 \leq b \leq 90$ $8 \leq e \leq 13$
Joists (in small gaps)			
	T (a = b)	T b.e	$30 \leq a \leq 100$ $4 \leq e \leq 11$

PROFILE TYPE	NOMENCLATURE	DEFINITION
solid profiles		
	Circular shaped section: Big: $5 \leq \emptyset \leq 200\ mm$ Small: $\emptyset < 5\ mm$	Ø
	Square shaped section: $8 \leq a \leq 100\ mm$	a
	Rectangular shaped section: Thick: $4 \leq e \leq 10\ mm$ Thin: $e > 10\ mm$	b.e

	Plate: $e = 4\ mm$ $150 \leq b \leq 400\ m$	b.e
	Sheet ($b > 400\ mm$): Thin: $e \leq 4\ mm$ Thick: $e > 4\ mm$	b.e
Empty profiles		
	Circular shaped section: $50 \leq \emptyset \leq 200\ mm$; $2 \leq e \leq 9\ mm$ $\emptyset > 200$ con acero A-52 ST	Ø.e
	Square shaped section: $40 \leq a \leq 175\ mm$ $2 \leq e \leq 9\ mm$	a.e
	Rectangular shaped section: $50x30 \leq bxh \leq 250x100\ mm$ $2 \leq e \leq 9\ mm$	b.h.e

Rolled beam vs. forged beam



In some cases the profile we need for our solution is not available in the wide steel profile market because we need specific properties (size, resistance, stability…). If that happens we also have the possibility of creating a custom profile with the exact properties we need in our project. This way we optimize the amount of material, but it also has a higher economic cost. The project and structure designer is going to be the one in charge to decide which solution is the best for each case.

7. CTE – LIMIT STATES METHOD IN STEEL STRUCTURES

The CTE uses the limit state method to verify the building structure. It identifies two limit states: Serviceability limit state (SLS) and Ultimate limit state (ULS). In this course and also in these notes we will learn how to verify the most important state limits for steel structures.

7.1. CTE'S NOMENCLATURE

As we have seen in the chapter 3, each Limit State has a specific expression to establish the loading.

In the case of steel, these are the most important verifications:

- **SLS - Serviceability limit state**
  - Deflection
  - Horizontal deflection

- **ULS - Ultimate limit state**
  - Section's resistance
    - Normal stress
    - Tangential stress
    - Combined stress
  - Bar's stability
    - Buckling
    - Lateral torsional buckling
    - Dent
    - Flexo-compression

The main difference between SLS and ULS is the use of security factors. When verifying Serviceability Limit States magnification factors are not used. We call **characteristic values**, the values of loads and strengths without magnification factors. Strengths in characteristic values have subindex “K”.

When verifying Ultimate Limit States, the regulation applies an additional security measure: the application of magnification factors in loads, and reduction factors in materiel’s yield strength.

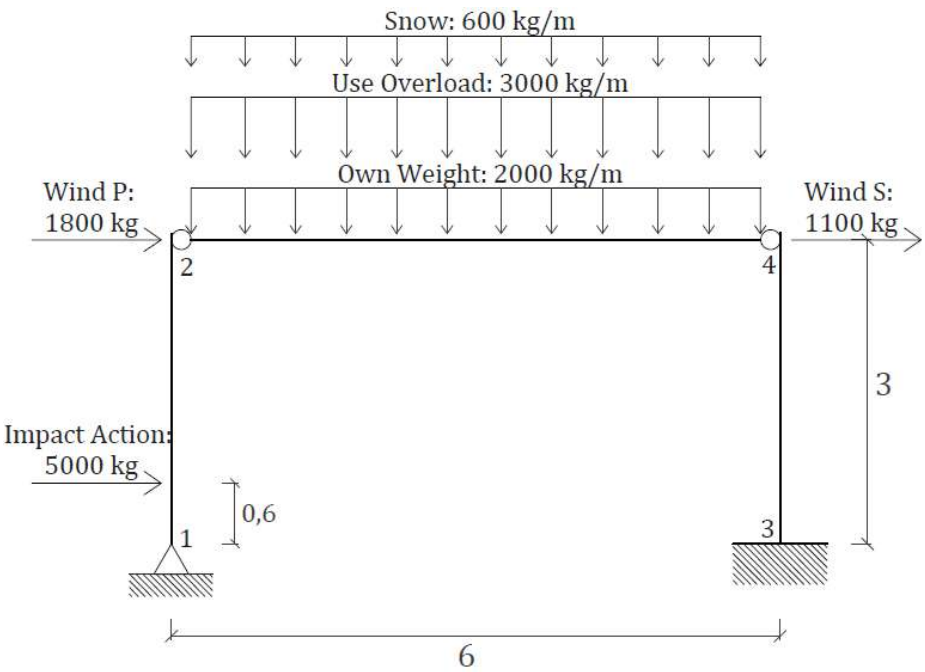
We call **calculation values**, the values of loads and strengths with magnification factors. Strengths in calculation values have subindex “ED”.

Materials’ yield strength are represented with the f symbol. When applying reduction factor to the materials we introduce subindex “D”. In the case of the steel, the symbol for the yield strength with reduction factor is f_{yD}($f_{cD}$for the concrete).

7.2. SERVICEABILITY LIMIT STATE

In this chapter, in the following exercise we will learn how to calculate and verify the vertical and horizontal deflection of a structural frame. As we will see, this verification can also be used as a pre dimensioning method. If we know the maximum deflection allowed for our structure we can determine the profile we need to prevent it. This pre dimensioning method along with others can be found in the annexes of these notes (Annex 1: pre dimensioning methods).

Example. SLS (Serviceability Limit State). Using the load combination explained in the CTE regulation, dimension the beam and the column 3-4 of the following frame (The beam with an IPE profile and the column with a HBE profile).

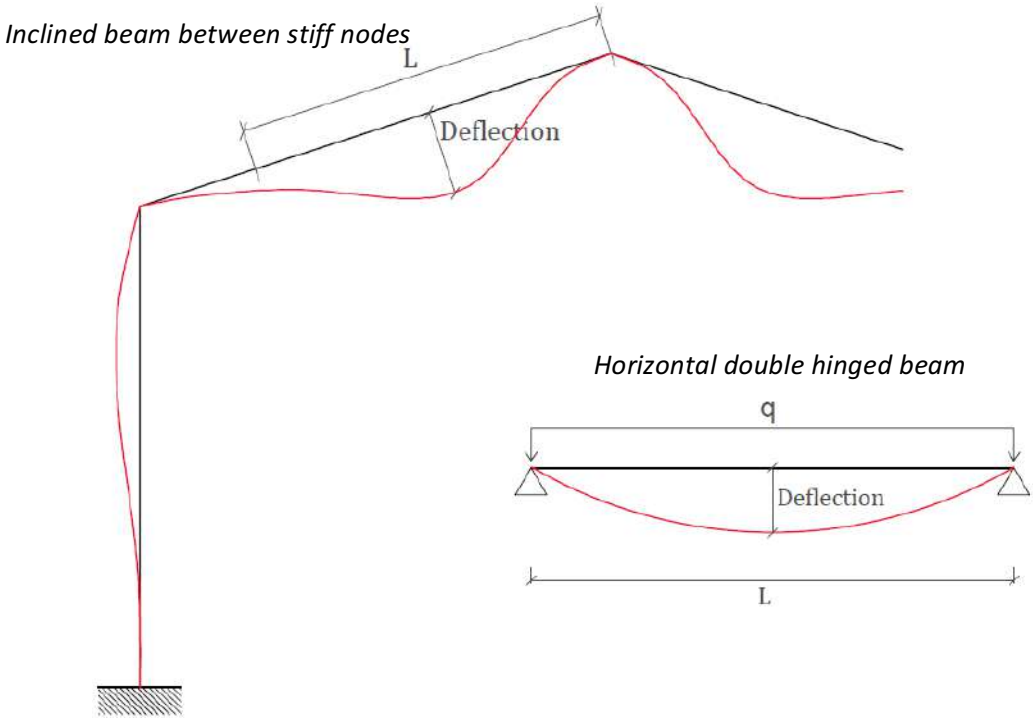


Data:

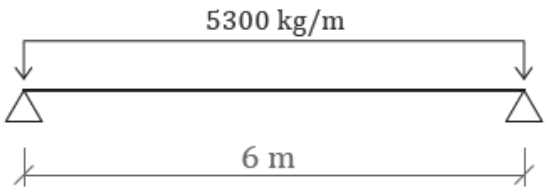
- S275 Steel.
- Beam's deflection in the middle of the beam $\delta = L/500$; L being the beam length.
- Column's horizontal deflection $\delta = H/250$; H being the column height

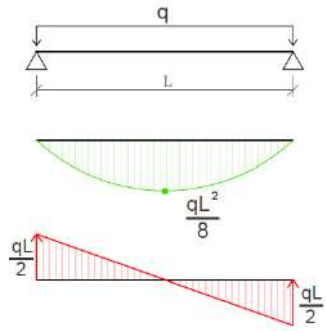
7.2.1. Vertical deflection

Vertical deflection is the maximum deflection in horizontal or inclined beams.



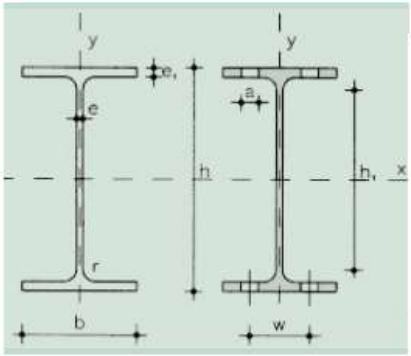
After analysing the load combination we know that the worst hypothesis for the beam is the following one:





$$\delta_{MAX} = {}^{L}/_{500} = {}^{600}/_{500} = 1{,}2\ cm$$

$$\delta = \frac{5}{384} \cdot \frac{ql^4}{EI_y};\ I_y = \frac{5}{384} \cdot \frac{ql^4}{E \cdot \delta_{max}} = \frac{5 \cdot 53 \cdot 600^4}{384 \cdot 2{,}1 \cdot 10^6 \cdot 1{,}2} = 35491 cm^4 \rightarrow \boxed{IPE\ 500}$$

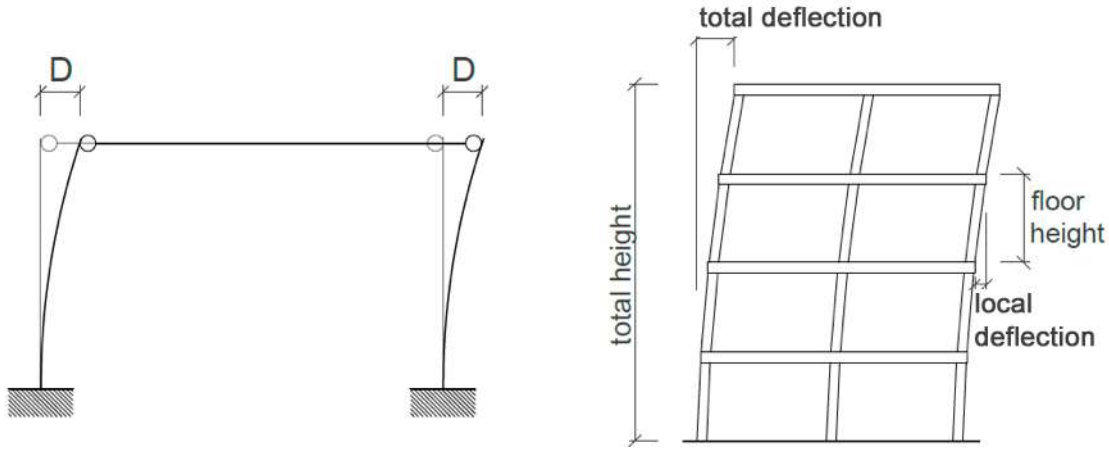


A = Section's area
S_x = Half section's static moment against X axis
I_x = Section's inertia moment against X axis
$W_x = 2I_x : h$. Section's resistant module against X axis
$i_x = \sqrt{I_x : A}$. Section's rotation radious against X axis
I_y = Section's inertia moment against Y axis
$W_y = 2I_y : b$. Section's resistant module against Y axis
$i_y = \sqrt{I_y : A}$. Section's rotation radious against Y axis
I_t = Section's torsion module
I_w = Section's warping module
u = Section's perimeter
a = Rivet's hole diameter
w = Distance between holes' axis
h_1 = Soul's flat part's height
p = Weight per meter

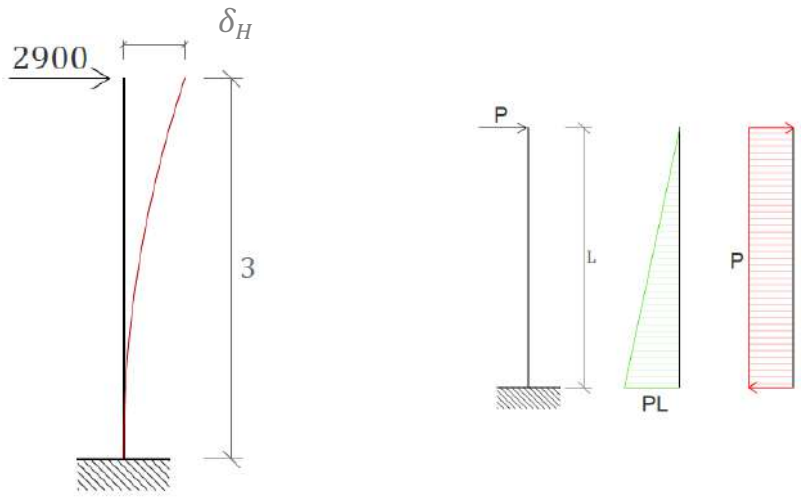
Profile	Dimensions							Section's terms										Weight
	h mm	b mm	e mm	e_1 mm	r_1 mm	h_1 mm	u mm	A cm^2	S_x cm^3	I_x cm^4	W_x cm^3	i_x cm	I_y cm^4	W_y cm^3	i_y cm	I_t cm^4	I_w cm^6	p kp/m
IPE 80	80	46	3,8	5,2	5	60	328	7,64	11,6	80,1	20,0	3,24	8,49	3,69	1,05	0,721	118	6,00
IPE 100	100	55	4,1	5,7	7	75	400	10,30	19,7	171,0	34,2	4,07	15,90	5,79	1,24	1,140	351	8,10
IPE 120	120	64	4,4	6,3	7	93	475	13,20	30,4	318,0	53,0	4,90	27,70	8,65	1,45	1,770	890	10,40
IPE 140	140	73	4,7	6,9	7	112	551	16,40	44,2	541,0	77,3	5,74	44,90	12,30	1,65	2,630	1.981	12,90
IPE 160	160	82	5,0	7,4	9	127	623	20,10	61,9	869,0	109,0	6,58	68,30	16,70	1,84	3,640	3.959	15,80
IPE 180	180	91	5,3	8,0	9	146	698	23,90	83,2	1.320,0	146,0	7,42	101,00	22,20	2,05	5,060	7.431	18,80
IPE 200	200	100	5,6	8,5	12	159	788	28,50	110,0	1.940,0	194,0	8,26	142,00	28,50	2,24	6,670	12.990	22,40
IPE 220	220	110	5,9	9,2	12	178	848	33,40	143	2.770	252	9,11	205	37,3	2,48	9,15	22.670	26,20
IPE 240	240	120	6,2	9,8	15	190	922	39,10	183	3.890	324	9,97	284	47,3	2,69	12,00	37.390	30,70
IPE 270	270	135	6,6	10,2	15	220	1.040	45,90	242	5.790	429	11,20	420	62,2	3,02	15,40	70.580	36,10
IPE 300	300	150	7,1	10,7	15	249	1.160	53,80	314	8.360	557	12,50	604	80,5	3,35	20,10	125.900	42,20
IPE 330	330	160	7,5	11,5	18	271	1.250	62,60	402	11.770	713	13,70	788	98,5	3,55	26,50	199.100	49,10
IPE 360	360	170	8,0	12,7	18	299	1.350	72,70	510	16.270	904	15,00	1.040	123,0	3,79	37,30	313.600	57,10
IPE 400	400	180	8,6	13,5	21	331	1.470	84,50	654	23.130	1.160	16,50	1.320	146,0	3,95	48,30	490.000	66,30
IPE 450	450	190	9,4	14,6	21	379	1.610	98,80	851	33.740	1.500	18,50	1.680	176,0	4,12	65,90	791.000	77,60
IPE 500	500	200	10,2	16,0	21	426	1.740	116,00	1.100	48.200	1.930	20,40	2.140	214,0	4,31	91,80	1.249.000	90,70
IPE 550	550	210	11,1	17,2	24	468	1.880	134,00	1.390	67.120	2.440	22,30	2.670	254,0	4,45	122,00	1.884.000	106,00
IPE 600	600	220	12,0	19,0	24	514	2.010	155,00	1.760	92.080	3.070	24,30	3.390	308,0	4,66	172,00	2.846.000	122,0

7.2.2. Horizontal deflection

The relative horizontal displacement of the ends of a column is called horizontal deflection. Overall, this movement is generated by the wind load. We will have to verify the total deflection (comparing the total displacement with the total height of the building) and the local deflection (comparing the local displacement with the height of just one floor).

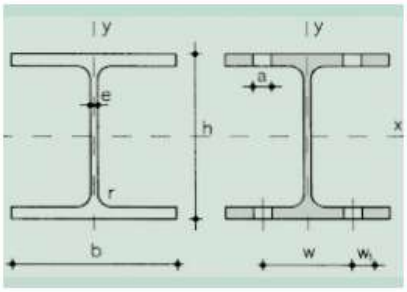


After analysing the load combinations of this frame, we know that the worst hypothesis is the wind load:



$$\delta_{MAX} = {}^{H}/_{250} = {}^{300}/_{250} = 1{,}2\ cm$$

$$\delta_H = \frac{Pl^3}{3EI_y};\ I_y = \frac{P \cdot l^3}{3 \cdot E \cdot \delta_{MAX}} = \frac{2900 \cdot 300^3}{3 \cdot 2{,}1 \cdot 10^6 \cdot 1{,}2} = 10357\ cm^4 \rightarrow \boxed{HEB\ 240}$$

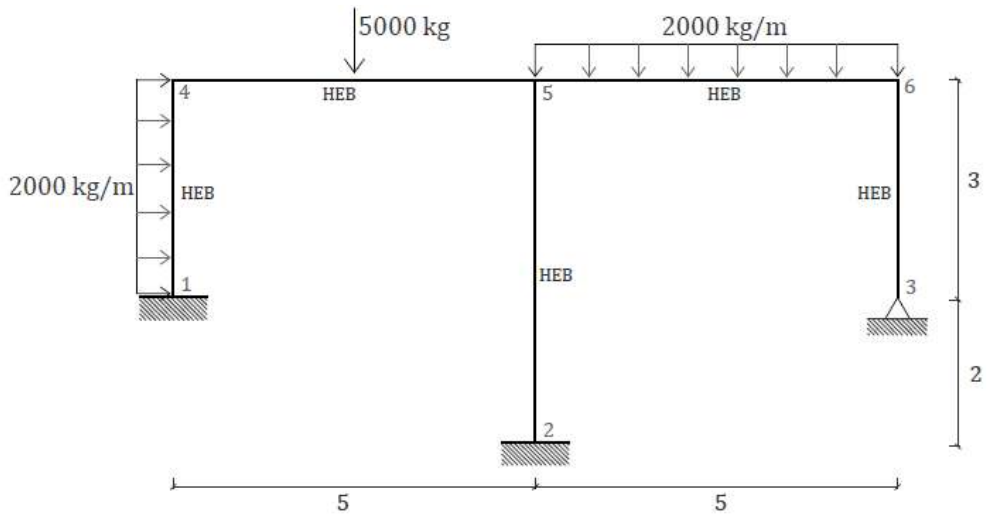


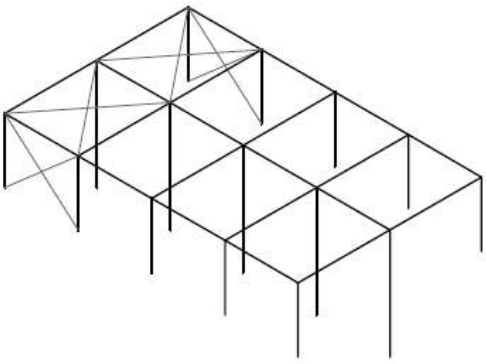
A = Section's area
S_x = Half section's static moment against X axis
I_x = Section's inertia moment against X axis
$W_x = 2I_x : h$. Section's resistant module against X axis
$i_x = \sqrt{I_x : A}$. Section's rotation radious against X axis
I_y = Section's inertia moment against Y axis
$W_y = 2I_y : b$. Section's resistant module against Y axis
$i_y = \sqrt{I_y : A}$. Section's rotation radious against Y axis
I_t = Section's torsion module
I_a = Section's warping module
u = Section's perimeter
a = Rivet's hole diameter
w = Distance between holes' axis
h_1 = Soul's flat part's height
p = Weight per meter

Profile	Dimensions							Section's terms										Weight
	h mm	b mm	e mm	e_1 mm	r_1 mm	h_1 mm	u mm	A cm^2	S_x cm^3	I_x cm^4	W_x cm^3	i_x cm	I_y cm^4	W_y cm^3	i_y cm	I_t cm^4	I_a cm^6	p kp/m
HEB 100	100	100	6,0	10,0	12	56	567	26,0	52,1	450	90	4,16	167	33	2,53	9,34	3.375	20,4
HEB 120	120	120	6,5	11,0	12	74	686	34,0	82,6	864	144	5,04	318	53	3,06	14,90	9.410	26,7
HEB 140	140	140	7,0	12,0	12	92	805	43,0	123,0	1.509	216	5,93	550	79	3,58	22,50	22.480	33,7
HEB 160	160	160	8,0	13,0	15	104	918	54,3	177,0	2.492	311	6,78	889	111	4,05	33,20	47.940	42,6
HEB 180	180	180	8,5	14,0	15	122	1.040	65,3	241,0	3.831	426	7,66	1.363	151	4,57	46,50	93.750	51,2
HEB 200	200	200	9,0	15,0	18	134	1.150	78,1	321,0	5.696	570	8,54	2.003	200	5,07	63,40	171.100	61,3
HEB 220	220	220	9,5	16,0	18	152	1.270	91,0	414,0	8.091	736	9,43	2.843	258	5,59	84,40	295.400	71,5
HEB 240	240	240	10,0	17,0	21	164	1.380	106,0	527,0	11.259	938	0,30	3.923	327	6,08	110,00	486.900	83,2
HEB 260	260	260	10,0	17,5	24	177	1.500	118,4	641,0	14.919	1.150	11,20	5.135	395	6,58	130,00	753.700	93,0
HEB 280	280	280	10,5	18,0	24	196	1.620	131,4	767,0	19.270	1.380	12,10	6.595	471	7,09	153,00	1.130.000	103,0
HEB 300	300	300	11,0	19,0	27	208	1.730	149,1	934,0	25.166	1.680	13,00	8.563	571	7,58	192,00	1.688.000	117,0
HEB 320	320	300	11,5	20,5	27	225	1.770	161,3	1.070,0	30.823	1.930	13,80	9.239	616	7,57	241,00	2.069.000	127,0
HEB 340	340	300	12,0	21,5	27	243	1.810	170,9	1.200,0	36.656	2.160	14,60	9.690	646	7,53	278,00	2.454.000	134,0
HEB 360	300	300	12,5	22,5	27	261	1.850	180,6	1.340,0	43.193	2.400	15,50	10.140	676	7,49	320,00	2.883.000	142,0
HEB 400	400	300	13,5	24,0	27	298	1.930	197,8	1.620,0	57.680	2.880	17,10	10.819	721	7,40	394,00	3.817.000	155,0
HEB 450	450	300	14,0	26,0	27	344	2.030	218,0	1.990,0	79.887	3.550	19,10	11.721	781	7,33	500,00	5.258.000	171,0
HEB 500	500	300	14,5	28,0	27	390	2.120	238,6	2.410,0	107.176	4.290	21,20	12.624	842	7,27	625,00	7.018.000	187,0
HEB 550	550	300	15,0	29,0	27	438	2.220	254,1	2.800,0	136.691	4.970	23,20	13.077	872	7,17	701,00	8.856.000	199,0
HEB 600	600	300	15,5	30,0	27	486	2.320	270,0	3.210,0	171.041	5.700	25,20	13.530	902	7,08	783,00	10.965.000	212,0

7.2.3. Examples

Dimension the beams and columns of this frame using the same HBE profile for all of them. Verify the serviceability limit states (SLS).





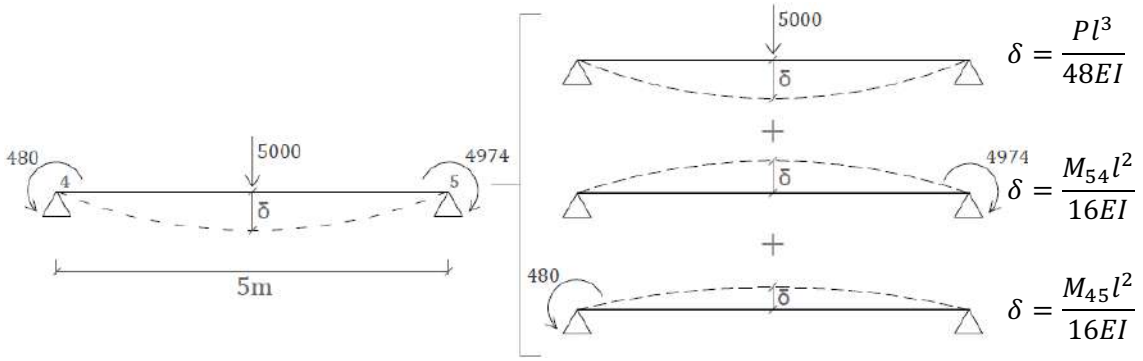
1	4		5			6		2	3
14	41	45	54	52	56	65	63	25	36
4432	-480	480	-4974	1043	3931	-3586	3586	1393	0

Data:

- S275 steel.
- Don´t use simultaneity factors, only one load combination.
- Beam's deflection in the middle of the beam $\delta = L/500$; L being the beam length.
- Column's horizontal deflection $\delta = H/250$; H being the column height

Beam's maximum deflection

4-5 beam:



$$\delta = \frac{Pl^3}{48EI}$$

$$\delta = \frac{M_{54}l^2}{16EI}$$

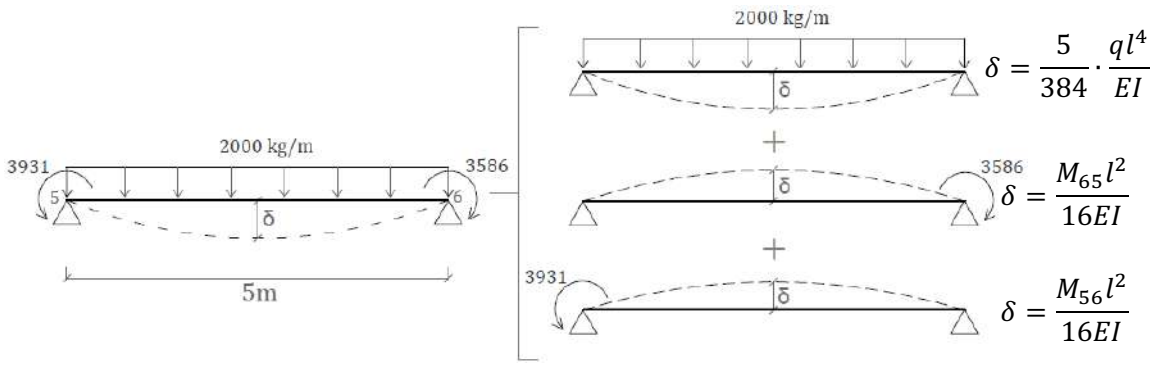
$$\delta = \frac{M_{45}l^2}{16EI}$$

$$\delta_{MAX} = {}^{L}/_{500} = {}^{500}/_{500} = 1\ cm$$

$$\delta = \frac{Pl^3}{48EI} - \frac{M_{54}l^2}{16EI} - \frac{M_{45}l^2}{16EI} = \frac{5000 \cdot 500^3}{48 \cdot 2{,}1 \cdot 10^6 \cdot I} - \frac{497400 \cdot 500^2}{16 \cdot 2{,}1 \cdot 10^6 \cdot I} - \frac{48000 \cdot 500^2}{16 \cdot 2{,}1 \cdot 10^6 \cdot I}$$

$$\delta = \frac{2142{,}36}{I} \rightarrow I_{45} = \frac{2142{,}36}{1} = \boxed{2142{,}36\ cm^4}$$

5-6 beam:



$$\delta = \frac{5}{384} \cdot \frac{ql^4}{EI}$$

$$\delta = \frac{M_{65}l^2}{16EI}$$

$$\delta = \frac{M_{56}l^2}{16EI}$$

$$\delta_{MAX} = {}^{L}/_{500} = {}^{500}/_{500} = 1\ cm$$

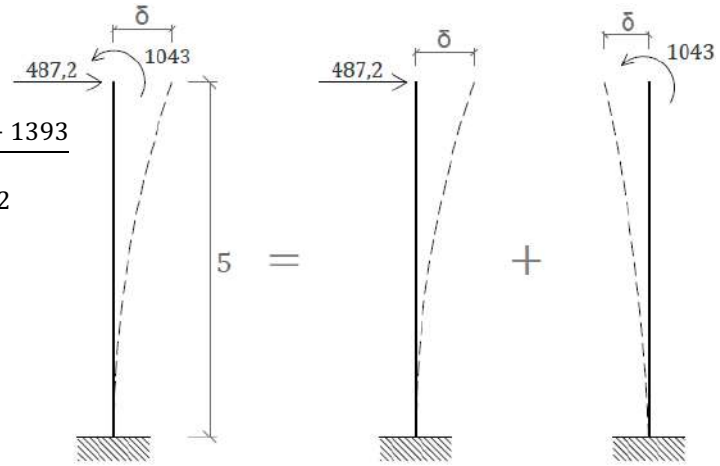
$$\delta = \frac{5}{384} \cdot \frac{ql^4}{EI} - \frac{M_{65}l^2}{16EI} - \frac{M_{56}l^2}{16EI} = \frac{5 \cdot 20 \cdot 500^4}{384 \cdot 2{,}1 \cdot 10^6 \cdot I} - \frac{358600 \cdot 500^2}{16 \cdot 2{,}1 \cdot 10^6 \cdot I} - \frac{393100 \cdot 500^2}{16 \cdot 2{,}1 \cdot 10^6 \cdot I}$$

$$\delta = \frac{2157}{I} \rightarrow I_{56} = \frac{2157}{1} = \boxed{2157\ cm^4}$$

Column's maximum deflection

2-5 column:

$$\frac{1043 + 1393}{5} = 487{,}2$$



$$\delta = \frac{Pl^3}{3EI} \qquad \delta = \frac{M \cdot a}{2EI}(2l - a) = \frac{M \cdot 3l}{2EI}(2l - l) = \frac{ml^2}{2EI}$$

$$\delta_{MAX} = {}^{H}/_{250} = {}^{300}/_{250} = 1{,}2\ cm \longrightarrow$$

The deflection of all the columns is going to be the same, that's why we have to choose the shortest to define the limit of deflection, because that is going to be the most restrictive.

$$\delta = \frac{Pl^3}{3EI} - \frac{M_{52}l^2}{2EI} = \frac{487{,}2 \cdot 500^3}{3 \cdot 2{,}1 \cdot 10^6 \cdot I} - \frac{104300 \cdot 500^2}{2 \cdot 2{,}1 \cdot 10^6 \cdot I}$$

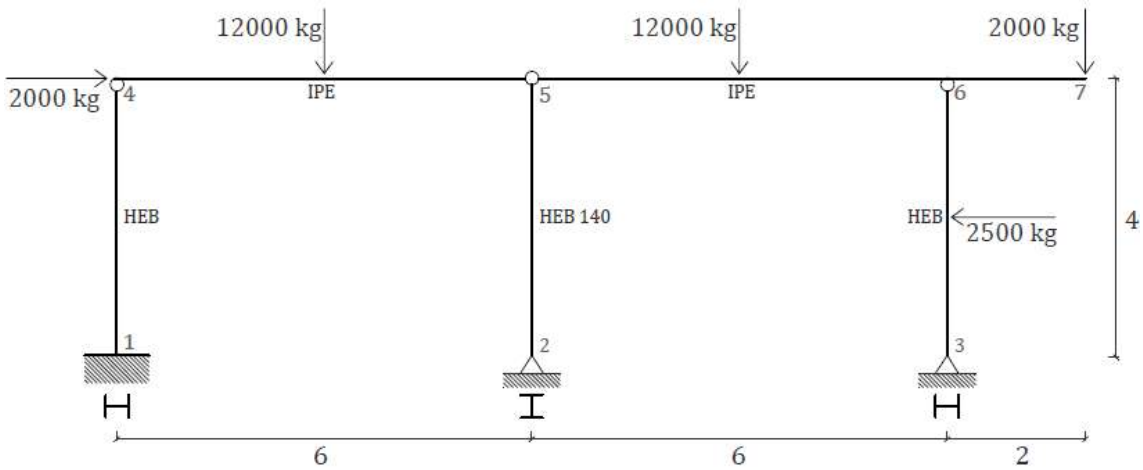
$$\delta = \frac{3454{,}4}{I} \rightarrow I_{25} = \frac{3454{,}4}{1{,}2} = \boxed{2879\ cm^4}$$

$$\left.\begin{array}{l} I_{45} = 2142\ cm^4 \\ I_{56} = 2157\ cm^4 \\ I_{25} = 2879\ cm^4 \end{array}\right\}$$

As all the profiles have to be the same, we dimension it taking the highest inertia:

$$\boxed{I = 2879\ cm^4\ \rightarrow HEB\ 180}$$

Exercise. Draw the frame's diagrams and dimension all the beams and column 1-4 according to Serviceability limit states (SLS).

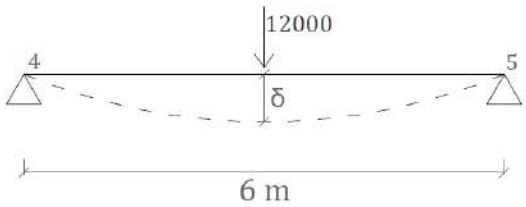


Data:

- S275 steel.
- Don´t use simultaneity factors, only one load combination.
- Beam's deflection in the middle of the beam $\delta = L/500$; L being the beam length.
- Column's horizontal deflection $\delta = H/250$; H being the column height

Beam´s maximum deflection

4-5 beam:

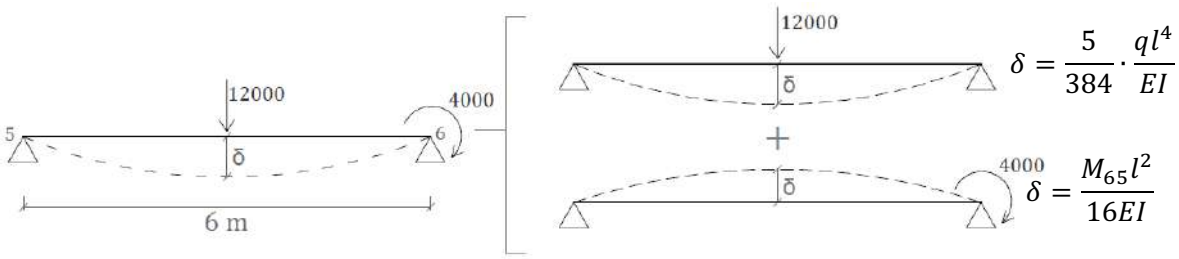


$$\delta = \frac{Pl^3}{48EI}$$

$$\delta_{MAX} = {}^{L}/_{500} = {}^{600}/_{500} = 1{,}2\ cm$$

$$\delta = \frac{Pl^3}{48EI} = \frac{12000 \cdot 600^3}{48 \cdot 2{,}1 \cdot 10^6 \cdot I} = \frac{25714{,}29}{I} \rightarrow I_{45} = \frac{25714{,}29}{1{,}2} = 21428{,}57\ cm^4 \ \rightarrow \boxed{IPE\ 400}$$

5-6 beam:



$$\delta = \frac{5}{384} \cdot \frac{ql^4}{EI}$$

$$\delta = \frac{M_{65}l^2}{16EI}$$

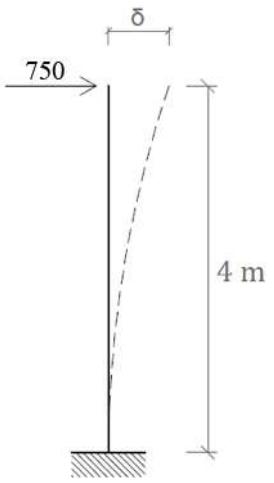
$$\delta_{MAX} = {}^{L}/_{500} = {}^{600}/_{500} = 1{,}2\ cm$$

$$\delta = \frac{Pl^3}{48EI} - \frac{M_{65}l^2}{16EI} = \frac{12000 \cdot 600^3}{48 \cdot 2{,}1 \cdot 10^6 \cdot I} - \frac{400000 \cdot 600^2}{16 \cdot 2{,}1 \cdot 10^6 \cdot I}$$

$$\delta = \frac{21428{,}57}{I} \rightarrow I_{56} = \frac{21428{,}57}{1{,}2} = 17857{,}14\ cm^4 \ \rightarrow \boxed{IPE\ 400}$$

Column's maximum deflection

<u>1-4 column:</u>



$$\delta = \frac{Pl^3}{3EI}$$

$$\delta_{MAX} = {}^{H}/_{250} = {}^{400}/_{250} = 1{,}6\ cm$$

$$\delta = \frac{Pl^3}{3EI} = \frac{750 \cdot 400^3}{3 \cdot 2{,}1 \cdot 10^6 \cdot I} = \frac{7619{,}05}{I} \rightarrow I_{14} = \frac{7619{,}05}{1{,}6} = 4761{,}9\ cm^4 \rightarrow \boxed{HEB\ 200}$$

7.3. ULTIMATE LIMIT STATE

7.3.1. Section's resistance

In this chapter, along some examples, we will apply the theory learnt in chapter 5 for verifying normal and tangential stress in cross sections. For this, we will use the nomenclature and the formulas of the CTE.

- CTE's formula for verifying normal stress:

$$\frac{N_{ED}}{A} + \frac{M_{y,ED}}{W_{y,EL}} + \frac{M_{z,ED}}{W_{z,EL}} \leq f_{yD}$$

  - N_{ED} : $Axial\ strength\ in\ calculation\ value$
  - $M_{y,ED}$: $Bending\ moment\ in\ calculation\ value\ about\ "y"\ axis$
  - $M_{z,ED}$: $Bending\ moment\ in\ calculation\ value\ about\ "z"\ axis$
  - f_{yD} : $Steel's\ yield\ strength\ in\ calculation\ value$
  - $W_{y,EL}$: $Elastic\ resistance\ module\ about\ "y"\ axis$
  - $W_{z,EL}$: $Elastic\ resistance\ module\ about\ "z"\ axis$
  -

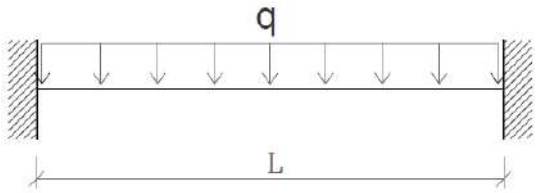
- CTE's formula for verifying tangential stress :

$$\frac{V_{ED} \cdot S_y}{I_y \cdot b} \leq \frac{f_{yD}}{\sqrt{3}}$$

  - V_{ED} : $Shear\ strength\ in\ calculation\ value$
  - S_y : $Middle\ section' static\ moment$
  - I_y : $Inertia\ moment\ about\ "y"\ axis$
  - b : $Section's\ thickness\ (in\ rolled\ profiles, web\ thickness)$
  - $\frac{f_{yD}}{\sqrt{3}}$: $Steel's\ tangential\ yield\ strength\ in\ calculation\ value$

Stress combination verification: The regulations defines that we only have to verify the stress combination if tangential stress is bigger than half of the tangential yield strength.

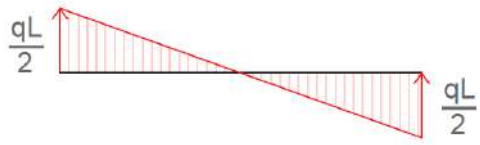
Example. Verify the following beam according to its section resistance (ULS).



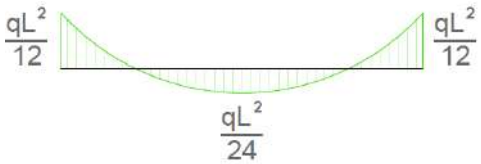
Data:

- $q = 1500\ kg/m\ (increased)$
- $l = 5\ m$
- $Beam\ \rightarrow IPE\ 270 \begin{cases} S_y = 242\ cm^3 \\ I_y = 5790\ cm^4 \\ W_y = 429\ cm^3 \end{cases}$
- $Steel\ S275\ ;\ \gamma_M : steel's\ reduction\ factor = 1{,}05$

<u>Diagrams:</u>



$$V_{MAX} = \frac{ql}{2} = \frac{1500 \cdot 5}{2} = 3750\ kg$$



$$M_{MAX} = \frac{ql^2}{12} = \frac{1500 \cdot 5^2}{12} = 3125\ kg \cdot m$$

-Normal stress verification:

$$\frac{M_{ED}}{W_y} \leq f_{yD}\ ;\ \frac{312500}{429} \leq \frac{2750}{1{,}05}\ ; 728 < 2619\ \checkmark$$

-Tangential stress verification:

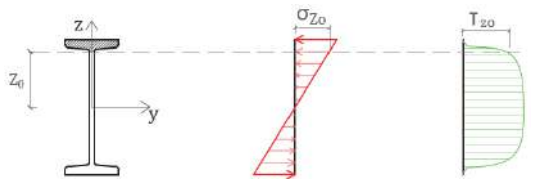
$$V_{MAX} = \frac{ql}{2} = \frac{1500 \cdot 5}{2} = 3750\ kg$$

$$\frac{S_y \cdot V_{ED}}{b \cdot I_y} \leq \frac{f_{yD}}{\sqrt{3}}\ ;\ \frac{242 \cdot 3750}{0{,}66 \cdot 5790} \leq \frac{2750}{1{,}05 \cdot \sqrt{3}}\ ; 237 < 1512\ \checkmark$$

In this case we don´t need to calculate σ_{CO}: $\left(\tau \ngtr \frac{f_{yD}}{2\sqrt{3}}\right)$

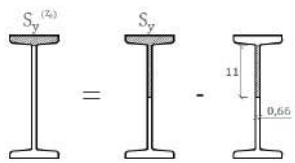
-Combined stress verification (calculated as an example)

$$\sigma_{CO} = \sqrt{{\sigma_{Zo}}^2 + 3{\tau_{Zo}}^2} \leq f_{yD}$$



$$\sigma_{z0} = \frac{M_{ED} \cdot Z_0}{I_y} = \frac{312500 \cdot 11}{5790} = 593{,}7\ kg/cm^2$$

$$\tau_{z0} = \frac{V_{ED} \cdot {S_y}^{(Z_0)}}{b \cdot I_y} = \frac{3750 \cdot 202}{0{,}66 \cdot 5790} = 198{,}2\ kg/cm^2$$



$$- (11 \cdot 0{,}66) \cdot \frac{11}{2} = 202\ cm^3$$

$$\sigma_{CO} = \sqrt{{\sigma_{Zo}}^2 + 3{\tau_{Zo}}^2} = \sqrt{593{,}7^2 + 3 \cdot 198{,}2^2} = 686\ kg/cm^2$$

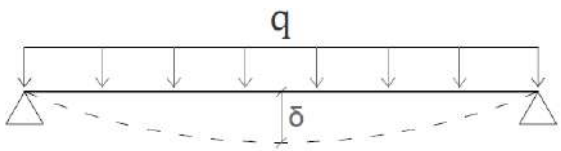
$$\sigma_{CO} \leq f_{yD}\ ; 686 \leq \frac{2750}{1{,}05}\ ; 686 < 2619\ \checkmark$$

7.3.2. Mixed sections. Connectors

7.3.2.1. Steiner applied in mixed sections

In bars working under bending moment, EI is the linear behaviour constant between flexion and deflection. Therefore, in sections of different materials, if EI value is the same, sections are equivalent.

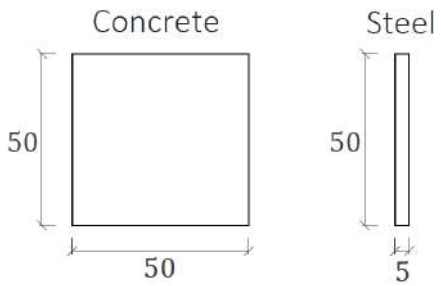
In a double hinged beam under a distributed load:



Maximum deflection value:

$$\delta = \frac{5}{384} \cdot \frac{ql^4}{EI}$$

We are going to compare the sections of different materials creating a beam:



$$E_C = 2{,}1 \cdot 10^5\ kg/cm^2$$

$$E_S = 2{,}1 \cdot 10^6\ kg/cm^2$$

$$I_{yC} = \frac{b \cdot h^3}{12} = \frac{50 \cdot 50^3}{12} = 520833{,}\hat{3}\ cm^4$$

$$I_{yS} = \frac{b \cdot h^3}{12} = \frac{5 \cdot 50^3}{12} = 52083{,}\hat{3}\ cm^4$$

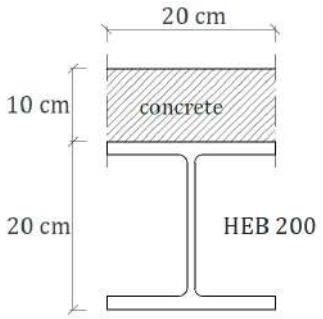
$$\left.\begin{aligned} EI_C &= 520833{,}\hat{3} \cdot 2{,}1 \cdot 10^5 = 1{,}09375\ kg \cdot cm^2 \\ EI_S &= 52083{,}\hat{3} \cdot 2{,}1 \cdot 10^6 = 1{,}09375\ kg \cdot cm^2 \end{aligned}\right\} \quad EI_S = EI_C$$

This means that both sections are equivalent. In any case, when comparing a section with two different materials we have to first define the relation between the elasticity modulus (the relation between concrete and steel is ($E_S/E_C = 10$), and then apply the inverse proportion to the Moments of inertia of these materials.

This relation is really effective when we analyse mixed sections. To define the geometric characteristics of a mixed section, first of all we define an equivalent section made of just one material, and then we follow the normal procedure.

7.3.2.2. Example. Mixed section

Define the centre of gravity (G.C.) and inertia of this mixed section:

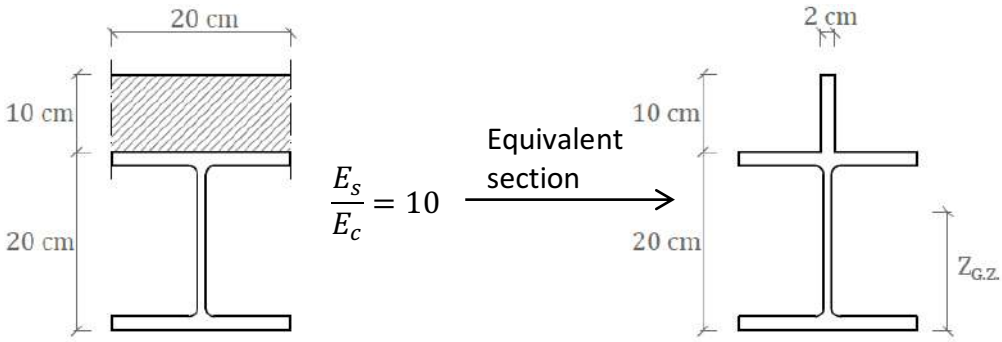


$$E_c = 2{,}1 \cdot 10^5\ kg/cm^2$$

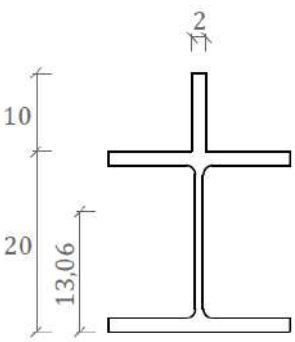
$$E_s = 2{,}1 \cdot 10^6\ kg/cm^2$$

$$I_{y,HEB\ 200} = 5696\ cm^4$$

$$A_{HEB\ 200} = 78{,}1\ cm^2$$



$$Z_{G.C.} = \frac{\sum_i^N Z_{G.C.i} \cdot A_i}{A_i} = \frac{10 \cdot 78{,}1 + 25 \cdot (10 \cdot 2)}{78{,}1 + (10 \cdot 2)} = 13{,}06\ cm$$

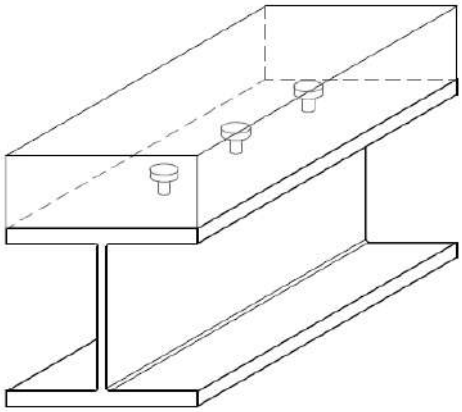


$$I_y = \left(I_{y1} + A_1 \cdot {d_1}^2\right) + \left(I_{y2} + A_2 \cdot {d_2}^2\right)$$

$$I_y = (5696 + 78{,}1 \cdot 3{,}06^2) + \left(\frac{2 \cdot 10^3}{12} + 2 \cdot 10 \cdot 11{,}94^2\right)$$

$$\boxed{I_y = 9445\ cm^4}$$

7.3.2.3. Calculation of Connector



The connectors in this mixed beam are working under pure shear strength.

This means that they don´t receive bending moment.

The tangential stress produced by pure shear strength is this one:

$$\tau = \frac{V}{A}$$

$$\tau = Tangential\ stress$$

$$V = Shear\ strength$$

$$A = Section´s\ area$$

This stress can't exceed the tangential yield strength of the material. This value can change depending on the type of steel we use.

Referring to the CTE regulation, this would be the general expression:

$$\frac{V_{ED}}{A} \leq \frac{f_{yD}}{\sqrt{3}}$$

V_{ED} ⋮ $Shear\ strength\ in\ calculation\ value$

A ⋮ $Area$

$\frac{f_{yD}}{\sqrt{3}}$ ⋮ $Tangential\ yield\ strength$

$*\ Reduction\ factor\ for\ Connectors: \gamma_M = 1{,}25$

Therefore, the shear strength that connectors can support:

$$\frac{V_{ED}}{A} \leq \frac{f_{yD}}{\sqrt{3}} \; ; \; V_{ED} \leq A_{Con} \cdot \frac{f_{yD}}{\sqrt{3}} \; ; \; V_{Con} = A_{Con} \cdot \frac{f_{yD}}{\sqrt{3}}$$

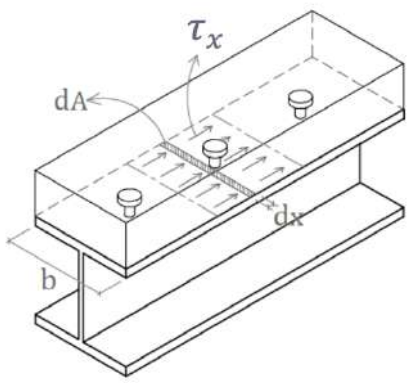
V_{Con} ⋮ $The\ maximum\ shear\ strength\ that\ each\ connector\ can\ receive$

Taking into account this definition, we have to verify if the shear strength generated in the bar exceeds the connectors' sustainable shear strength:

$$V_{X,ED} \leq n \cdot V_{Con}$$

$V_{X,ED} = Shear\ strength\ produced\ by\ \tau_X\ longitudinal\ stress$

$n = Number\ of\ connectors$



For that purpose we have to calculate $V_{X,ED}$. Adding all the longitudinal tangential stress τ_x existing in the load area of each connector, we can quantify how much shear strength they receive:

$$V_{X,ED} = \int_A \tau_X \cdot dA = b \int_0^l \tau_X \cdot dx$$

τ_X ⋮ $tangential\ tension$
b ⋮ $fiber\ thickness$

To calculate τ_X, we use Collignon's expression:

$$\tau_X = \tau_Z = \frac{V_{ED} \cdot S_y{}^{z_0}}{b \cdot I_y}$$

τ_X ⋮ $Longitudinal\ tangential\ stress$
τ_Z ⋮ $Cross\ tangential\ tension$
V_{ED} ⋮ $Shear\ strength\ of\ the\ cross\ section$
$S_y{}^{z_0}$ ⋮ $the\ static\ moment\ of\ the\ area\ that\ is\ above\ the\ fiber\ we\ are\ analysing$
b ⋮ $Thickness\ of\ the\ fiber$
I_y ⋮ $Inertia\ moment$

When using this expression we can have a problem, because the shear strength is often variable. If V_{ED} is variable, τ_z cross tangential stress will also be variable, and therefore, τ_x longitudinal tangential stress will be variable too:

$$V_{X,ED} = b\int_0^l \tau_x \cdot dx = b\int_0^l \frac{V_{ED} \cdot S_y{}^{z_0}}{b \cdot I_y} dx = \frac{S_y{}^{z_0}}{I_y}\int_0^l V_{ED} dx$$

But we can simplify the previous expression if we realize that the integer represents the area of the shear strength diagram:

$$\Delta_v = \int_0^l V_{ED} dx$$

This means that we can use this expression to calculate $V_{X,ED}$:

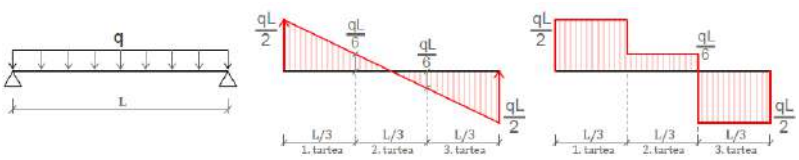
$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y}\Delta_v$$

Therefore, summarizing the information we have learnt in this chapter, these are the formulas we are going to use for calculating the number of connectors we need:

$$V_{X,ED} \leq n \cdot V_{kon}$$

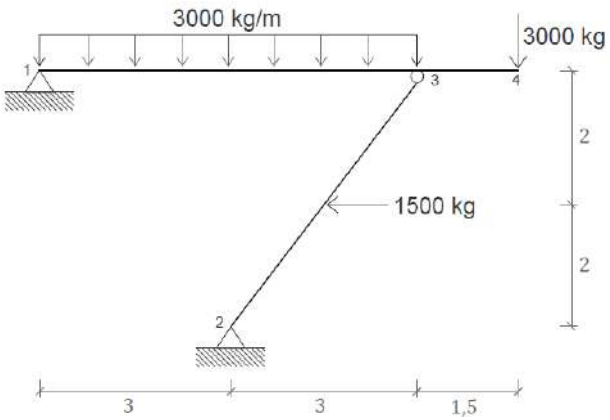
$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y}\Delta_v;\ V_{Con} = A_{Con} \cdot \frac{f_{yD}}{\sqrt{3}}$$

This way, when we know the shear strength diagram, we can calculate the number of connectors we need. But when shear strength is variable, the distance between them should be variable too, being closer where shear strength is bigger. To simplify this situation (rounding up the area of shear strength), we will analyse the bars in different sections, and we will represent constant shear strength in each section with the biggest value of the section. To apply this method in regular beams, we will divide it in three sections:

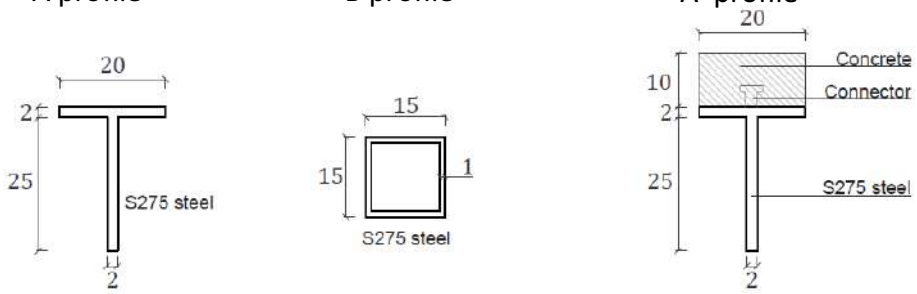


7.3.2.4. Example

1. Draw the strength diagrams.
2. Verify the Ultimate Limit States (ULS) referring to the section's resistance for A and B type of profile. A profile for 1-3 bar and B profile for 2-3 bar.
3. Verify the Ultimate Limit States (ULS) referring to the section's resistance (only normal stress) for A' profile in 1- bar.
4. How many connectors should we put in the mixed beams so as the steel and concrete can work together?



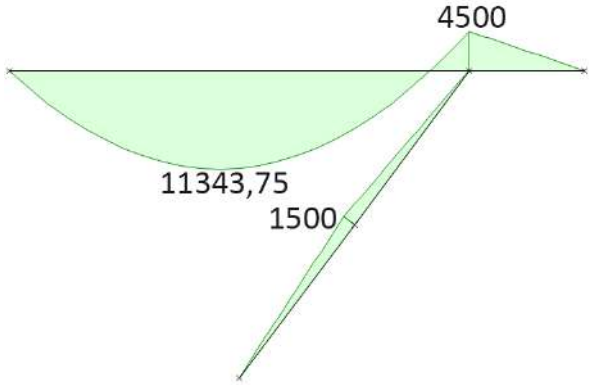
A profile B profile A' profile



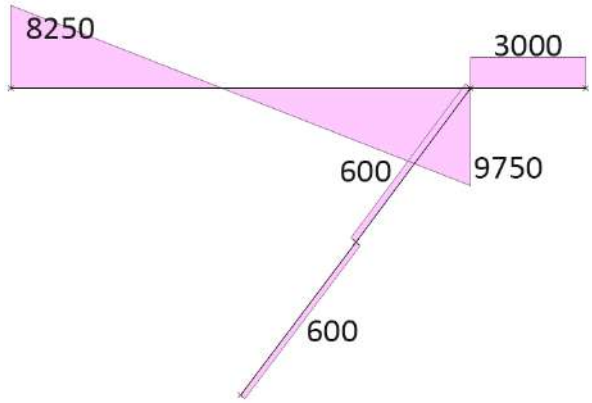
Data:

- The actions include magnification factor
- Steel's yield strength for traction and compression: 2.750 kg/cm^2
- Steel's elasticity modulus: 2.100.000 kg/cm^2
- Concrete's yield strength for compression: 250 kg/cm^2
- Concrete's yield strength for traction: 105 kg/cm^2
- Concrete's elasticity modulus: 210.000 kg/cm^2
- Connector's diameter: 2 cm
- Steel's reduction factor: 1,05
- Connector's reduction factor: 1,25
- Concrete's reduction factor: 1,5

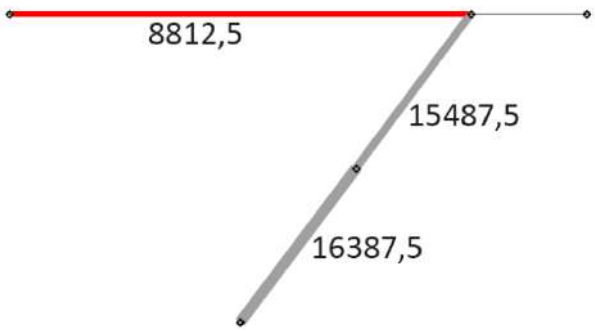
Diagrams:



Bending moment



Shear strength

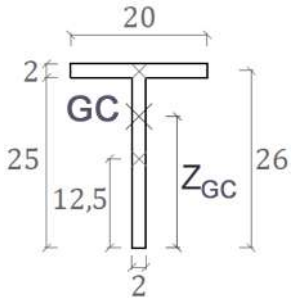


Axial strength

ULS verification in A and B type of profiles

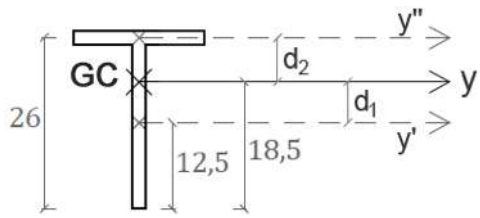
A profile

-Centre of gravity location:



$$Z_{G.C.} = \frac{\sum_i^n Z_{GC_i} \cdot A}{\sum_i^n A} = \frac{12{,}5(25 \cdot 2) + 26(20 \cdot 2)}{25 \cdot 2 + 20 \cdot 2} = 18{,}5\ cm$$

-Moment of inertia (Steniner):



$$I_y = \left(I_{y'} + A_1 \cdot {d_1}^2\right) + \left(I_{y''} + A_2 \cdot {d_2}^2\right)$$

$$I_y = \left(\frac{2 \cdot 25^3}{12} + (2 \cdot 25) \cdot 6^2\right) + \left(\frac{20 \cdot 2^3}{12} + (20 \cdot 2) \cdot 7{,}5^2\right) = 6667{,}5\ cm^4$$

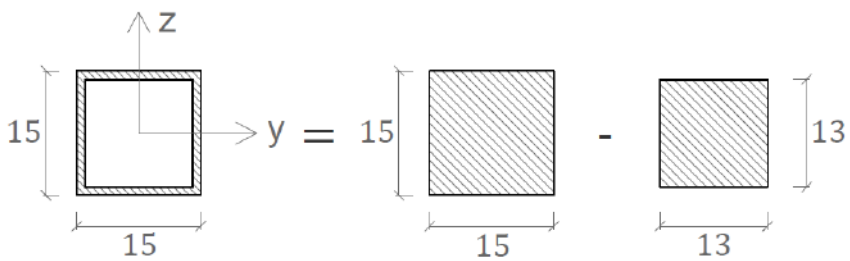
-Normal stress verification:

$$\frac{N_{ED}}{A} + \frac{M_{y,ED} \cdot Z_{MAX}}{I_y} \leq f_{yD}\ ;\ \frac{8812{,}5}{(20 \cdot 2) + (25 \cdot 2)} + \frac{1139375 \cdot 18{,}5}{6667{,}5} \leq \frac{2750}{1{,}05}$$

$$3245 \nless 2619\ \times \ \rightarrow this\ profile\ is\ not\ valid!$$

B profile

-Moment of inertia (Steiner):



$$I_y = \left(I_{y'} + A_1 \cdot {d_1}^2\right) - \left(I_{y''} + A_2 \cdot {d_2}^2\right) \;\rightarrow\; [d_1 = d_2 = 0]$$

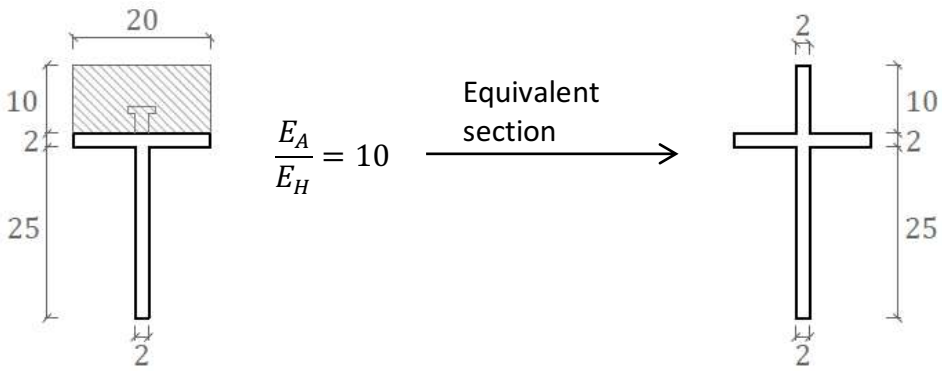
$$I_y = \left(\frac{15^4}{12} - \frac{13^4}{12}\right) = 1839\ cm^4$$

-Normal stress:

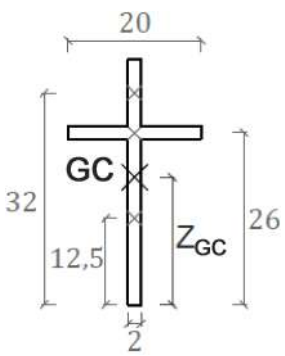
$$\frac{N_{ED}}{A} + \frac{M_{y,ED} \cdot Z_{MAX}}{I_y} \leq f_{yD}\ ;\ \frac{16387{,}5}{(15^2 - 13^2)} + \frac{1500 \cdot 100 \cdot 7{,}5}{1839} \leq \frac{2750}{1{,}05}$$

$$292{,}64 + 612 \leq 2619\ ; 909 < 2619\ \checkmark \rightarrow we\ can\ use\ this\ profile$$

<u>A' mixed profile</u>

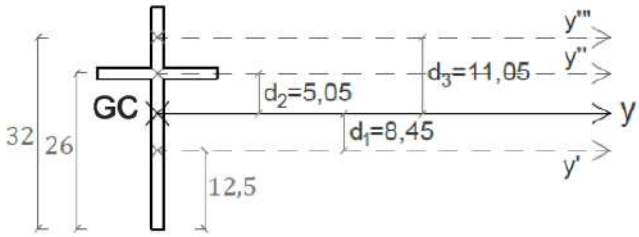


-Centre of gravity location:



$$Z_{G.C.} = \frac{\sum_i^n Z_{GC_i} \cdot A}{\sum_i^n A} = \frac{12{,}5(25 \cdot 2) + 26(20 \cdot 2) + 32(2 \cdot 10)}{(25 \cdot 2) + (20 \cdot 2) + (2 \cdot 10)}$$
$$= 20{,}95\ cm$$

-Moment of inertia (Steniner):



$$I_y = \left(I_{y'} + A_1 \cdot {d_1}^2\right) + \left(I_{y''} + A_2 \cdot {d_2}^2\right) + \left(I_{y'''} + A_3 \cdot {d_3}^2\right)$$

$$I_y = \left(\frac{2 \cdot 25^3}{12} + (2 \cdot 25) \cdot 8{,}45^2\right) + \left(\frac{20 \cdot 2^3}{12} + (20 \cdot 2) \cdot 5{,}05^2\right) + \left(\frac{2 \cdot 10^3}{12} + (2 \cdot 10) \cdot 11{,}05^2\right) = 6174{,}3 + 1033{,}43 + 2608{,}7$$

$$I_y = 9816{,}43\ cm^4$$

-Normal stress verification:

In the case of mixed sections, as concrete's resistance is different under compression and traction, we have to verify two situations:

1. Maximum negative bending moment (concrete under traction)
2. Maximum positive bending moment (concrete under compression)

Sign criteria:

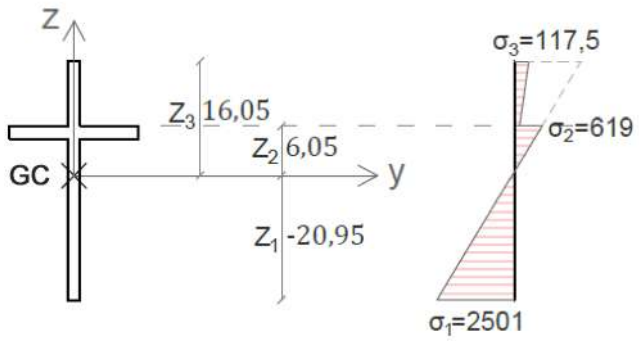
- Traction $(-)$, compression $(+)$

Maximum positive bending moment ($M_{MAX}^{+} = 11343{,}75\ kg \cdot m$):

$$\sigma_1 = \frac{N_{ED}}{A} + \frac{M_{ED} \cdot Z_1}{I_y} = \frac{-8812{,}5}{110} + \frac{1134375 \cdot (-20{,}95)}{9816{,}43} = -2501\ kg/cm^2$$

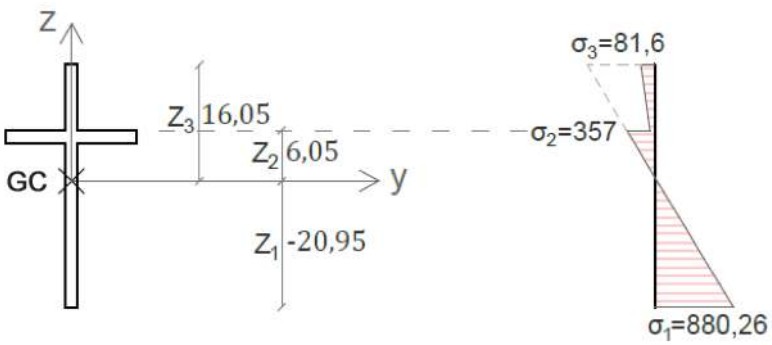
$$\sigma_2 = \frac{N_{ED}}{A} + \frac{M_{ED} \cdot Z_2}{I_y} = \frac{-8812{,}5}{110} + \frac{1134375 \cdot (6{,}05)}{9816{,}43} = 619\ kg/cm^2$$

$$\sigma_3 = \frac{N_{ED}}{A} + \frac{M_{ED} \cdot Z_3}{I_y} = \frac{-8812{,}5}{110} + \frac{1134375 \cdot (16{,}05)}{9816{,}43} = 1175\ kg/cm^2$$



$$\boxed{\sigma_1 = 2501\ kg/cm^2 < f_{yD}\ ;\ \sigma_2 = 619\ kg/cm^2 < f_{yD}\ ;\ \sigma_3 = 117{,}5\ kg/cm^2 < f_{yD}}$$

<u>Maximum negative bending moment ($M^-_{MAX} = -4500\ kg \cdot m$)</u>



$$\sigma_1 = \frac{N_{ED}}{A} + \frac{M_{ED} \cdot Z_1}{I_y} = \frac{-8812{,}5}{110} + \frac{-450000 \cdot (-20{,}95)}{9816{,}43} = 880{,}26\ kg/cm^2$$

$$\sigma_2 = \frac{N_{ED}}{A} + \frac{M_{ED} \cdot Z_2}{I_y} = \frac{-8812{,}5}{110} + \frac{1134375 \cdot (6{,}05)}{9816{,}43} = -357{,}45\ kg/cm^2$$

$$\sigma_3 = \frac{N_{ED}}{A} + \frac{M_{ED} \cdot Z_3}{I_y} = \frac{-8812{,}5}{110} + \frac{1134375 \cdot (16{,}05)}{9816{,}43} = -816\ kg/cm^2$$

$$\sigma_1 \le f_{yD} \quad 880 < \frac{2750}{1{,}05} = 2619 \ \checkmark \ / \ \sigma_2 \le f_{yD} \quad 357 < 2619 \ \checkmark \ / \ \sigma_2 \le f_{CD} \quad 35{,}7 < \frac{105}{1{,}5} = 70 \ \checkmark \ / \ \sigma_3 \le f_{CD} \quad 81{,}6 \not< 70 \ \times$$

(we should reinforce it)

Connector's calculation

$$V_{X,ED} \leq n \cdot V_{Con}$$

$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y} \Delta_v$$

$$V_{Con} = A_{Con} \cdot \frac{f_{yD}}{\sqrt{3}}$$

First of all, we will calculate how much load can bear each connector:

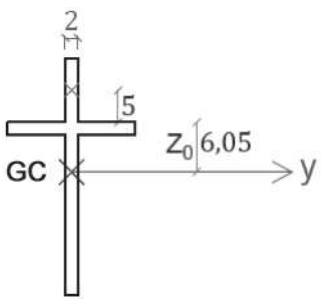
$$V_{Con} = A_{Con} \cdot \frac{f_{yD}}{\sqrt{3}} = A_{Con} \cdot \frac{f_{yD}}{\gamma_M \cdot \sqrt{3}} = 3{,}1416 \cdot \frac{2750}{1{,}25 \cdot \sqrt{3}} = 3990\ kg$$

$$A_{Con} = \pi \cdot r^2 = 3{,}1416 \cdot 1^2 = 3{,}1416\ cm^2$$
$$r = Radius\ of\ the\ connector = 1\ cm$$
$$\gamma_M = Reduction\ factor\ for\ the\ connectors = 1{,}25$$

Later on we will calculate what the shear strength that connectors should bar is:

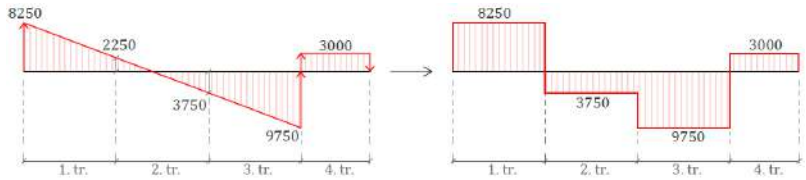
$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y} \Delta_v$$

And for applying this formula we need to calculate the value of static moment of the area above the fibre of the connectors:



$$S_y{}^{z_0} = (10 \cdot 2) \cdot (5 + 6{,}05) = 221\ cm^3$$

We should too calculate the shear strength diagram area. For doing this we will divide the beam in four sections, three between supports, and a fourth one for the cantilever. In each section we will represent a constant value of shear strength.



This way we can calculate the amount of connectors in each section.

1. Section

$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y} \cdot \Delta_{v_1} = \frac{221}{9816} \cdot 8250 \cdot 200 = 37149\ kg$$

$$nº \cdot V_{Con} = V_{X,ED}\ ;\ nº = \frac{37149}{3990} = 9{,}31\ \simeq 10\ \boxed{1.\,section \rightarrow 10\ connectors}$$

2.Section

$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y} \cdot \Delta_{v_2} = \frac{221}{9816} \cdot 3750 \cdot 200 = 16886\ kg$$

$$nº \cdot V_{Con} = V_{X,ED}\ ;\ nº = \frac{16886}{3990} = 4{,}23\ \simeq 5\ \boxed{2.\,section \rightarrow 5\ connectors}$$

3. Section

$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y} \cdot \Delta_{v_3} = \frac{221}{9816} \cdot 9750 \cdot 200 = 43903\ kg$$

$$nº \cdot V_{Con} = V_{X,ED}\ ;\ nº = \frac{43903}{3990} = 11\ \boxed{3.\,section \rightarrow 11\ connectors}$$

4. Section

$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y} \cdot \Delta_{v_4} = \frac{221}{9816} \cdot 150 \cdot 3000 = 10131\ kg$$

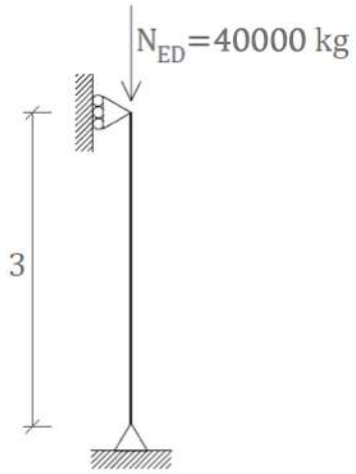
$$nº \cdot V_{Con} = V_{X,ED}\ ;\ nº = \frac{10131}{3990} = 2{,}6\ \simeq 3\ \boxed{4.\,section \rightarrow 3\ connectors}$$

7.3.3. Stability

On this chapter, following with the Ultimate Limite States, we are going to verify the stability of our structure. We are going to start verifying the buckling, produced by the axial strength.

7.3.3.1. Buckling

When we have verified the resistance of the section, we have seen that using steel profiles, cross sections with small surface can bear big strengths. This means that taking into account just the resistance of the section, very slender bars can bear big strengths, as we can see in the following example:



$$\frac{N_{ED}}{A} \leq f_{yD}$$

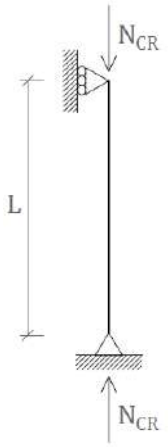
$$f_{yD} = \frac{2750}{\gamma_M} = \frac{2750}{1{,}05}\ kg/cm^2\ \ (yield\ strength)$$

$$A : surface\ of\ the\ section$$

$$\frac{N_{ED}}{f_{yD}} \leq A\ ; \frac{40000}{\frac{2750}{1{,}05}} \leq A\ ; A \geq 15{,}27\ cm^2\ \rightarrow\ \boxed{HEB\ 100}$$

However, we will see that taking into account the bar's slenderness, an instability phenomenon that can happen: the buckling. To quantify this risk, we are going to learn, in the following chapter, the Euler expression for the critical axial strength of buckling.

Euler's expression



$$N_{CR} = \frac{\pi^2}{l^2} \cdot EI$$

N_{CR} : $Critical\ tension\ to\ buckling$

l : $Bar's\ length$

I : $Profile's\ lowest\ inertia$

E : $Material's\ elasticity\ module$

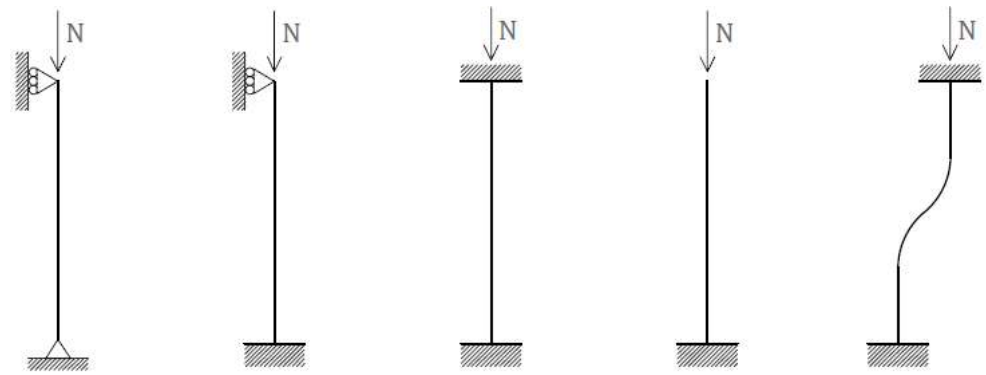
This expression reflects a hinged bar in an ideal state. The ideal state means that these following conditions are fulfilled.

- Bar's axis is completely straight.
- Axial strength's axis meets bar's axis.
- Elasticity modulus and moment of inertia are constant.
- There are no residual tensions.

Apart from the ideal state, the other condition was for the bar to have hinged joints. Therefore, in the case of having different joints we should rewrite the expression this way:

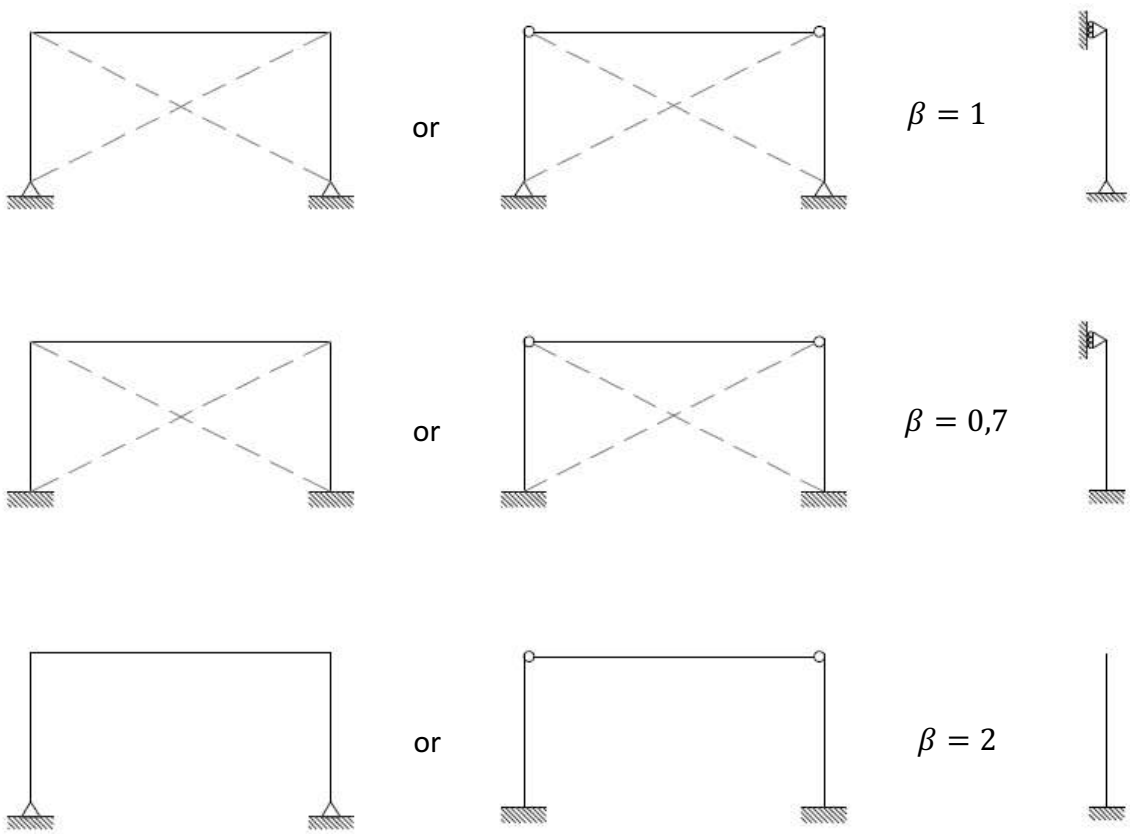
$$N_{CR} = \frac{\pi^2}{{l_k}^2} \cdot EI \quad \rightarrow \quad l_k = l \cdot \beta \ \ (buckling\ length)$$

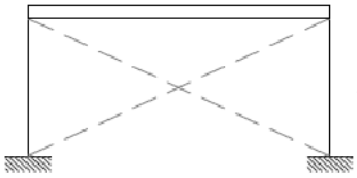
β is the buckling coefficient, which is related to the bar's supports (joints) and to their translational degree of freedom:



$\beta = 1$	$\beta = 0{,}7$	$\beta = 0{,}5$	$\beta = 2$	$\beta = 1$
Double hinged	Hinged fixed	Double fixed	Cantilever	Double fixed displaceable

Nevertheless, the bars we usually analyse aren´t isolated bars, they are part of a frame. Therefore, we are going to see some basic cases so that we can differentiate how much the β coefficient is going to be in each case:



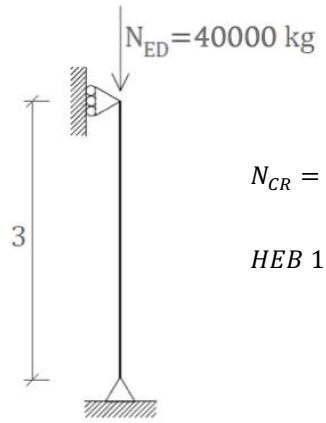


$\beta = 0{,}5$

$\beta = 1$

*The beam has much more stiffness than the columns.

Therefore, introducing Euler's expression, we can restart the column's new calculation:



$$N_{CR} = \frac{\pi^2}{{l_k}^2} \cdot EI_z \; ; \; I_z = \frac{N_{CR} \cdot {l_k}^2}{\pi^2 \cdot E} = \frac{40000 \cdot 300^2}{3{,}14^2 \cdot 2{,}1 \cdot 10^6} = 173{,}69 \; cm^4$$

$$HEB\ 120 \;\rightarrow\; \checkmark \quad (I_z = 318 \; cm^4)$$

As we can see, HEB 100 is not enough anymore, and we have to redimension the bar. But, as we know, Euler's expression analyses columns in ideal states. However, mechanical and geometric imperfections in real pieces and profiles diminish N_{CR} critical tension value.

For that reason, CECM organization ("Convención Europea de Construcción Metálica") in order to solve this deficiency and to unite Europe's regulations, they organised an experimental program:

- More than 1000 trials.
- Analysing different types of profiles
- With a mechanical slenderness (λ) of between 20 and 160

After this research they defined the European Buckling curves ("Curvas Europeas de Pandeo", CTE-DE-SE-A, Chart 6.2). By means of this chart, depending on the reduced slenderness ($\bar{\lambda}$) and the buckling curve of the piece, we define the buckling reduction factor, represented by χ letter. And finally, the expression of the verification of the bar for buckling is (CTE-DB-SE-A 6.3.2 Stability under compression strength):

$$\frac{N_{C,ED}}{\chi \cdot A} < f_{yD}$$

$$\chi : Buckling\ reduction\ factor \rightarrow \text{ Chart 6.3}$$

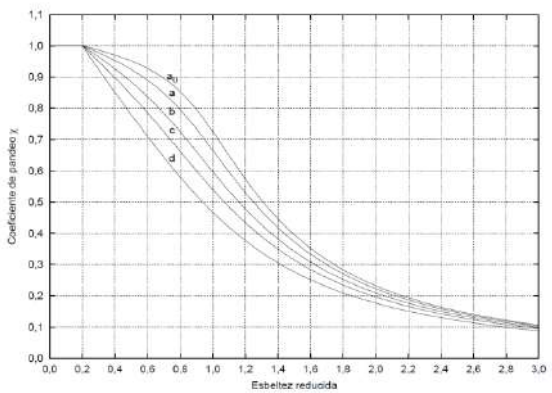


Figura 6.3 Curvas de pandeo

Tabla 6.3 Valores del coeficiente de pandeo (χ)

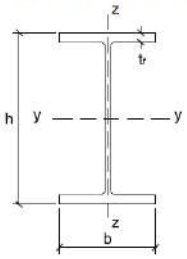
Esbeltez reducida	Curva de pandeo a_0	a	b	c	d
Coeficiente (α) de imperfección	0,13	0,21	0,34	0,49	0,76
≤ 0,20	1,00	1,00	1,00	1,00	1,00
0,30	0,99	0,98	0,96	0,95	0,92
0,40	0,97	0,95	0,93	0,90	0,85
0,50	0,95	0,92	0,88	0,84	0,78
0,60	0,93	0,89	0,84	0,79	0,71
0,70	0,90	0,85	0,78	0,72	0,64
0,80	0,85	0,80	0,72	0,66	0,58
0,90	0,80	0,73	0,66	0,60	0,52
1,00	0,73	0,67	0,60	0,54	0,47
1,10	0,65	0,60	0,54	0,48	0,42
1,20	0,57	0,53	0,48	0,43	0,38
1,30	0,51	0,47	0,43	0,39	0,34
1,40	0,45	0,42	0,38	0,35	0,31
1,50	0,40	0,37	0,34	0,31	0,28
1,60	0,35	0,32	0,31	0,28	0,25
1,80	0,28	0,27	0,25	0,23	0,21
2,00 (1)	0,23	0,22	0,21	0,20	0,18
2,20 (1)	0,19	0,19	0,18	0,17	0,15
2,40 (1)	0,16	0,16	0,15	0,14	0,13
2,70 (2)	0,13	0,13	0,12	0,12	0,11
3,00 (2)	0,11	0,10	0,10	0,10	0,09

(1) esbeltez intolerable en los elementos principales
(2) esbeltez intolerable incluso en elementos de arriostramiento

In the Chart 6.3 we can obtain the buckling reduction factor χ depending on two elements: **buckling curve** and the **reduced slenderness** $\bar{\lambda}$. The chart is presented in two different ways: in the left we can see the buckling curves (in this chart, once we have chosen the buckling curve of our profile, we can obtain the reduction factor), and in the right chart we have the same information but done in numerical values, to obtain the value of the reduction factor faster. But, as we have seen, the two chart give the same information.

The buckling curve can be easily defined with the Chart 6.2, which basically depends on the section's geometry.

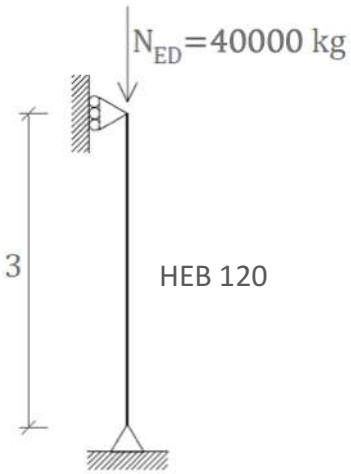
Tabla 6.2 Curva de pandeo en función de la sección transversal

Tipo de sección		Tipo de acero	S235 a S355		S450	
		Eje de pandeo (1)	y	z	y	z
Perfiles laminados en I 	h/b > 1,2	t ≤ 40 mm	a	b	a_0	a_0
	40 mm < t ≤ 100 mm		b	c	a	a
	h/b ≤ 1,2	t ≤ 100 mm	b	c	a	a
	t > 100 mm		d	d	c	c

The reduced slenderness $\bar{\lambda}$ is a geometric feature of the bar that measures the bar's tendency to suffer buckling. **For main structural pieces the regulation doesn´t allow values bigger than 2** (as we can see in de chart 6.3). To calculate the value of reduced slenderness, we have the following formula in the CTE:

$$\bar{\lambda} = \sqrt{\frac{A \cdot f_y}{N_{CR}}}$$

Let's apply this formula in our column (**HEB 120** profile).



$$\frac{N}{A \cdot \chi_z} \leq f_{yD}$$

We are going to calculate χ_z factor for Z axis, because is more restrictive.

$$N_{CR,z} = \frac{\pi^2}{l_{k,z}} \cdot EI_z = \frac{\pi^2}{300^2} \cdot 2{,}1 \cdot 10^6 \cdot 318 = 73232\ kg$$

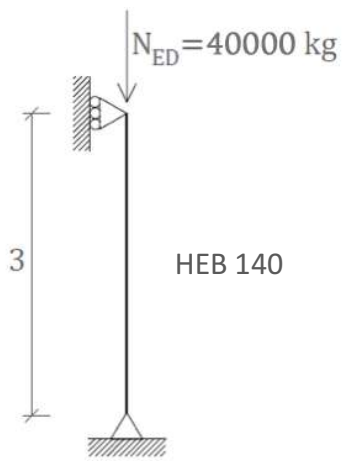
$$\bar{\lambda}_z = \sqrt{\frac{A \cdot f_y}{N_{CR,z}}} = \sqrt{\frac{34 \cdot 2750}{73232}} = 1{,}3$$

- Chart 6.2 application: $HEB\ 120 \rightarrow C\ curve\ (z\ axis)$
- Chart 6.3 application: $\left.\begin{array}{l}\bar{\lambda}_z = 1{,}3 \\ C\ curve\end{array}\right\} \chi_z = 0{,}39$

Thus,

$$\frac{N}{A \cdot \chi_z} \leq f_{yD}\ ;\ \frac{40000}{34 \cdot 0{,}39} \leq \frac{2750}{1{,}05};\ 3017 < 2619\ ✗$$

This means that we need a bigger profile (**HEB 140)**:



$$\frac{N}{A \cdot \chi_z} \leq f_{yD}$$

$$N_{CR,z} = \frac{\pi^2}{l_{k,z}} \cdot EI_z = \frac{\pi^2}{300^2} \cdot 2{,}1 \cdot 10^6 \cdot 550 = 126660\ kg$$

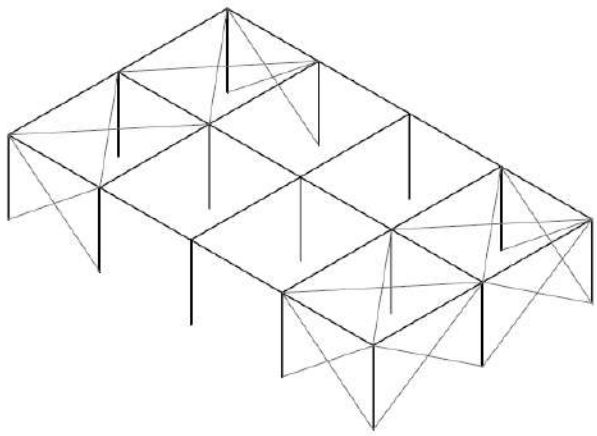
$$\bar{\lambda}_z = \sqrt{\frac{A \cdot f_y}{N_{CR,z}}} = \sqrt{\frac{43 \cdot 2750}{126660}} = 0{,}97$$

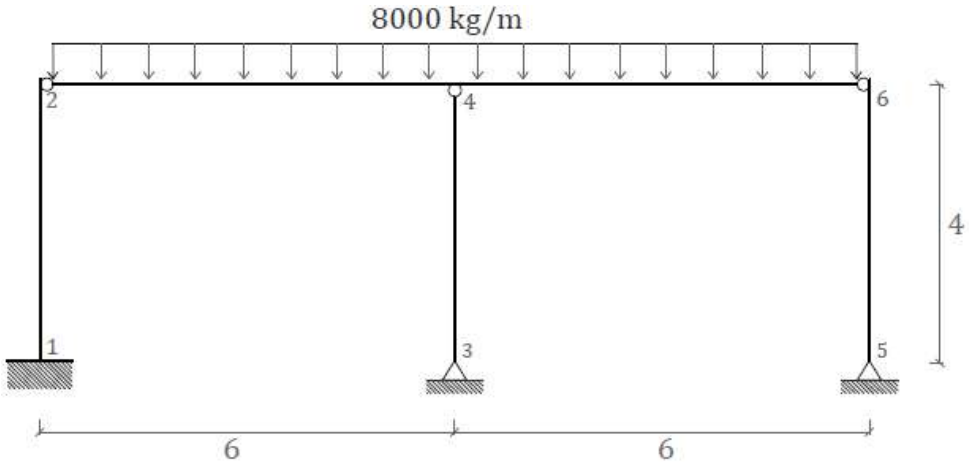
- Chart 6.2 application: $HEB\ 140 \rightarrow C\ \ curve\ (z\ axis)$
- Chart 6.3 application: $\left.\begin{array}{l}\bar{\lambda}_z = 0{,}97 \\ C\ curve\end{array}\right\} \chi_z = 0{,}54$

Therefore,

$$\frac{N}{A \cdot \chi_z} \leq f_{yD}\ ;\ \frac{40000}{43 \cdot 0{,}54} \leq \frac{2750}{1{,}05};\ 1722 < 2619\ ✓$$

Exercise. Dimension the columns of this structural frame using HEB profiles. Verify the Ultimate limit States (ULS) referring to the section's resistance and bar's stability.



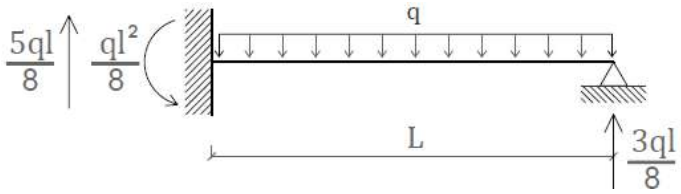


Data:

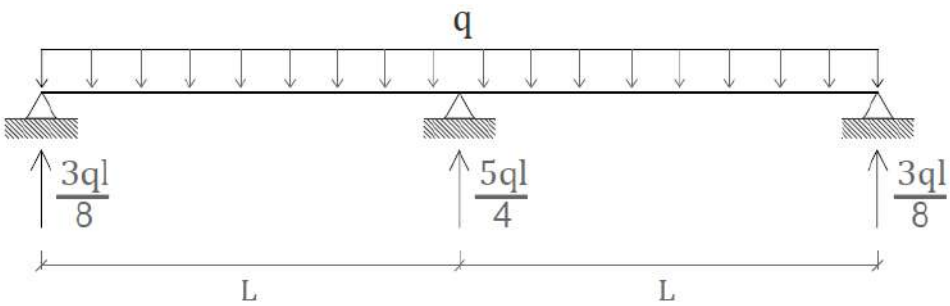
- S275 steel
- Load's magnification factor is 1,5.
- Material's reduction factor is 1,05.
- Don´t use simultaneity factors, only one load combination.

Due to symmetry:

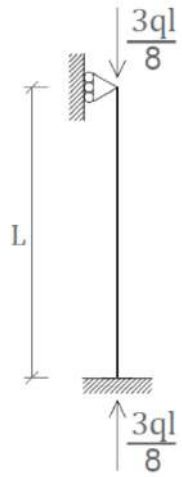
4-6 bar



Thus, 2-4-6 bar:



<u>1-2 columns</u>



$$N_{ED} = \frac{3ql}{8} \cdot 1{,}5 = \frac{3 \cdot 8000 \cdot 6}{8} \cdot 1{,}5 = 27000\ kg$$

- <u>Pre dimensioning (Annex 1: Pre dimensioning methods)</u>

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z \rightarrow\ I_z = \frac{{l_k}^2 \cdot N_{CR,Z}}{\pi^2 \cdot E}; I_z = \frac{{l_k}^2 \cdot N_{CR,Z}}{\pi^2 \cdot E} = \frac{(400 \cdot 0{,}7)^2 \cdot 27000}{\pi^2 \cdot 2{,}1 \cdot 10^6} = 102{,}13\ cm^4$$

$$I_z = 102{,}13 cm^4 \rightarrow \boxed{HEB\ 100}$$

-Verification. Section resistance:

$$\frac{N_{ED}}{A} \leq f_{yD}\ ;\ \frac{N_{ED}}{f_{yD}} \leq A\ ;\ \frac{27000}{26} \leq \frac{2750}{1{,}05}; 1038 < 2619\ \checkmark$$

- Verification. Buckling:

$$\frac{N_{ED}}{\chi_Z \cdot A} \leq f_{yD}\ ;\ \frac{27000}{0{,}4 \cdot 26} \leq \frac{2750}{1{,}05}\ ; 2596 < 2619\ \checkmark\ \rightarrow valid$$

- χ_z calculation:

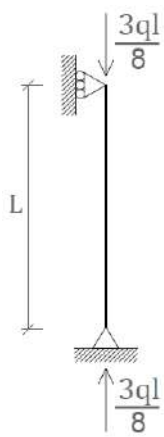
As β coefficient for both axes is the same, directly we are going to verify the column in z axis.

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z = \frac{\pi^2}{(400 \cdot 0{,}7)^2} \cdot 2{,}1 \cdot 10^6 \cdot 167 = 44148\ kg$$

$$l_{k,z} = 0{,}7l$$

$$\left.\begin{array}{l} \bar{\lambda}_z = \sqrt{\dfrac{f_y \cdot A}{N_{CR,z}}} = \sqrt{\dfrac{2750 \cdot 26}{44148}} = 1{,}27 \\ C\ curve \end{array}\right\} \quad \chi_z = 0{,}4$$

-5-6 column



$$N_{ED} = \frac{3ql}{8} \cdot 1{,}5 = \frac{3 \cdot 8000 \cdot 6}{8} \cdot 1{,}5 = 27000\ kg$$

- Pre dimensioning

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z \rightarrow \; I_z = \frac{{l_k}^2 \cdot N_{CR,Z}}{\pi^2 \cdot E}; I_z = \frac{{l_k}^2 \cdot N_{CR,Z}}{\pi^2 \cdot E} = \frac{(400)^2 \cdot 27000}{\pi^2 \cdot 2{,}1 \cdot 10^6} = 208{,}43\ cm^4$$

$$I_z = 208{,}43 cm^4 \rightarrow \boxed{HEB\ 120}$$

-Verification. Section resistance:

$$\frac{N_{ED}}{A} \leq f_{yD}\ ;\ \frac{N_{ED}}{f_{yD}} \leq A\ ;\ \frac{27000}{34} \leq \frac{2750}{1{,}05}; 794 < 2619\ \checkmark$$

- Verification. Buckling:

$$\frac{N_{ED}}{\chi_Z \cdot A} \leq f_{yD}\ ;\ \frac{27000}{0{,}33 \cdot 26} \leq \frac{2750}{1{,}05}\ ; 3147 < 2619\ \times\ \rightarrow not\ valid$$

- χ_z calculation:

As β coefficient for both axes is the same, directly we are going to verify the column in z axis.

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z = \frac{\pi^2}{(400)^2} \cdot 2{,}1 \cdot 10^6 \cdot 318 = 44193\ kg$$

$$l_{k,z} = l$$

$$\left.\begin{array}{l} \bar{\lambda}_z = \sqrt{\frac{f_y \cdot A}{N_{CR,z}}} = \sqrt{\frac{2750 \cdot 34}{44193}} = 1{,}45 \\ C\ curve \end{array}\right\} \chi_z = 0{,}33$$

- Resize -> $HEB\ 140$

$$\frac{N_{ED}}{\chi_Z \cdot A} \leq f_{yD}\ ;\ \frac{27000}{0{,}41 \cdot 43} \leq \frac{2750}{1{,}05}\ ; 1531 < 2619\ \checkmark\ \rightarrow valid$$

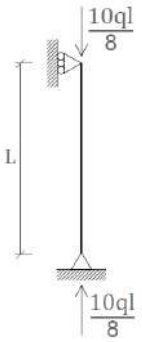
- χ_z calculation:

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z = \frac{\pi^2}{400^2} \cdot 2{,}1 \cdot 10^6 \cdot 550 = 71246\ kg$$

$$l_{k,z} = l$$

$$\left.\begin{array}{l} \overline{\lambda_z} = \sqrt{\dfrac{f_y \cdot A}{N_{CR,z}}} = \sqrt{\dfrac{2750 \cdot 43}{71246}} = 1{,}23 \\ \qquad\qquad\qquad\qquad C\ curve \end{array}\right\} \quad \chi_z = 0{,}41$$

<u>3-4 column</u>



$$N_{ED} = \frac{10ql}{8} \cdot 1{,}5 = \frac{10 \cdot 8000 \cdot 6}{8} \cdot 1{,}5 = 90000\ kg$$

- <u>Pre dimensioning</u>

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z \rightarrow\ I_z = \frac{{l_k}^2 \cdot N_{CR,Z}}{\pi^2 \cdot E};\ I_z = \frac{{l_k}^2 \cdot N_{CR,Z}}{\pi^2 \cdot E} = \frac{(400)^2 \cdot 90000}{\pi^2 \cdot 2{,}1 \cdot 10^6} = 695\ cm^4$$

$$I_z = 695\ cm^4 \rightarrow \boxed{HEB\ 160}$$

<u>-Verification. Section resistance:</u>

$$\frac{N_{ED}}{A} \leq f_{yD}\ ;\ \frac{N_{ED}}{f_{yD}} \leq A\ ;\ \frac{90000}{54{,}3} \leq \frac{2750}{1{,}05};\ 1658 < 2619\ \checkmark$$

- Verification. Buckling:

$$\frac{N_{ED}}{\chi_Z \cdot A} \leq f_{yD}\ ;\ \frac{90000}{0{,}45 \cdot 54{,}3} \leq \frac{2750}{1{,}05}\ ; 3683 < 2619\ \times\ \rightarrow not\ valid$$

- χ_z calculation:

As β coefficient for both axes is the same, directly we are going to verify the column in z axis.

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z = \frac{\pi^2}{(400)^2} \cdot 2{,}1 \cdot 10^6 \cdot 889 = 115160\ kg$$

$$l_{k,z} = l$$

$$\left.\begin{array}{r} \overline{\lambda_z} = \sqrt{\frac{f_y \cdot A}{N_{CR,z}}} = \sqrt{\frac{2750 \cdot 54{,}3}{115160}} = 1{,}14 \\ C\ curve \end{array}\right\} \quad \chi_z = 0,.45$$

- Resize -> HEB 180

$$\frac{N_{ED}}{\chi_Z \cdot A} \leq f_{yD}\ ;\ \frac{90000}{0{,}54 \cdot 65{,}3} \leq \frac{2750}{1{,}05}\ ; 2552 < 2619\ \checkmark\ \rightarrow\ valid$$

- χ_z calculation:

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z = \frac{\pi^2}{400^2} \cdot 2{,}1 \cdot 10^6 \cdot 1363 = 176561\ kg$$

$$l_{k,z} = l$$

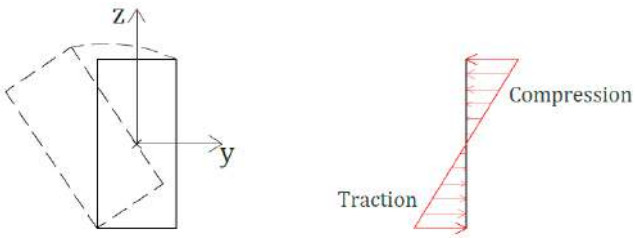
$$\left.\begin{array}{r} \overline{\lambda_z} = \sqrt{\frac{f_y \cdot A}{N_{CR,z}}} = \sqrt{\frac{2750 \cdot 65{,}3}{176561}} = 1 \\ C\ curve \end{array}\right\} \quad \chi_z = 0{,}54$$

Thus:

1-2 column → HEB 100 ; 5-6 column → HEB 120 ; 3-4 column → HEB 180

7.3.3.2. Lateral torsional buckling

In bars working under bending moment, when the inertia about the weak axis is too low comparing with the strong one (in beams), before reaching the section's plastification point we can lose the stability. This happens because the part of the section working under compression can suffer buckling (about the weak axis). This phenomenon is called lateral torsional buckling.



The procedures to check axial buckling and lateral torsional buckling are quite similar. First we define the maximum bending moment for the ideal beam to support (M_{CR} : critical moment). Once we have defined M_{CR}, we have to specify how much should be the beam's lateral torsional reduced slenderness $(\bar{\lambda}_{LT})$. As in the axial buckling, when we have calculated the reduced slenderness and defined the buckling curve, we can obtain the χ_{LT} in the Chart 6.3.

$$\frac{M_{ED}}{\chi_{LT} \cdot W_{EL,y}} \leq f_{yD}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{EL,y} \cdot f_y}{M_{CR}}}$$

χ_{LT} : Lateral torsional reduction factor

$\bar{\lambda}_{LT}$: Lateral torsional reduced slenderness

$W_{EL,y}$: *Bending resistance about y axis*

M_{CR}

To calculate the critical moment of lateral torsional buckling, we use the following expression for a double hinged beam, with rectangular cross section:

$$M_{CR} = \frac{\pi}{l}\sqrt{I_z \cdot G \cdot I_t \cdot E}$$

I_z ⋮ $Moment\ of\ inertia\ about\ z\ axis$
G ⋮ $Shear\ modulus$
I_t ⋮ $Torsion\ constant$
E ⋮ $Young's modulus$
l ⋮ $Length\ of\ the\ bar$

But this formula has some limitations. Hence why we have to introduce two elements in this formula: **moment equivalent factor C_1**, to take into account different types of bending moment diagram, and **lateral buckling length l_C**, suitable when we have different types of supports and struts.

As a result of introducing in this formula the previous factors, and using the CTE's nomenclature, we obtain the following expression:

$$M_{LT,V} = C_1 \frac{\pi}{l_C}\sqrt{I_z \cdot G \cdot I_t \cdot E}$$

l_C ⋮ $Lateral\ buckling\ Length$
C_1 ⋮ $Equivalent\ moment\ factor$

Apart from this formula CTE regulation gathers another formula to obtain the critical moment of lateral torsional buckling. This new expression takes into account the effect of warping.

$$M_{LT,W} = C_1 \cdot W_{EL,y} \cdot \frac{\pi^2 \cdot E}{{l_C}^2} \cdot {i_{f,z}}^2$$

$i_{f,z}$ ⋮ Rotation radius of the compressed part of the section. This is made up with the compressed flange and the third part of the web. Is similar to i_z.

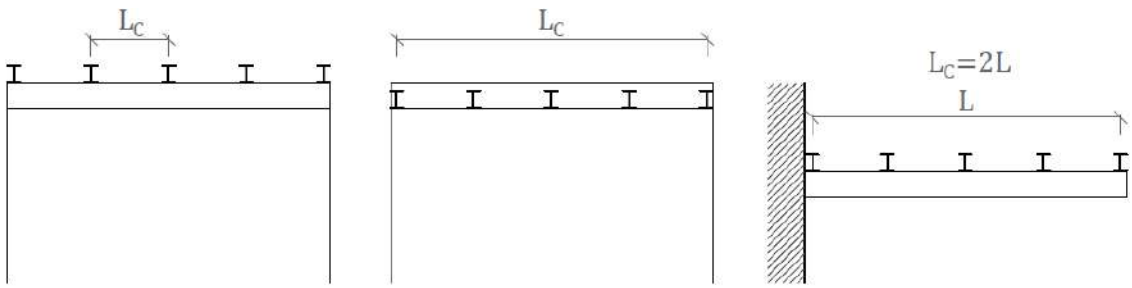
Finally, in order to complete the calculation of the critical moment M_{CR} we combine the previous two expressions:

$$M_{CR} = \sqrt{{M_{LTv}}^2 + {M_{LTw}}^2}$$

To solve this formula, we have the *critical moment of lateral torsional buckling* chart in the 4th annex of these lecture notes.

<u>L_C lateral buckling length</u>

L_C buckling length measures the distance between horizontal struts. For that purpose, first we have to analyse which is the compressed side of the beam (which part of the section is going to be working under compression).

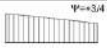
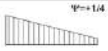
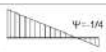
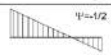


In the left scheme of the previous examples we have a regular beam. If this beam is working under positive bending moment, the upper side of the cross section is compressed. Taking into account that the secondary beams are located in the upper side of the primary beam, we can consider that the lateral buckling length is the distance between secondary beams. In the example in the middle, secondary beams are attached to the lower side of the primary beam. If working under positive bending moment, like in the previous example, they are attached to the side working under tensile strength. In that case we don't consider secondary beams like struts, and L_C is the whole length of the bar. In the last example, if the cantilever works, as usual, under negative bending moment, the lower side of the beam is compressed. Therefore, secondary beams don't function as struts and lateral buckling length is $2L$ (buckling length of a cantilever).

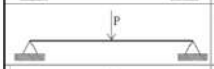
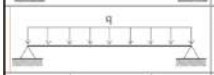
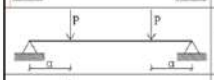
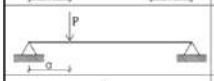
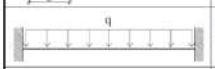
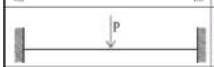
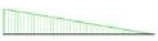
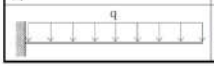
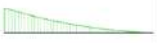
Equivalent moment factor C_1

C_1 factor is used to calculate an equivalent uniform moment diagram. We can obtain its value in the chart 6.7 of the CTE. This table represents simplified moment diagrams between struts.

Tabla 6.7 Valor del factor C_1 correspondiente a los valores del factor k_e (k_e=1)

Condiciones de apoyo y tipo de solicitación	Diagrama de momentos flectores	C_1
	Ψ=+1	1
	 Ψ=+3/4	1,14
	Ψ=+1/2	1,32
	 Ψ=+1/4	1,56
	Ψ=0	1,88
	 Ψ=-1/4	2,28
	 Ψ=-1/2	2,7
	Ψ=-3/4	2,93
	Ψ=-1	2,75

When having other types of moment diagrams, we can use the following table to obtain C_1values:

Karga eta euskarri mota	Momentu diagrama	Momentu koef.
		1.00
		1.75
		2.56
		1.35
		1.13
		$1 + \alpha^2$
		$1.35 + 1.95\,(0.5 - \alpha)^2$
		1.30
		1.70
		1.28
		2.04

Lateral torsional buckling reduction factor χ_{LT}

Summarizing the information of this chapter, these are the steps we have to follow to verify lateral torsional buckling:

1.) $M_{CR} = \sqrt{{M_{LTv}}^2 + {M_{LTw}}^2}$

$M_{LTv} = C_1 \cdot \frac{M}{l_C} \sqrt{I_z \cdot G \cdot I_t \cdot E}$ → lateral torsional buckling 's critical moment with uniform torsion

$M_{LTv} = C_1 \cdot W_{EC,y} \cdot \frac{M^2 \cdot E}{{l_C}^2} \cdot {i_{f,z}}^2$ → lateral torsional buckling 's critical moment with non-uniform torsion

2.) $\overline{\lambda}_{LT} = \sqrt{\frac{W_{EL,y} \cdot f_y}{M_{CR}}}$

$\overline{\lambda}_{LT} : lateral\ torsional\ reduced\ slenderness$

When lateral torsional reduced slenderness is smaller than 0,4, CTE regulation accepts $\chi_{LT} = 1$

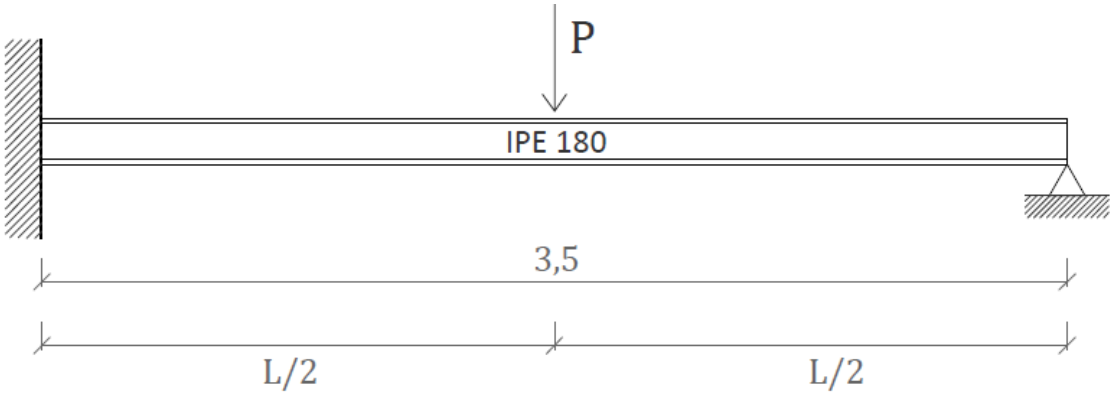
3.) $\left.\begin{array}{l} \overline{\lambda}_{LT} \\ Buckling\ curve\ (chart\ 6.6) \end{array}\right\}$ χ_{LT} lateral buckling reduction factor. (chart 6.3)

Tabla 6.6 Factor de imperfección α_{LT}

Elemento	Límites	Curva de pandeo	α_{LT}
Perfil laminado con sección en doble T	h/b ≤ 2	a	0,21
	h/b > 2	b	0,34

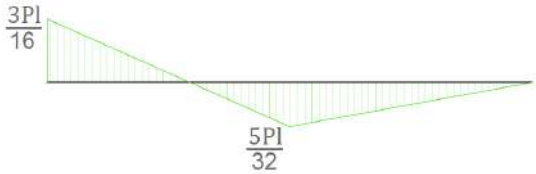
$$\frac{M_{ED}}{\chi_{LT} \cdot W_{EL,y}} \leq f_{yD}$$

EXERCISE. Verify this beam's stability against lateral torsional buckling.



Data:

- P = 2500 kg
- S25 steel
- Actions' magnification factor is 1,5.
- Material's reduction factor is 1,05.



$$P_k = 2500\ kg\ ;\ P_D = 2500 \cdot 1{,}5 = 3750\ kg$$

$$M_D = \frac{3PL}{16} = \frac{3 \cdot 3750 \cdot 3{,}5}{16} = 2461\ kg \cdot m$$

Bar's stability. Lateral torsional buckling:

$$\frac{M_{ED}}{\chi_{LT} \cdot W_y} \leq f_{yD}$$

$\underline{\chi_{LT}}$:

$$M_{LT,v} = b_{LT,v} \cdot \frac{C_1}{L_c} = 9 \cdot 10^{10}\,\frac{1{,}35}{3500} = 3{,}47 \cdot 10^7\ N \cdot mm = 3{,}47 \cdot 10^5\ kg \cdot cm$$

- $b_{LT,v}$ (lat. tors. buckling charts, IPE 180) $= 90121 \cdot 10^6\ N \cdot mm^2 \simeq 9 \cdot 10^{10} N \cdot mm^2$
- C_1 (4. annex. Equivalent moment factors) $= 1{,}35$
- L_C (without joists) $= 3{,}5\ m = 3500\ mmm$

$$M_{LT,w} = b_{LT,w} \cdot \frac{C_1}{{L_c}^2} = 1{,}49 \cdot 10^{14}\,\frac{1{,}35}{3500^2} = 1{,}64 \cdot 10^7\ N \cdot mm = 1{,}64 \cdot 10^5\ kg \cdot cm$$

- $b_{LT,w} = 1{,}49 \cdot 10^{14} N \cdot mm^3$
- $C_1 = 1{,}35$
- $L_C = 3{,}5\ m = 3500\ mmm$

$$M_{CR} = \sqrt{{M_{LT,v}}^2 + {M_{LT,w}}^2} = \sqrt{(3{,}47 \cdot 10^5)^2 + (1{,}64 \cdot 10^5)^2} = 3{,}84 \cdot 10^5\ kg \cdot cm$$

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{CR}}} = \sqrt{\frac{146 \cdot 2750}{3{,}84 \cdot 10^5}} = 1{,}02$$

$$\left.\begin{array}{l} \overline{\lambda}_{LT} = 1{,}02 \\ Buckling\ curve\ \rightarrow A \end{array}\right\} \chi_{LT} = 0{,}67$$

Verification:

$$\frac{M_{ED}}{\chi_{LT} \cdot W_y} \leq f_{yD}\ ;\ \frac{246100\ (kg \cdot cm)}{0{,}67 \cdot 146} = 2516 < \frac{2750}{1{,}05} = 2619\ \mathrm{kg}/cm^2\ \checkmark$$

7.3.3.3. Bending compression stress

Applied to any piece:

$$\frac{N_{ED}}{\chi_y \cdot A^* \cdot f_{yD}} + k_y \cdot \frac{C_{m,y} \cdot M_{y,ED} + e_{N,y} \cdot N_{ED}}{\chi_{LT} \cdot W_y \cdot f_{yD}} + \alpha_z \cdot k_z \cdot \frac{c_{m,z} \cdot M_{z,ED} + e_{N,z} \cdot N_{ED}}{W_z \cdot f_{yD}} \leq 1$$

Pieces that cannot suffer buckling generated by torsion:

$$\frac{N_{ED}}{\chi_z \cdot A^* \cdot f_{yD}} + k_y \cdot \frac{C_{m,y} \cdot M_{y,ED} + e_{N,y} \cdot N_{ED}}{W_y \cdot f_{yD}} + k_z \cdot \frac{c_{m,z} \cdot M_{z,ED} + e_{N,z} \cdot N_{ED}}{W_z \cdot f_{yD}} \leq 1$$

Pieces that can suffer buckling generated by torsion:

$$\frac{N_{ED}}{\chi_z \cdot A^* \cdot f_{yD}} + k_{yLT} \cdot \frac{M_{y,ED} + e_{N,y} \cdot N_{ED}}{\chi_{LT} \cdot W_y \cdot f_{yD}} + k_z \cdot \frac{c_{m,z} \cdot M_{z,ED} + e_{N,z} \cdot N_{ED}}{W_z \cdot f_{yD}} \leq 1$$

$N_{ED}, M_{y,ED}$ and $M_{z,ED}$ the highest value of the axial and bending strengths in a bar.

$f_{yD} = {f_y}/{\gamma_{M1}}$

A^*; W_y; W_z; α_y; α_z; $e_{N,y}$; $e_{N,z}$ values are taken from chart 6.8.

χ_y *and* χ_z	buckling reduction factors about y and z axis
χ_{LT}	lateral torsional buckling reduction factor according to chart 6.3.3. If there is no risk of suffering from buckling generated by torsion, we take the value 1,00.
$e_{N,y}$ *and* $e_{N,z}$	Eccentricity of the effective centre of gravity of the cross section in reference to the centre of gravity of the whole section.
k_y, k_z *and* k_{yLT}	Interaction factors. They are indicated in the chart 3.9.
$C_{m,y}, C_{mz,}$ *eta* C_{mLT}	Equivalent moment factors. They are taken from the chart 6.10, depending on the bending moment diagram's form.

- A^*; W_y; W_z; α_y; α_z; $e_{N,y}$; $e_{N,z}$ factors:

Tabla 6.8 Términos de comprobación, según peor clase de sección en la pieza

Clase	A^*	W_y	W_z	α_y	α_z	$e_{N,y}$	$e_{N,z}$
1	A	$W_{pl,y}$	$W_{pl,z}$	0,6	0,6	0	0
2	A	$W_{pl,y}$	$W_{pl,z}$	0,6	0,6	0	0
3	A	$W_{el,y}$	$W_{el,z}$	0,8	1	0	0
4	A_{eff}	$W_{eff,y}$	$W_{eff,z}$	0,8	1	Según pieza y tensiones	Según pieza y tensiones

- k_y, k_z *and* k_{yLT} factors:

Tabla 6.9 Coeficientes de interacción según peor clase de sección en la pieza

Clase	Tipo de sección	k_y	k_z	k_{yLT}
1 y 2	I, H, abiertas	$1+(\bar{\lambda}_y-0{,}2)\cdot\frac{N_{Ed}}{\chi_y N_{C,Rd}}$	$1+(2\cdot\bar{\lambda}_z-0{,}6)\cdot\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	el menor de $1-\frac{0{,}1\cdot\bar{\lambda}_z}{(c_{mLT}-0{,}25)}\frac{N_{Ed}}{\chi_z N_{C,Rd}}$
	Hueca delgada		$1+(\bar{\lambda}_z-0{,}2)\cdot\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	$0{,}6+\bar{\lambda}_z$
3 y 4	Todas	$1+0{,}6\cdot\bar{\lambda}_y\cdot\frac{N_{Ed}}{\chi_y N_{C,Rd}}$	$1+0{,}6\cdot\bar{\lambda}_z\cdot\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	$1-\frac{0{,}05\cdot\bar{\lambda}_z}{(c_{mLT}-0{,}25)}\frac{N_{Ed}}{\chi_z N_{C,Rd}}$

siendo

$\bar{\lambda}_y$ y $\bar{\lambda}_z$ valores de las esbelteces reducidas para los ejes y – y y z – z, no mayores que 1,00.

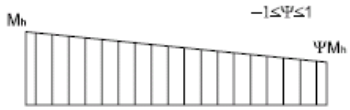
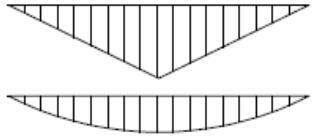
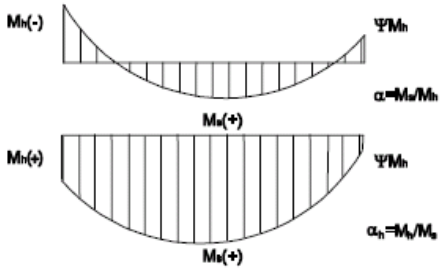
$N_{C,Rd} = A^* \cdot \frac{f_y}{\gamma_{M1}}$

- $C_{m,y}, C_{m,z}$ *and* C_{mLT} factors (depending on the moment diagram's form):

*On structures without bracing, if buckling length is bigger than bar's length we have to take $c_m = 0{,}9$ value.

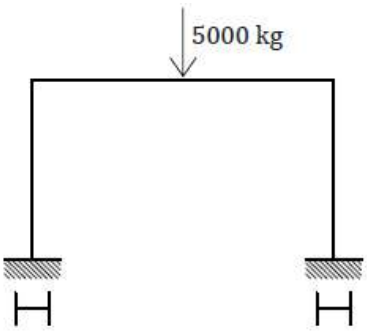
Tabla 6.10 Coeficientes del momento equivalente

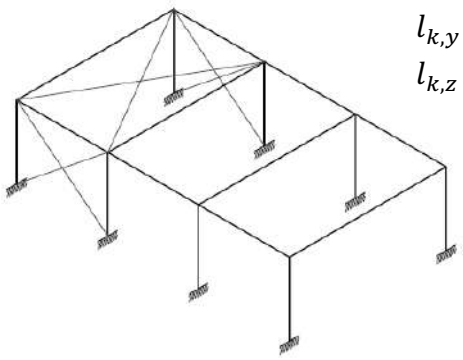
Factor de momento flector	Eje de flexión	Puntos arriostrados en dirección
$c_{m,y}$	y–y	z - z
$c_{m,z}$	z - z	y–y
$c_{m,LT}$	y – y	y–y

Diagrama de Flectores	Factor de momento uniforme equivalente $c_{m,y} = c_{m,i}$ (i =y) $c_{m,z} = c_{m,i}$ (i =z) $c_{m,LT} = c_{m,i}$ (i =LT)
Momentos de extremo  	$c_{m,i} = 0{,}6 + 0{,}4 \cdot \psi \geq 0{,}4$
Momento debido a cargas laterales coplanarias 	$c_{m,i} = 0{,}9$ $c_{m,i} = 0{,}95$
Momentos debidos a cargas laterales y momentos de extremos  	$c_{m,i} = 0{,}1 - 0{,}8 \cdot \alpha \geq 0{,}4$ si $-1 \leq \alpha \leq 0$ $c_{m,i} = 0{,}2 + 0{,}8 \cdot \alpha \geq 0{,}4$ si $0 \leq \alpha \leq 1$ $c_{m,i} = 0{,}95 + 0{,}05 \cdot \alpha_h$ con $-1 \leq \alpha_h \leq 1$

- <u>C_m equivalent moment's factor calculation</u>

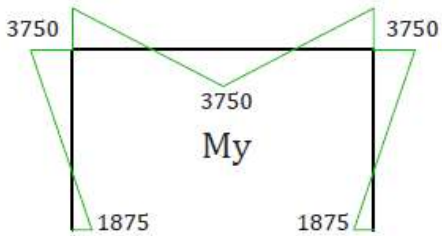
a)





$l_{k,y} = 2l$

$l_{k,z} = 0{,}7l$



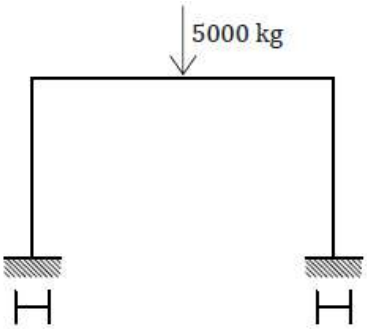
$\boxed{c_{m,y} = 0{,}9}$ $(l_{k,y} > l)$

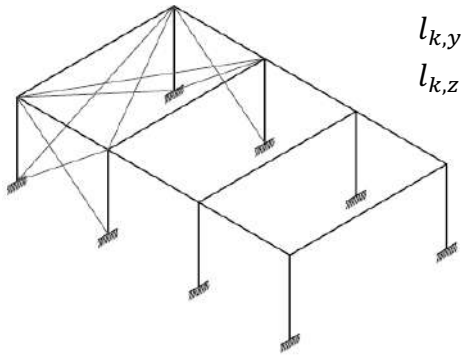
$$\psi = \frac{1875}{-3750} = -0{,}5$$

$c_{m,LT} = 0{,}6 + 0{,}4\psi = 0{,}6 + 0{,}4(-0{,}5)$;

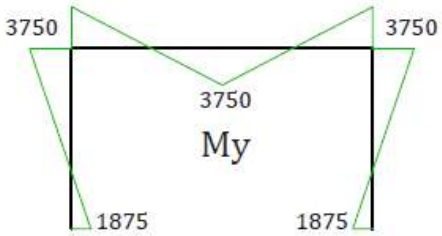
$\boxed{c_{m,LT} = 0{,}4}$

b)





$$l_{k,y} = 0{,}7l$$
$$l_{k,z} = 0{,}7l$$

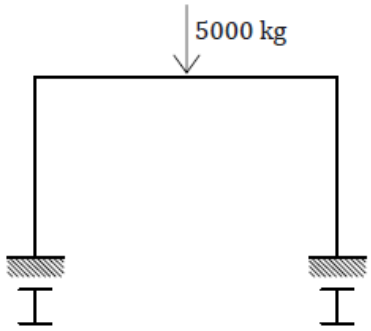


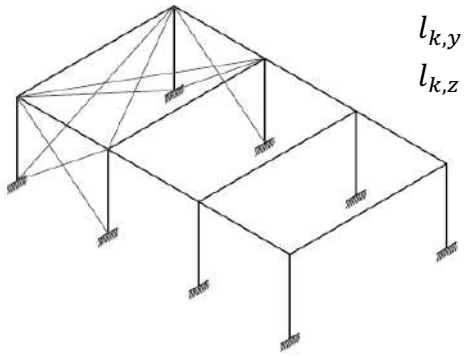
$$\psi = \frac{1875}{-3750} = -0{,}5$$

$$c_{m,y} = 0{,}6 + 0{,}4\psi \;\rightarrow\; \boxed{c_{m,y} = 0{,}4}$$

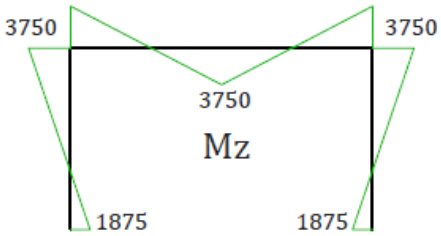
$$c_{m,LT} = 0{,}6 + 0{,}4\psi \;\rightarrow\; \boxed{c_{m,LT} = 0{,}4}$$

c)





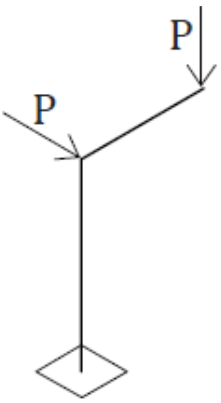
$$l_{k,y} = 0{,}7l$$
$$l_{k,z} = 0{,}7l$$



$$\psi = \frac{1875}{-3750} = -0{,}5$$

$$c_{m,z} = 0{,}6 + 0{,}4\psi \;\rightarrow\; \boxed{c_{m,z} = 0{,}4}$$

d)



$l_{k,y} = 2l$

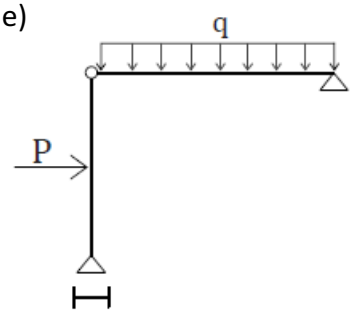
$l_{k,z} = 2l$

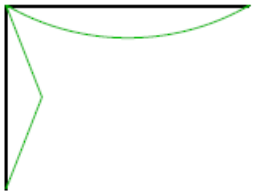
$\boxed{c_{m,y} = 0{,}9}$

$\boxed{c_{m,z} = 0{,}9}$

$\boxed{c_{m,LT} = 0{,}9}$

e)



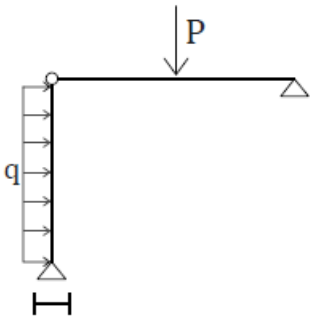


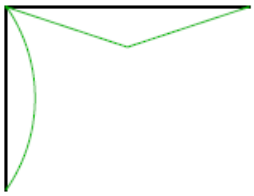
$l_{k,y} = l$

$\boxed{c_{m,y} = 0{,}9}$

$\boxed{c_{m,LT} = 0{,}9}$

f)



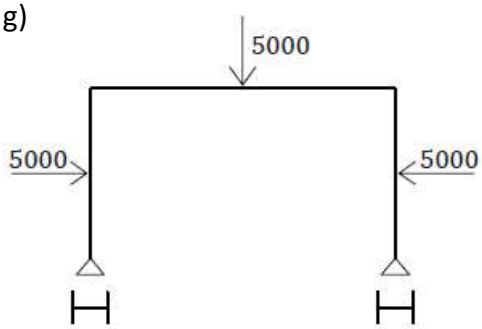


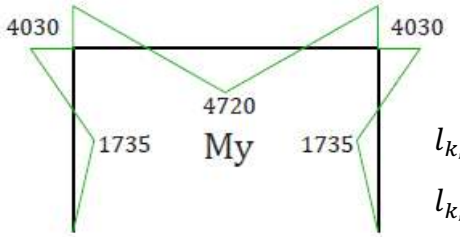
$l_{k,y} = l$

$\boxed{c_{m,y} = 0{,}95}$

$\boxed{c_{m,LT} = 0{,}95}$

g)





$l_{k,y} = 2l$

$l_{k,z} = 0{,}7l$

$\boxed{c_{m,y} = 0{,}9}$ $(l_{k,y} > l)$

$\boldsymbol{c_{m,LT}}$

The expression we have to apply to know α value is:

$$\alpha = \frac{M_s}{M_H} \quad (M_S < M_H) \ ; or \quad \alpha_h = \frac{M_H}{M_S} \quad (M_H < M_S)$$

M_S: Biggest bending moment along the bar. Absolute value

M_H: Biggest bending moment in any end of the bar. Absolute value

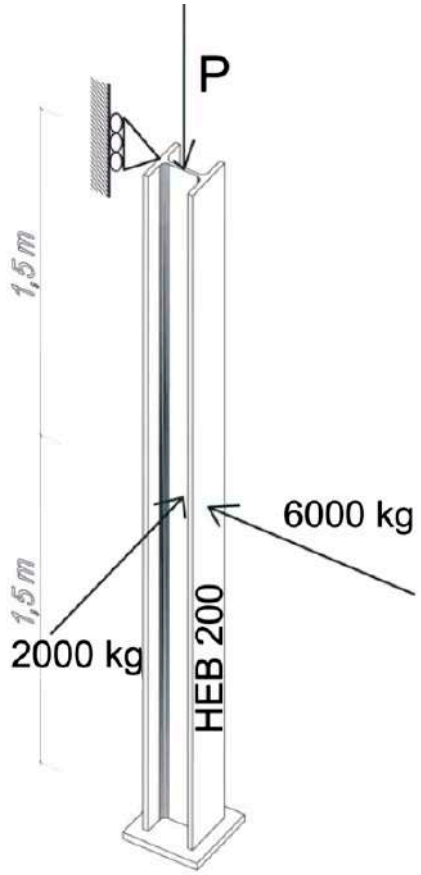
In our case $M_S < M_H$, thus:

$$\alpha = \frac{1735}{-4030} = -0{,}43$$

$$c_{m,LT} = 0{,}1 - 0{,}8\alpha \geq 0{,}4 \quad (-1 \leq \alpha \leq 0)$$

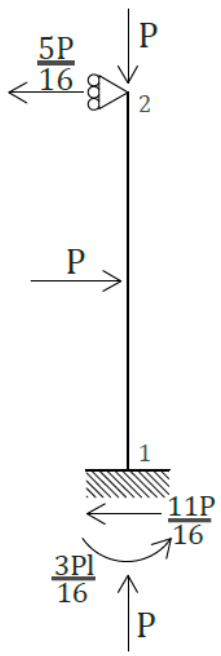
$$c_{m,LT} = 0{,}1 - 0{,}8(-0{,}43) = 0{,}44 \ ; \quad \boxed{c_{m,LT} = 0{,}44}$$

Exercise. Define which is the biggest P value that this column can support.



Data:

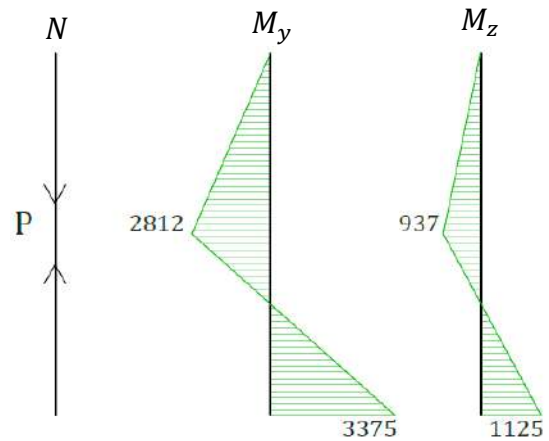
- S275 steel
- There are no ultimate limit state limitations.
- Material's reduction factor is 1,05.
- Don´t take into account magnification and simultaneity factors.



$$P = V_1 + V_2\ ;\ V_1 = P - V_2 = P - \frac{5P}{16} = \frac{11P}{16}$$

$$\Sigma M_1 = 0\ ; P \cdot \frac{l}{2} = V_2 \cdot l + \frac{3Pl}{16}\ ;\ V_2 = \left(\frac{9Pl}{16} - \frac{3Pl}{16}\right)\frac{1}{l} = \frac{5P}{16}$$

Then,



Section's resistance (worst section in 1 support):

$$\frac{N_{ED}}{A} + \frac{M_{y,ED}}{W_{y,EL}} + \frac{M_{z,ED}}{W_{z,EL}} \leq f_{yD}$$

$$\frac{P}{78{,}1} + \frac{337500}{570} + \frac{112500}{200} \leq \frac{2750}{1{,}05};\ \frac{P}{78{,}1} \leq 2619 - 592{,}1 - 562{,}5\ ;$$

$$P \leq 78{,}1 \cdot 1464{,}4\ ;\quad P \leq 114369{,}6\ kg$$

Stability:

$$1)\quad \frac{N_{ED}}{\chi_y \cdot A \cdot f_{yD}} + k_y \cdot \frac{C_{m,y} \cdot M_{y,ED}}{\chi_{LT} \cdot W_y \cdot f_{yD}} + \alpha_z \cdot k_z \cdot \frac{c_{m,z} \cdot M_{z,ED}}{W_z \cdot f_{yD}} \leq 1$$

$$2)\quad \frac{N_{ED}}{\chi_z \cdot A \cdot f_{yD}} + k_{y,LT} \cdot \frac{M_{y,ED}}{\chi_{LT} \cdot W_y \cdot f_{yD}} + k_z \cdot \frac{c_{m,z} \cdot M_{z,ED}}{W_z \cdot f_{yD}} \leq 1$$

$\underline{\chi_y}$

$$N_{CRy} = \frac{\pi^2}{{l_k}^2} E \cdot I_y = \frac{\pi^2}{210^2} \cdot 2{,}1 \cdot 10^6 \cdot 5696 = 2{,}68 \cdot 10^6\ kg$$

$$l_k = \beta \cdot l = 0{,}7 \cdot 300 = 210\ cm$$

$$I_y = 5696\ cm^4$$

$$\bar{\lambda}_y = \sqrt{\frac{A \cdot f_y}{N_{CRy}}} = \sqrt{\frac{78{,}1 \cdot 2750}{2{,}68 \cdot 10^6}} = 0{,}28$$

Chart 6.2:

$$HEB\ 200 : \frac{h}{b} < 1{,}2\ ; t < 100\ mm \ \rightarrow\ B\ curve\ (y\ axis)$$

Chart 6.3:

$$\left.\begin{array}{l} \bar{\lambda} = 0{,}28 \\ B\ curve \end{array}\right\} \quad \chi_y = 0{,}96$$

$\underline{\chi_z}$

$$N_{CRz} = \frac{\pi^2}{{l_k}^2} E \cdot I_z = \frac{\pi^2}{210^2} \cdot 2{,}1 \cdot 10^6 \cdot 2003 = 9{,}4 \cdot 10^5\ kg$$

$$l_k = \beta \cdot l = 0{,}7 \cdot 300 = 210\ cm$$

$$I_z = 2003\ cm^4$$

$$\bar{\lambda}_z = \sqrt{\frac{A \cdot f_y}{N_{CRz}}} = \sqrt{\frac{78{,}1 \cdot 2750}{9{,}4 \cdot 10^5}} = 0{,}48$$

Chart 6.2:

$$HEB\ 200 : \frac{h}{b} < 1{,}2\ ; t < 100\ mm \ \rightarrow\ C\ curve\ (z\ axis)$$

Chart 6.3:

$$\left.\begin{array}{l}\bar{\lambda} = 0{,}48 \\ C\ curve\end{array}\right\} \quad \chi_z = 0{,}85$$

$\underline{\chi_{LT}}$

$$M_{LT,v} = b_{LT,v} \cdot \frac{C_1}{L_c} = 1{,}46 \cdot 10^{12}\,\frac{1{,}35}{3000} = 6{,}57 \cdot 10^7\ N \cdot mm = 6{,}57 \cdot 10^6\ kg \cdot cm$$

- $b_{LT,v}$ (lateral torsional buckling chart, HEB 200) $= 1{,}46 \cdot 10^{12}\ N \cdot mm^2$
- C_1 (4. annex equivalent moment C_1 factor) $= 1{,}35$
- L_C (without struts) $= 3\ m = 3000\ mmm$

$$M_{LT,w} = b_{LT,w} \cdot \frac{C_1}{{L_c}^2} = 3{,}37 \cdot 10^{15}\,\frac{1{,}35}{3500^2} = 5{,}05 \cdot 10^8\ N \cdot mm = 5{,}05 \cdot 10^6\ kg \cdot cm$$

- $b_{LT,w} = 3{,}37 \cdot 10^{15} N \cdot mm^3$
- $C_1 = 1{,}35$
- $L_C = 3\ m = 3000\ mmm$

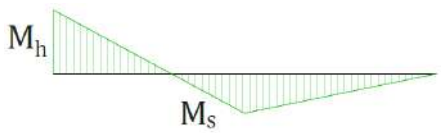
$$M_{CR} = \sqrt{{M_{LT,v}}^2 + {M_{LT,w}}^2} = \sqrt{(6{,}57 \cdot 10^6)^2 + (5{,}05 \cdot 10^6)^2} = 8{,}28 \cdot 10^6\ kg \cdot cm$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{CR}}} = \sqrt{\frac{570 \cdot 2750}{8{,}28 \cdot 10^6}} = 0{,}435$$

$$\left.\begin{array}{l}\bar{\lambda}_{LT} = 0{,}435 \\ Buckling\ curve\ \rightarrow A\end{array}\right\} \chi_{LT} = 0{,}93$$

$\underline{C_{m,y}, C_{m,z}\ amd\ C_{mLT}}$

6.10 chart ($M_y\ and\ M_z\ are\ equals;\ l_c = l;\ C_{m,y} = C_{m,z}\ C_{mLT}$)



$$M_s < M_h\ \rightarrow\ \alpha\frac{M_s}{M_h} = \frac{2812}{-3375} = -0{,}833$$

$$0 \geq \alpha \geq -1\ ;\ C_{m,i} = 0{,}1 - 0{,}8\alpha = 0{,}77$$

$\underline{\alpha_y \; and \; \alpha_z}$

6.8 chart; $\alpha_y = \alpha_z = 0{,}6 \;\; (1.class)$

$\underline{k_y, k_z \;\; and \; k_{LT}}$

6.9 chart

$$N_{C,RD} = A \cdot \frac{f_y}{l_M} = 78{,}1 \cdot \frac{2750}{1{,}05} = 204547{,}6 \; kg$$

$$k_y = 1 + \left(\bar{\lambda}_y - 0{,}2\right) \frac{N_{ED}}{\chi_y \cdot N_{C,RD}} = 1 + (0{,}28 - 0{,}2) \frac{114369{,}6}{0{,}96 \cdot 204547{,}6} = 1{,}05$$

$$k_z = 1 + \left(\bar{\lambda}_z - 0{,}6\right) \frac{N_{ED}}{\chi_z \cdot N_{C,RD}} = 1 + (2 \cdot 0{,}48 - 0{,}6) \frac{114369{,}6}{0{,}85 \cdot 204547{,}6} = 1{,}24$$

$$\boxed{k_{LT}} = 1 - \frac{0{,}1 \cdot \bar{\lambda}_z}{\left(C_{m,LT} - 0{,}25\right)} \cdot \frac{N_{ED}}{\chi_y \cdot N_{C,RD}} = 1 - \frac{0{,}1 \cdot 0{,}48}{(0{,}77 - 0{,}25)} \cdot \frac{114369{,}6}{0{,}85 \cdot 204547{,}6} = \boxed{0{,}94}$$

$$k_{LT} = 0{,}6 + \bar{\lambda}_z = 0{,}6 + 0{,}48 = 1{,}08$$

Verification

1) $$\frac{N_{ED}}{\chi_y \cdot A \cdot f_{yD}} + k_y \cdot \frac{C_{m,y} \cdot M_{y,ED}}{\chi_{LT} \cdot W_y \cdot f_{yD}} + \alpha_z \cdot k_z \cdot \frac{c_{m,z} \cdot M_{z,ED}}{W_z \cdot f_{yD}} \leq 1$$

$$\frac{114369{,}6}{0{,}96 \cdot 78{,}1 \cdot 2619} + 1{,}05 \frac{0{,}77 \cdot 337500}{0{,}93 \cdot 570 \cdot 2619} + 0{,}6 \cdot 1{,}24 \frac{0{,}77 \cdot 112500}{200 \cdot 2619} \leq 1$$

$0{,}58 + 0{,}2 + 0{,}12 \leq 1 \; ; \;\; 0{,}9 < 1 \;\; \rightarrow \;\; valid \;\; \checkmark$

2) $$\frac{N_{ED}}{\chi_z \cdot A \cdot f_{yD}} + k_{y,LT} \cdot \frac{M_{y,ED}}{\chi_{LT} \cdot W_y \cdot f_{yD}} + k_z \cdot \frac{c_{m,z} \cdot M_{z,ED}}{W_z \cdot f_{yD}} \leq 1$$

$$\frac{114369{,}6}{0{,}85 \cdot 78{,}1 \cdot 2619} + 0{,}94\frac{337500}{0{,}93 \cdot 570 \cdot 2619} + 1{,}24\frac{0{,}77 \cdot 112500}{200 \cdot 2619} \leq 1$$

$$0{,}66 + 0{,}23 + 0{,}2 \nless 1\ ;\ \ 1{,}09 \nless 1 \ \ \rightarrow \ \ not\ valid\ \times$$

<u>We are going to calculate the second equation under **P** value again:</u>

$$N_{ED} = P$$

$$k_z = 1 + \left(\bar{\lambda}_z - 0{,}6\right)\frac{N_{ED}}{\chi_z \cdot N_{C,RD}} = 1 + (2 \cdot 0{,}48 - 0{,}6)\frac{P}{0{,}85 \cdot 204547{,}6}$$
$$= 1 + P \cdot 2{,}07 \cdot 10^{-6}$$

$$\boxed{k_{LT}} = 1 - \frac{0{,}1 \cdot \bar{\lambda}_z}{\left(C_{m,LT} - 0{,}25\right)} \cdot \frac{N_{ED}}{\chi_y \cdot N_{C,RD}} = 1 - \frac{0{,}1 \cdot 0{,}48}{(0{,}77 - 0{,}25)} \cdot \frac{P}{0{,}85 \cdot 204547{,}6}$$
$$= \boxed{1 - P \cdot 5{,}31 \cdot 10^{-7}}$$

$$2)\quad \frac{N_{ED}}{\chi_y \cdot A \cdot f_{yD}} + k_{y,LT} \cdot \frac{M_{y,ED}}{\chi_{LT} \cdot W_y \cdot f_{yD}} + k_z \cdot \frac{c_{m,z} \cdot M_{z,ED}}{W_z \cdot f_{yD}} \leq 1$$

$$\frac{P}{0{,}85 \cdot 78{,}1 \cdot 2619} + (1 - P \cdot 5{,}31 \cdot 10^{-7})\frac{337500}{0{,}93 \cdot 570 \cdot 2619}$$
$$+(1 + P \cdot 2{,}07 \cdot 10^{-6})\frac{0{,}77 \cdot 112500}{200 \cdot 2619} \leq 1$$

$$5{,}75 \cdot 10^{-6} \cdot P + 0{,}2431 - P \cdot 1{,}3 \cdot 10^{-7} + 0{,}166 + P \cdot 3{,}42 \cdot 10^{-7} \leq 1$$

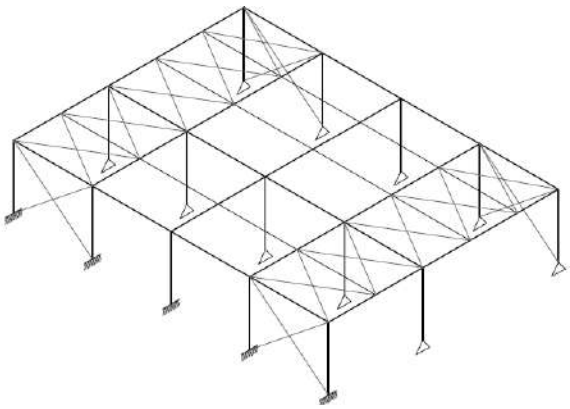
$$P(5{,}75 \cdot 10^{-6} + 3{,}42 \cdot 10^{-7} - 1{,}3 \cdot 10^{-7}) \leq 1 - 0{,}2431 - 0{,}166$$

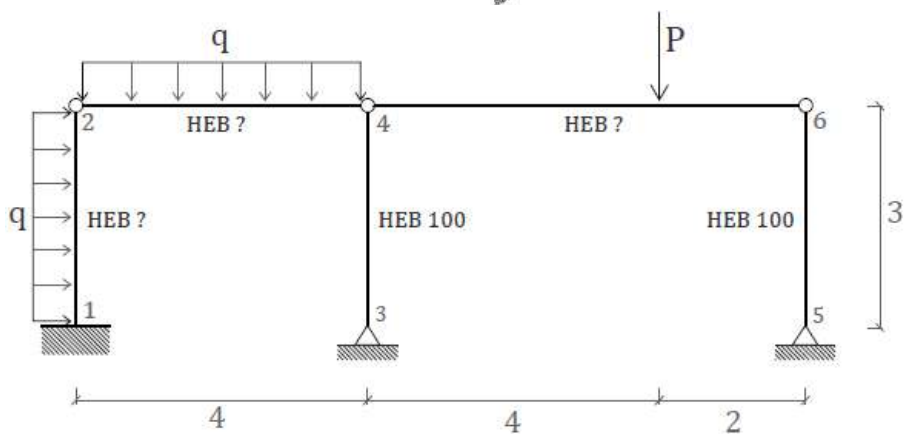
$$P \leq \frac{0{,}591}{5{,}96 \cdot 10^{-6}}\ ;\ \ P \leq 99161\ kg$$

$$\boxed{P = 99161\ kg} \ \rightarrow \ \ Maximum\ \boldsymbol{P}\ load\ supported\ by\ this\ column$$

7.3.4. Overall review exercise

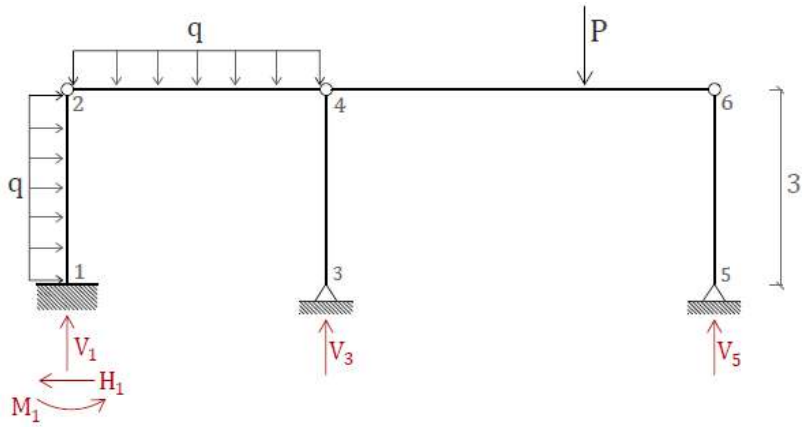
1. Through the Ultimate Limit States (ULS), analyse 4-3 and 6-5 columns and define the maximum **P** and **q** loads that the frame can support.
2. With **P** and **q** values:
   a. Draw the diagrams of strengths in characteristic value.
   b. Pre dimension beams 2-4 and 4-6 according to the Serviceability Limit States.
   c. Verify beam 2-4 lateral torsional buckling.
   d. Verify beam 4-6 section resistance.
   e. Pre dimension column 1-2 according to the Serviceability Limit States.





Data:

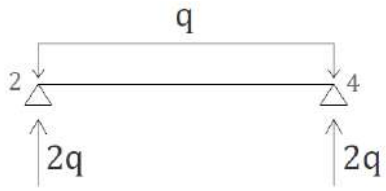
- S275 Steel
- Actions' magnification factor is 1,5.
- Material's reduction factor is 1,05.
- Don´t use simultaneity factors, only one load combination.
- Beam's deflection in the middle of the beam $\delta = L/300$; L being the beam length.
- Column's horizontal deflection $\delta = H/250$; H being the column height



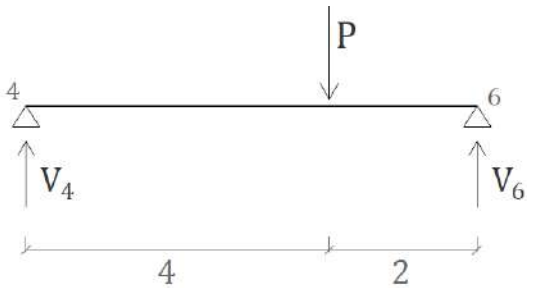
$\Sigma H = 0;\ H_1 = 3q$

$\Sigma M_1 = 0;\ -4{,}5q - 8q - 8P + 4V_3 + 10V_5 + M_1 = 0;\ M_1 = 4{,}5q$

<u>2-4 bi-hinged beam</u>



<u>4-6 bi-hinged beam</u>



$\Sigma M_2 = 0;\ 4P = 6V_6;\ V_6 = {}^{2P}/_{3}$

$V_4 = P - {}^{2P}/_{3} = {}^{P}/_{3}$

Thus,

$V_1 = 2q\ ;\ V_3 = 2q + {}^{P}/_{3}\ ;\ V_5 = {}^{2P}/_{3}\ ;\ H_1 = 3q\ ;\ M_1 = 4{,}5q$

Column 6-5 (only flection)

$$\frac{N_{ED}}{\chi_z \cdot A} \leq f_{yD}\ ;\ \frac{1{,}5 \cdot (2P/3)}{0{,}37 \cdot 26} \leq \frac{2750}{1{,}05}\ ;\ \boxed{P = \frac{0{,}37 \cdot 26 \cdot 2750 \cdot 3}{1{,}05 \cdot 1{,}5 \cdot 2} = 25195{,}24\ kg}$$

$$N_{CR,z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z = \frac{\pi^2}{300^2} \cdot 2{,}1 \cdot 10^6 \cdot 167 = 38458{,}6\ kg$$

$$\left.\begin{array}{l} \bar{\lambda}_z = \sqrt{\dfrac{A \cdot f_y}{N_{CR,z}}} = \sqrt{\dfrac{26 \cdot 2750}{38458{,}6}} = 1{,}36 \\ Buckling\ curve\ \rightarrow C \end{array}\right\} \quad \chi_z = 0{,}37$$

4-3 curve (axial strength)

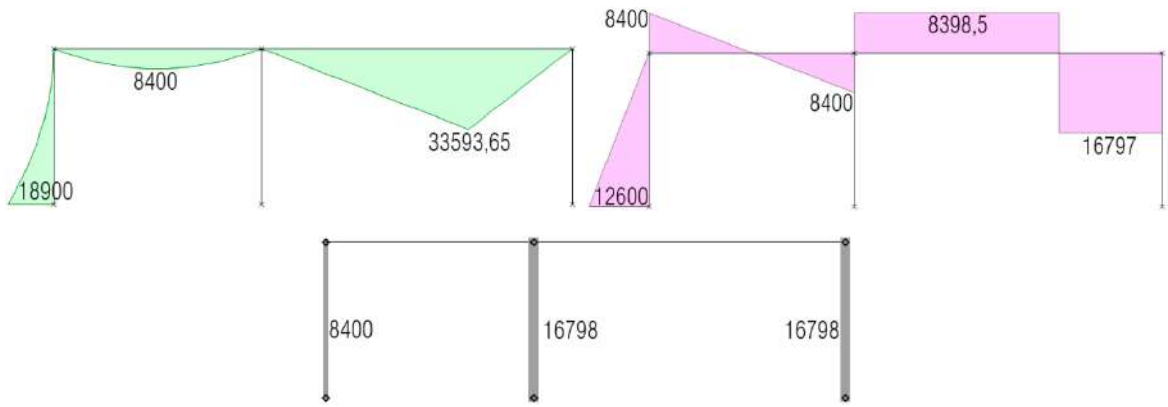
$$\frac{N_{ED}}{\chi_z \cdot A} \leq f_{yD}\ ;\ \frac{1{,}5 \cdot (2q + P/3)}{0{,}37 \cdot 26} \leq \frac{2750}{1{,}05}\ ;$$

$$\chi_z\ \rightarrow same\ to\ previous\ beam\ \rightarrow 0{,}37$$

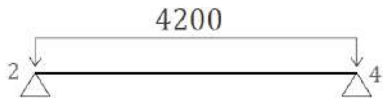
$$\frac{1{,}5 \cdot (2q + 25195/3)}{0{,}37 \cdot 26} \leq \frac{2750}{1{,}05}\ ;\ q = \left(\frac{0{,}37 \cdot 26 \cdot 2750}{1{,}05 \cdot 1{,}5} - \frac{25195{,}24}{3}\right) \cdot \frac{1}{2}\ ;$$

$$\boxed{q = (16796{,}82 - 8398{,}41) \cdot \frac{1}{2} \simeq 4200\ kjg}$$

Diagrams



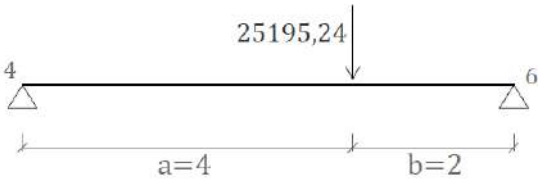
Beam 2-4



$$\delta = \frac{5}{384} \cdot \frac{q \cdot l^4}{EI} \; ; I = \frac{5}{384} \cdot \frac{q \cdot l^4}{E \cdot \delta} = \frac{5}{384} \cdot \frac{42 \cdot 400^4}{2{,}1 \cdot 10^6 \cdot 1{,}\hat{3}} = 5000\ cm^4 \ \rightarrow \boxed{HEB\ 200}$$

$$\delta_{MAX} = {}^{l}/_{300} = {}^{400}/_{300} = 1{,}\hat{3}\ cm$$

Beam 4-6



$$\delta = \frac{P \cdot b}{3l \cdot EI} \cdot \left[\frac{a \cdot (l+b)}{3}\right]^{3/2} \; ; \; I = \frac{P \cdot b}{3l \cdot E \cdot \delta} \cdot \left[\frac{a \cdot (l+b)}{3}\right]^{3/2} ;$$

$$I = \frac{25195{,}24 \cdot 200}{3 \cdot 600 \cdot 2{,}1 \cdot 10^6 \cdot 2} \cdot \left[\frac{400 \cdot (600+200)}{3}\right]^{3/2} = 6{,}66 \cdot 10^{-4} \cdot \left(10{,}\hat{6} \cdot 10^4\right)^{3/2}$$
$$= 23210\ cm^4 \ \rightarrow \ \boxed{HEB\ 300}$$

$$\delta_{MAX} = {}^{l}/_{300} = {}^{600}/_{300} = 2\ cm$$

Beam 2-4, lateral torsional buckling

$$\frac{M_{ED}}{\chi_{LT} \cdot W_y} \leq f_{yD} \; ; \; \frac{8400 \cdot 100 \cdot 1{,}5}{1 \cdot 570} \leq \frac{2750}{1{,}05} \; ; 2210 < 2619 \checkmark$$

- $M_{LT,v} = b_{LT,v} \cdot \frac{C}{l_c} = 1{,}46 \cdot 10^{12}\ {}^{1{,}32}/_{2000} = 9{,}64 \cdot 10^8 N \cdot mm = 9{,}64 \cdot 10^6\ kg \cdot cm$
  - $b_{LT,v} = 1{,}46 \cdot 10^{12} N \cdot mm^2$
  - $C = 1{,}32\ (6.7\ chart\ of\ the\ CTE. Central\ section\ of\ the\ beam)$
  - $L_C\ = 2000\ mmm$

- $M_{LT,w} = b_{LT,w} \cdot \frac{C}{{l_c}^2} = 3{,}37 \cdot 10^{15}\ {}^{1{,}32}/_{2000^2} = 1{,}11 \cdot 10^9 N \cdot mm = 1{,}11 \cdot 10^7\ kg \cdot cm$
  - $b_{LT,w} = 3{,}37 \cdot 10^{15} N \cdot mm^3$

- $C = 1{,}32$
- $L_C = 2000\ mmm$

- $M_{CR} = \sqrt{{M_{LT,v}}^2 + {M_{LT,w}}^2} = \sqrt{(9{,}64 \cdot 10^6)^2 + (1{,}11 \cdot 10^7)^2} = 1{,}47 \cdot 10^7\ kg \cdot cm$

- $\bar{\lambda}_{LT} = \sqrt{{W_y \cdot f_y}/{M_{CR}}} = \sqrt{{570 \cdot 2750}/{1{,}47 \cdot 10^7}} = 0{,}326 \rightarrow \boxed{\chi_{LT} = 1}$

Beam 4-6, section resistance

$$\frac{M_{ED}}{W_y} \leq f_{yD}\ ;\ \frac{33593 \cdot 100 \cdot 1{,}5}{1680} \leq \frac{2750}{1{,}05}\ ; 2999 \nless 2619\ ✗$$

We have to dimension it again!

$$\frac{33593 \cdot 100 \cdot 1{,}5}{W_y} \leq \frac{2750}{1{,}05}\ ;\ W_y = \frac{33593 \cdot 100 \cdot 1{,}5 \cdot 1{,}05}{2750} = 1924\ cm^3 \rightarrow \boxed{HEB\ 320}$$

Column 1-2 horizontal deflection

$$\delta_{MAX} = {h}/{250} = {300}/{250} = 1{,}2\ cm$$

$$\delta = \frac{q \cdot l^4}{8EI} = \frac{42 \cdot 300^4}{8 \cdot 2{,}1 \cdot 10^6 \cdot I}\ ; I = \frac{42 \cdot 300^4}{8 \cdot 2{,}1 \cdot 10^6 \cdot 1{,}2} = 16875\ cm^4$$

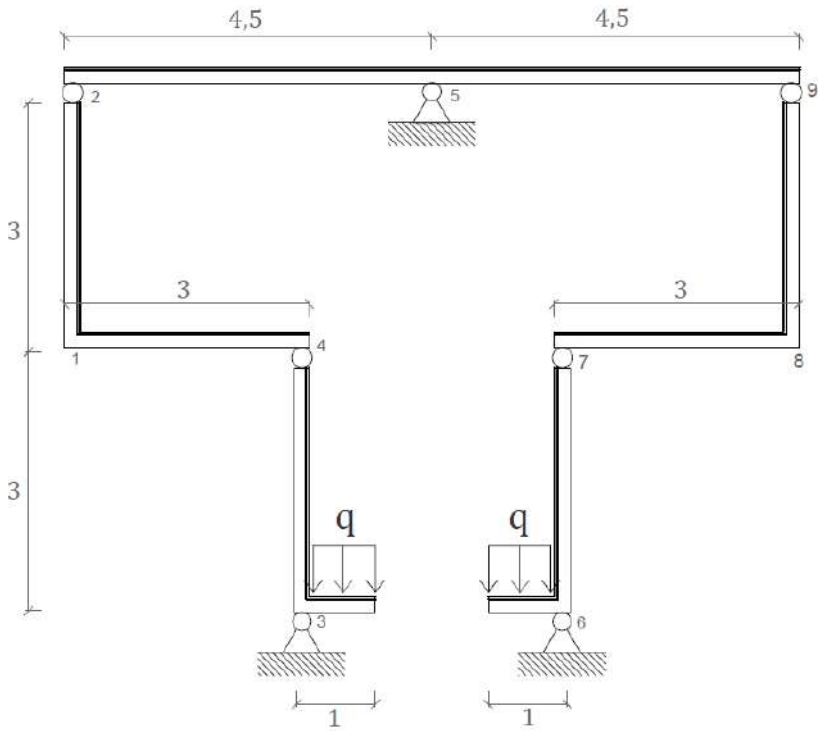
$$\rightarrow \boxed{HEB\ 280}$$

8. EXAM EXERCISES

8.1. VERIFICATIONS OF THE SECTIONS' RESISTANCE. MIXED SECTIONS.

Exercise

1. Draw the diagrams of the following frame under **q** distributed load.
2. Calculate the maximum q distributed load for the frame. Check the Ultimate limit states (ULS) referring to the sections' resistance (just Normal stress) and connectors tangential yield strength.

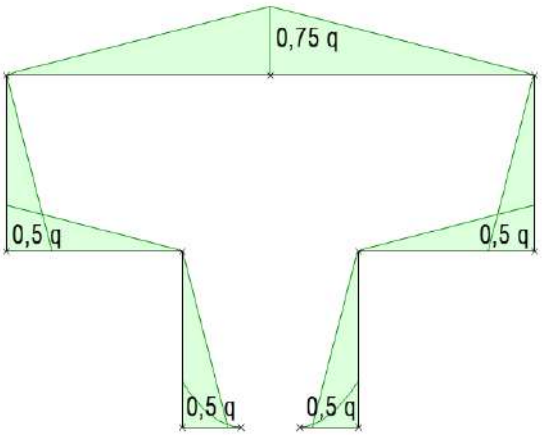


Data:

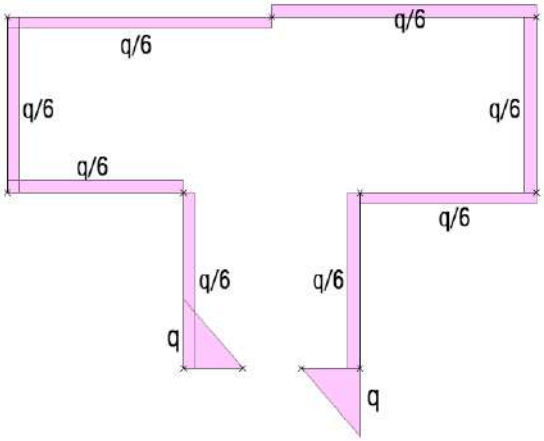
- All the actions include magnification factor.
- Steel's yield strength for traction and compression: 2.750 kg/cm^2
- Steel's elasticity modulus: 2.100.000 kg/cm^2
- Concrete's yield strength for compression: 250 kg/cm^2
- Concrete's yield strength for traction: 105 kg/cm^2

- Concrete's elasticity modulus: 210.000 kg/cm^2
- Connector's diameter: 2 cm
- In the cantilever are 4 connectors.
- In the rest of the frame's bars are 12 connectors.
- Steel's reduction factor: 1,05
- Connector's reduction factor: 1,25
- Concrete's reduction factor: 1,5

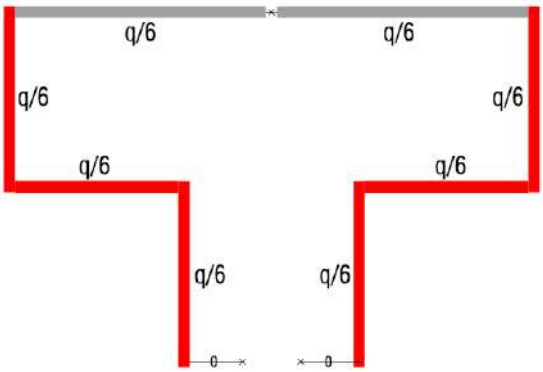
Diagrams:



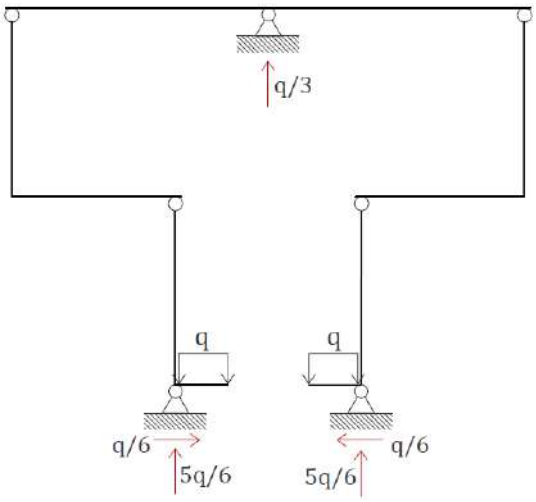
Bending moment



Cutting strength

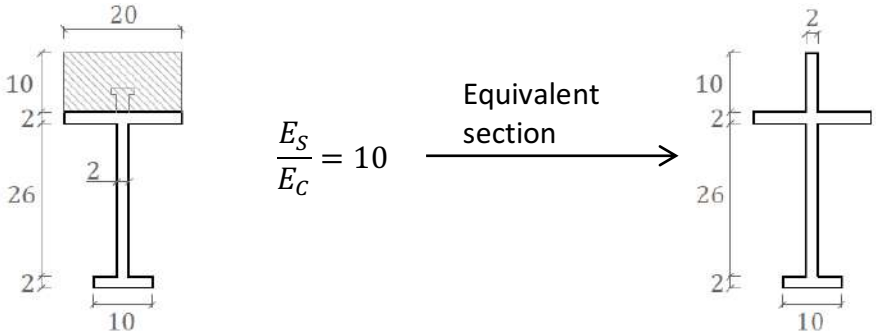


Axial strength

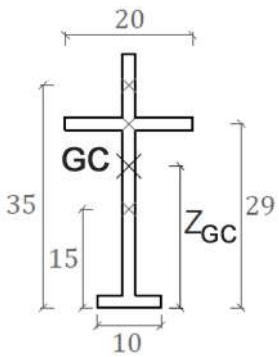


Reactions

Mixed section

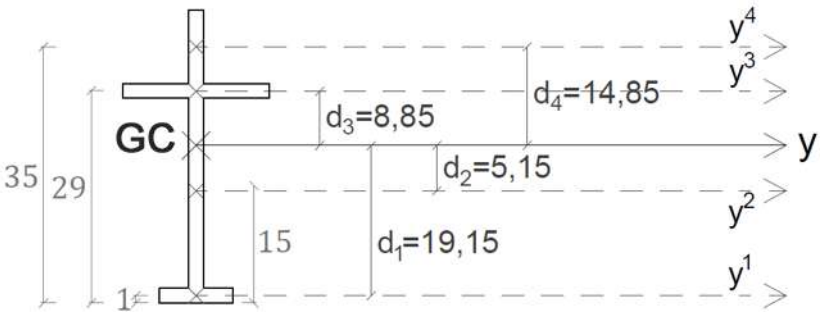


- Centre of gravity location:



$$Z_{G.C.} = \frac{\sum_i^n Z_{GC_i} \cdot A}{\sum_i^n A} = \frac{1(10 \cdot 2) + 15(26 \cdot 2) + 29(20 \cdot 2) + 35(10 \cdot 2)}{(10 \cdot 2) + (26 \cdot 2) + (20 \cdot 2) + (10 \cdot 2)}$$
$$= 20{,}15\ cm$$

-Moment of inertia (Steiner):



$$I_y = \left(I_{y^1} + A_1 \cdot {d_1}^2\right) + \left(I_{y^2} + A_2 \cdot {d_2}^2\right) + \left(I_{y^3} + A_3 \cdot {d_3}^2\right) + \left(I_{y^4} + A_4 \cdot {d_4}^2\right)$$

$$I_y = \left[\left(\frac{10 \cdot 2^3}{12}\right) + (10 \cdot 2) \cdot 19{,}15^2\right] + \left[\left(\frac{2 \cdot 26^3}{12}\right) + (26 \cdot 2) \cdot 5{,}15^2\right]$$
$$+ \left[\left(\frac{20 \cdot 2^3}{12}\right) + (20 \cdot 2) \cdot 8{,}85^2\right] + \left[\left(\frac{2 \cdot 10^3}{12}\right) + (2 \cdot 10) \cdot 14{,}85^2\right]$$

$$I_y = 7341{,}11\hat{6} + 4326{,}50\hat{3} + 3146{,}2\hat{3} + 4577{,}11\hat{6} = 19391\ cm^4$$

As all the bending moments have a negative sign we are going to check the biggest one (sign criteria: traction (-) and compression (+)):

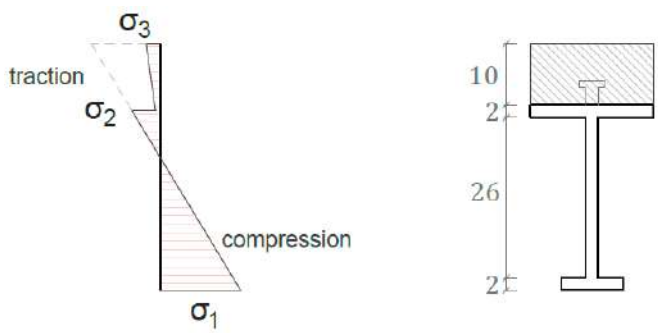
$M_{ED} = -0{,}75q$; $N_{ED} = {}^{q}/_{6}$

$$\sigma_1 = \frac{N_{ED}}{A} + \frac{M_{ED} \cdot z_1}{I_y} = \frac{q/6}{132} + \frac{(-0{,}75q) \cdot 100 \cdot (-20{,}15)}{19391} = q\left(\frac{1}{792} + \frac{1511{,}25}{19391}\right)$$
$$= 7{,}9 \cdot 10^{-2} \cdot q\ kg/cm^2$$

$$\sigma_2 = \frac{N_{ED}}{A} + \frac{M_{ED} \cdot z_2}{I_y} = \frac{q/6}{132} + \frac{(-0{,}75q) \cdot 100 \cdot (9{,}85)}{19391} = q\left(\frac{1}{792} - \frac{738{,}75}{19391}\right) = -3{,}7 \cdot 10^{-2} \cdot q\ kg/cm^2$$

$$\sigma_3 = \frac{N_{ED}}{A} + \frac{M_{ED} \cdot z_3}{I_y} = \frac{q/6}{132} + \frac{(-0{,}75q) \cdot 100 \cdot (19{,}85)}{19391} = q\left(\frac{1}{792} - \frac{1488{,}75}{19391}\right)$$
$$= -7{,}5 \cdot 10^{-2} \cdot q\ kg/cm^2 \rightarrow -7{,}5 \cdot 10^{-3} \cdot q\ kg/cm^2 \text{ (in concrete)}$$

Cross section



Verification in concrete:

$$\sigma_3 \leq f_{CD}; 7{,}5 \cdot 10^{-3} \cdot q \leq \frac{105}{1{,}5}\ ; \boxed{q = 9333\ kg/m}$$

Verification in steel:

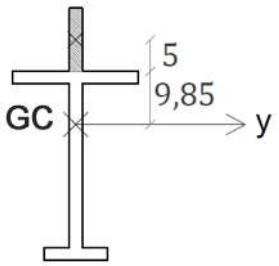
$$\sigma_1 \leq f_{yD}; 7{,}9 \cdot 10^{-2} \cdot q \leq \frac{2750}{1{,}05}\ ; q = 33152\ kg/m$$

The distance between connectors is already given, therefore we are going to analyse the part with the biggest shear stress (in this case the cantilever).

$V_{X,ED} \leq n \cdot V_{kon}$

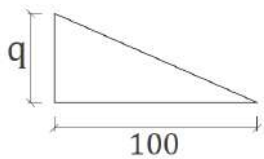
$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y} \cdot \Delta v = \frac{297}{19391} \cdot \frac{q \cdot 100}{2} = 0{,}766 \cdot q$$

<u>$S_y{}^{z_0}$ (Static moment)</u>



$$S_y{}^{z_0} = (2 \cdot 10) \cdot (9{,}85 + 5) = 297$$

<u>Δv (Shear strength diagram's area)</u>



$$\Delta_1 = \frac{q \cdot 100}{2}$$

$$V_{Con} = A_{Con} \cdot \frac{f_{yD}}{\sqrt{3}} = 2 \cdot \frac{2750}{\sqrt{3}} = 2540{,}34\ kg$$

<u>Verification:</u>

$V_{X,ED} \leq n \cdot V_{Con}$

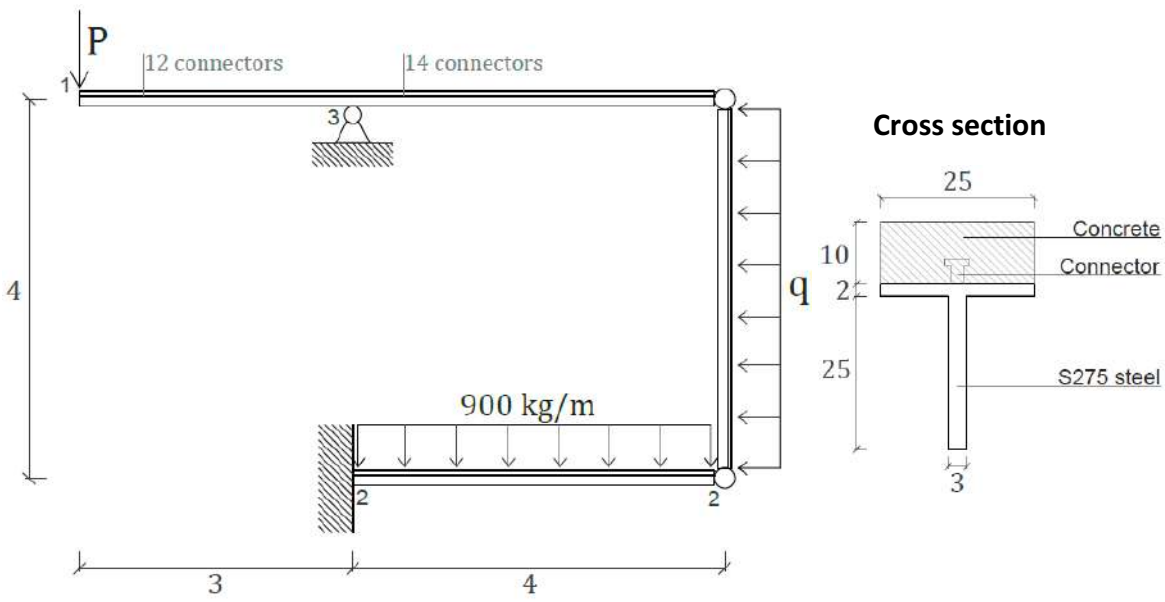
$0{,}766 \cdot q \leq 4 \cdot 2540{,}34\ ;\ q \leq 13265\ kg/m$

Thus, the maximum **q** load that the frame can resist is:

$$\boxed{q = 9333\ kg/m}$$

Exercise.

1. Calculate the maximum value of **P** until reaching the maximum resistance of the connectors in bars 1-3 and 3-5.
2. Calculate the maximum value of **q** distributed load in order to reach the bar 4-5 yield strength (considering the shear strength not relevant).
3. With **P** and **q** values draw the frame's diagrams.
4. Verify the bar 2-4 section resistance, considering shear strength not relevant.



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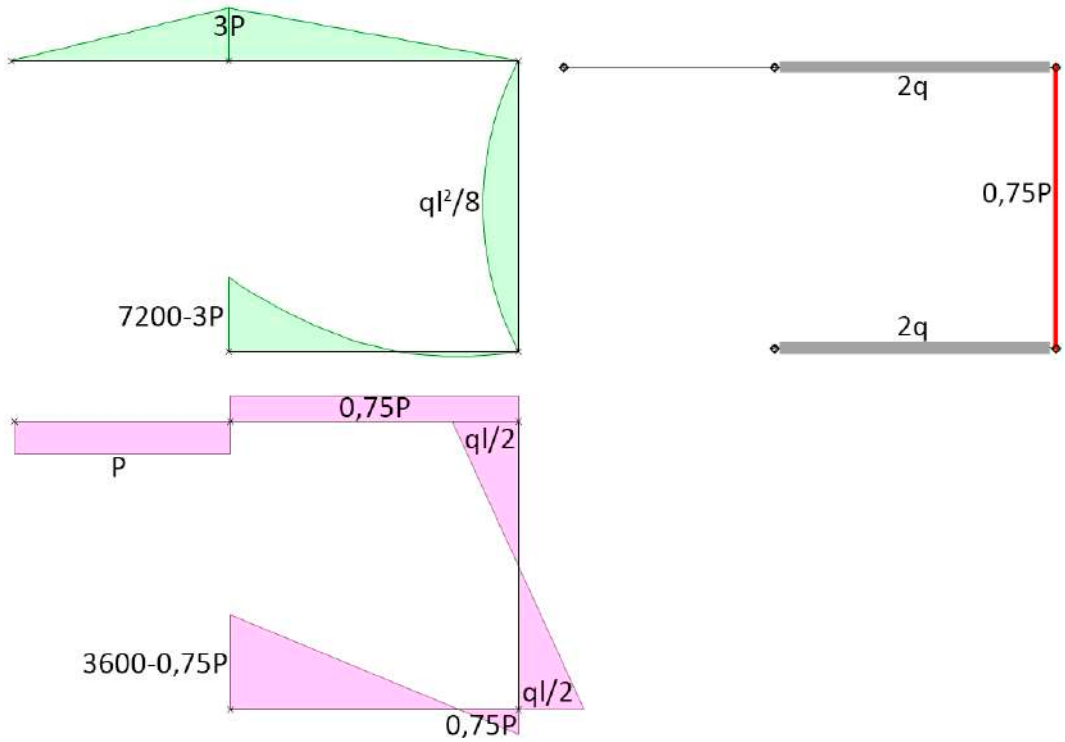
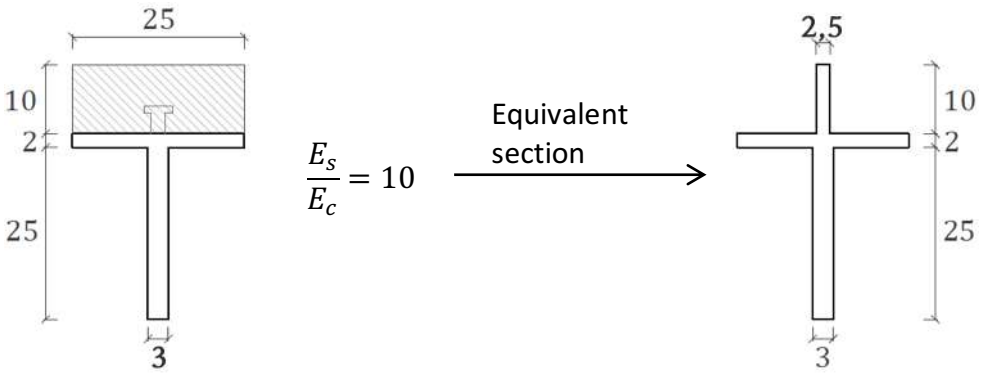
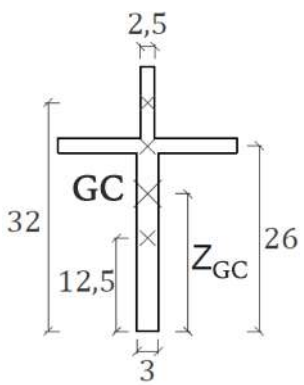
- Concrete's yield strength for traction: 105 kg/cm^2
- Concrete's elasticity modulus: 210.000 kg/cm^2
- Connector's area: 1 cm^2
- Steel's reduction factor: 1,05
- Connector's reduction factor: 1,25
- Concrete's reduction factor: 1,5



Mixed section

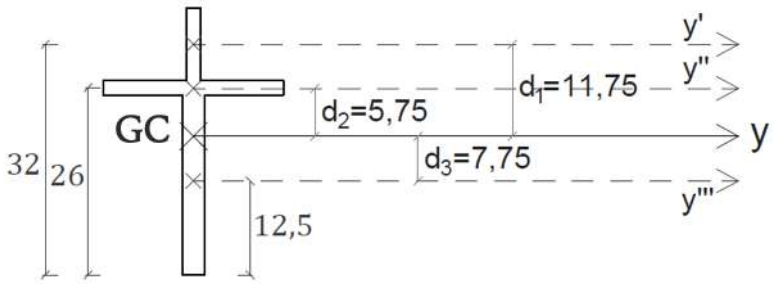


-Section's geometric characteristics:



$$Z_{G.C.} = \frac{\sum_i^n Z_{GC_i} \cdot A}{\sum_i^n A} = \frac{12{,}5(25 \cdot 3) + 26 \cdot (25 \cdot 2) + 32(2{,}5 \cdot 10)}{(25 \cdot 3) + (25 \cdot 2) + (2{,}5 \cdot 10)}$$
$$= 20{,}25\ cm$$

-Inertia:

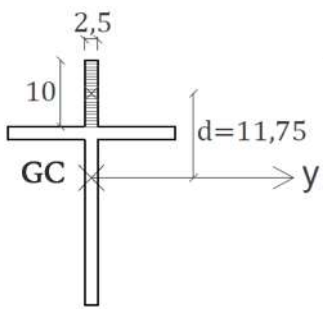


$$I_y = \left(I_{y'} + A_1 \cdot {d_1}^2\right) - \left(I_{y''} + A_2 \cdot {d_2}^2\right) + \left(I_{y'''} + A_3 \cdot {d_3}^2\right)$$

$$I_y = \left[\left(\frac{2{,}5 \cdot 10^3}{12}\right) + (2{,}5 \cdot 10) \cdot 11{,}75^2\right] - \left[\left(\frac{25 \cdot 2^3}{12}\right) + (25 \cdot 2) \cdot 5{,}75^2\right] + \left[\left(\frac{3 \cdot 25^3}{12}\right) + (3 \cdot 25) \cdot 7{,}75^2\right]$$

$$I_y = 13740{,}63\ cm^4$$

-${S_y}^{z_0}$:



$${S_y}^{z_0} = A \cdot d = (2{,}5 \cdot 10) \cdot 11{,}75 = 293{,}7\ cm^3$$

<u>P load's calculation $\left(V_{X,ED} \leq n \cdot V_{kon}\right)$ 1-3 bar</u>

$$V_{Con} = A_{Con} \cdot \frac{f_{yD}}{\sqrt{3}} = 1 \cdot \frac{2750}{1{,}25\sqrt{3}} = 1270{,}17\ kg$$

$$V_{X,ED} = \frac{{S_y}^{z_0}}{I_y} \cdot \Delta V_{1-3} = \frac{293{,}75}{13740{,}63} \cdot 300P = 6{,}41P$$

-ΔV_{1-3}:

$$\Delta V_{1-3} = 300 \cdot P = 300P$$

Verification:

$V_{X,ED} \leq n \cdot V_{Con}$; $6{,}41P \leq 12 \cdot 1270{,}17$; $\underline{P = 2378\ kg}$

<u>P load's calculation $\left(V_{ED,H} \leq n \cdot V_{kon}\right)$ 3-5 bar</u>

$$V_{Con} = A_{Con} \cdot \frac{f_{yD}}{\sqrt{3}} = 1 \cdot \frac{2750}{1{,}25\sqrt{3}} = 1270{,}17\ kg$$

$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y} \cdot \Delta V_{3-5} = \frac{293{,}75}{13740{,}63} \cdot 300P = 6{,}41P$$

$$\Delta V_{3-5} = 400 \cdot 0{,}75P = 300P$$

Verification:

$V_{X,ED} \leq n \cdot V_{Con}$; $6{,}41P \leq 16 \cdot 1270{,}17$; $\underline{P = 3170\ kg}$

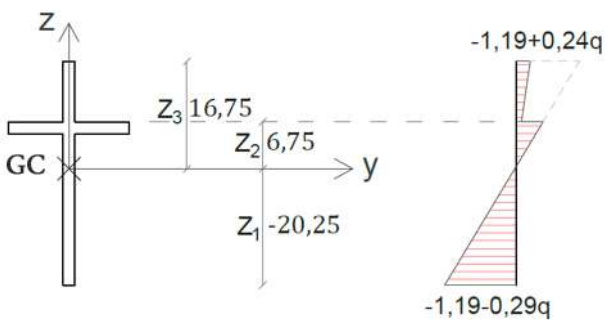
$\rightarrow$ $\boxed{P = 2378\ kg}$

<u>4-5 bar</u>

$$M_{max} = \frac{q \cdot l^2}{8} = \frac{q \cdot 4^2}{8} = 2q\ Kg \cdot m$$

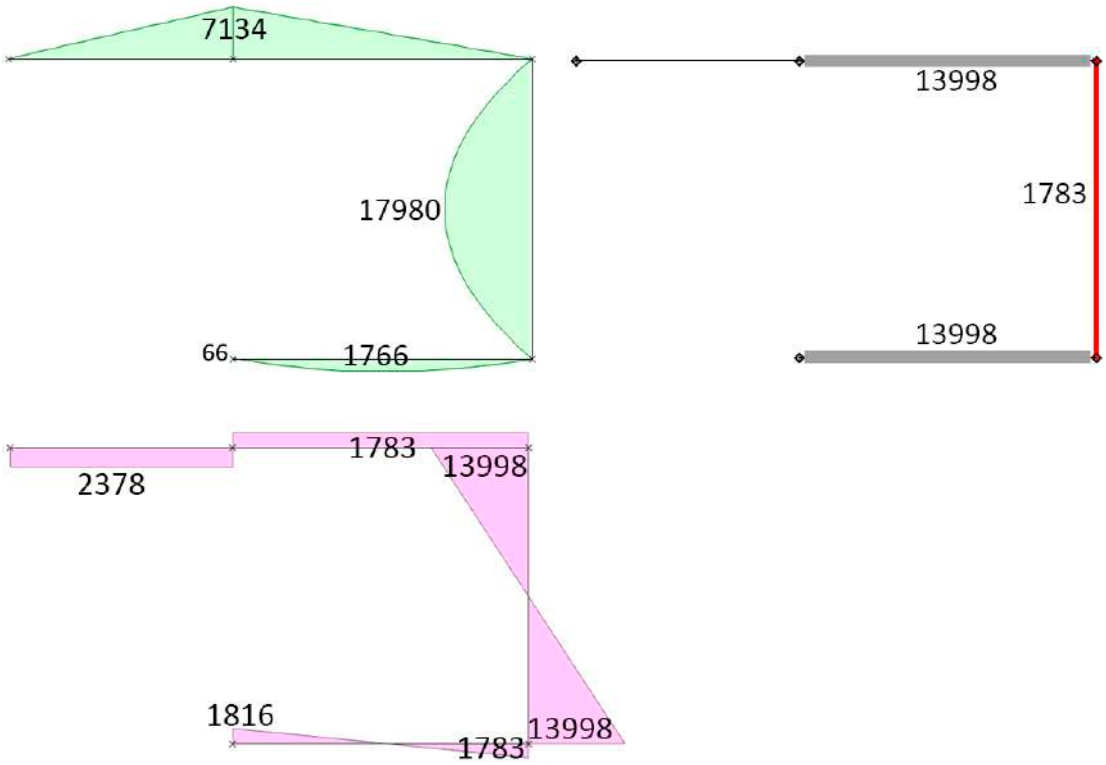
$$N_{ED} = 0{,}75P = -2378 \cdot 0{,}75 = -1783{,}5\ kg$$

$$\rightarrow \begin{cases} \sigma_{max}^{+} = \dfrac{N}{A} + \dfrac{M \cdot z_1}{I} = \dfrac{-1783{,}5}{150} + \dfrac{2q \cdot 100 \cdot 16{,}75}{13740{,}63} = -11{,}9 + 0{,}24q \\ \sigma_{max}^{-} = \dfrac{N}{A} + \dfrac{M \cdot z_2}{I} = \dfrac{-1783{,}5}{150} + \dfrac{2q \cdot 100 \cdot (-20{,}25)}{13740{,}63} = -11{,}9 - 0{,}29q \end{cases}$$



$$-1{,}19 + 0{,}024q \leq \frac{250}{1{,}5}\ ;\ \ q = 6994\ kg/m$$
$$-1{,}19 - 0{,}029q \leq \frac{2750}{1{,}05}\ ;\ \ q = 8990\ kg/m$$
$$\rightarrow \boxed{q = 6994\ kg/m}$$

Diagrams



Bar 2-4 verification

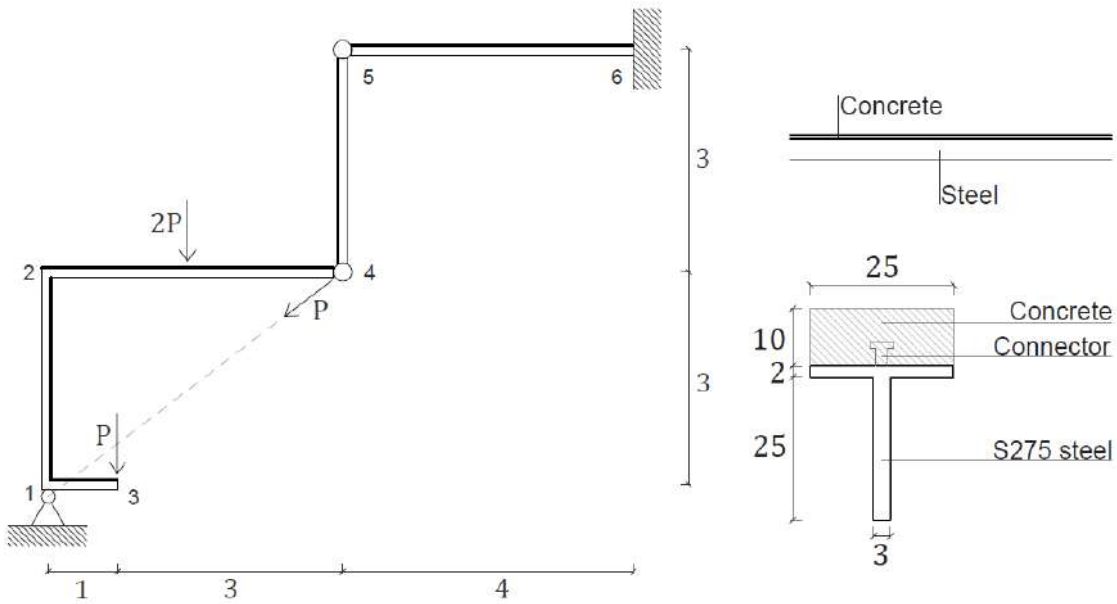
$$M^{+}_{max} = 1766\ kg \cdot m\ ;\ \ N = 13998\ kg$$

$$\sigma^{+}_{max} = \frac{13998}{150} + \frac{1766 \cdot 100 \cdot 16{,}75}{13740{,}63} = 309\,\frac{kg}{cm^2} \rightarrow 30{,}9\,\frac{kg}{cm^2} \leq \frac{250}{1{,}5} = 166{,}\hat{6}\ \checkmark$$

$$\sigma^{-}_{max} = \frac{13998}{150} + \frac{1766 \cdot 100 \cdot (-20{,}25)}{13740{,}63} = -166{,}94\,\frac{kg}{cm^2} \leq \frac{-2750}{1{,}05}\ \checkmark$$

Exercise

1. Solve the following frame under **P** load. Draw the diagrams and the deformation.
2. Define the maximum **P** load according to section's resistance (tangential stress is not relevant).
3. How many connectors do we need in the bar 5-6 in order to keep steel and concrete working together?
4. Draw the bar 2-4 worst section's Normal stress diagram and then verify the resistance.



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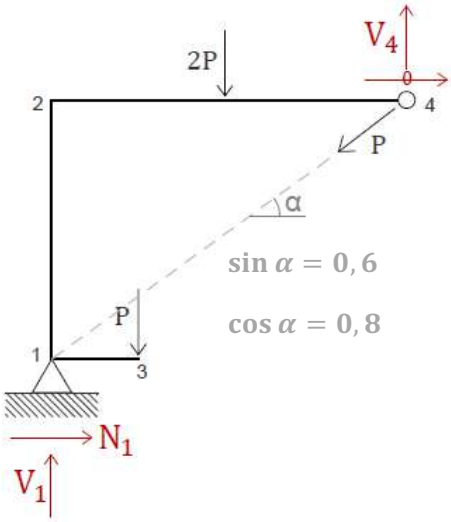
- Concrete's elasticity modulus: 210.000 kg/cm^2
- Connector's area: 1 cm^2
- Steel's reduction factor: 1,05
- Connector's reduction factor: 1,25
- Concrete's reduction factor: 1,5

To solve the frame, we are going to split it in three parts where the hinges are. This way the amount of equations and unknowns will be the same.



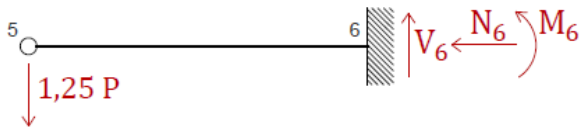
$\Sigma H = 0$; $\boxed{N_1 = 0{,}8\ P}$

$\Sigma M_4 = 0$

$2P \cdot 2 + P \cdot 3 + 3 \cdot 0{,}8P - 4V_1 = 0$; $\boxed{V_1 = 2{,}35\ P}$

$\Sigma V = 0$

$V_4 - 2P - P - 0{,}6P + 2{,}35P = 0$; $\boxed{V_4 = 1{,}25\ P}$

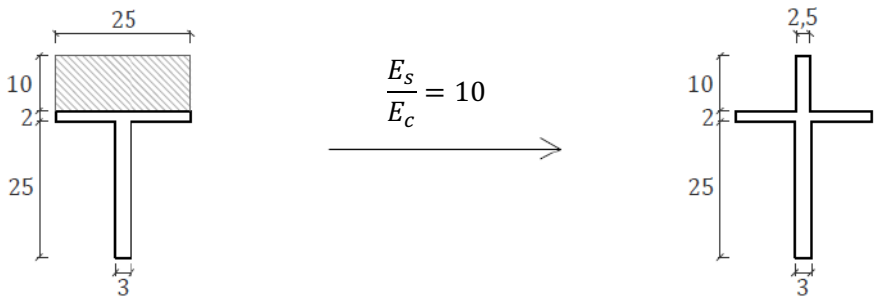


$\Sigma H = 0$; $\boxed{N_6 = 0}$

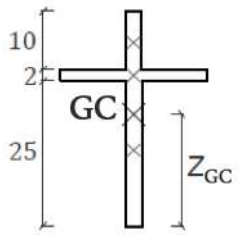
$\Sigma V = 0$; $\boxed{V_6 = 1{,}25\ P}$

$\Sigma M_6 = 0$; $1{,}25P \cdot 4 - M_6 = 0$; $\boxed{M_6 = 5\ P}$

-Mixed beam



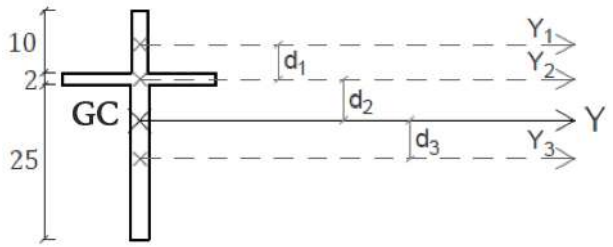
-Centre of gravity



$$Z_{GC} = \frac{\sum_i^n Z_{GC} A_i}{\sum_i^n A_i} = \frac{(10 \cdot 2{,}5) \cdot 32 + (25 \cdot 2) \cdot 26 + (25 \cdot 3) \cdot 12{,}5}{25 + 50 + 75}$$

$$Z_{GC} = \frac{3037{,}5}{150} = 20{,}25\ cm$$

-Moment of inertia



$d_1 = 11{,}75\ cm$

$d_2 = 5{,}75\ cm$

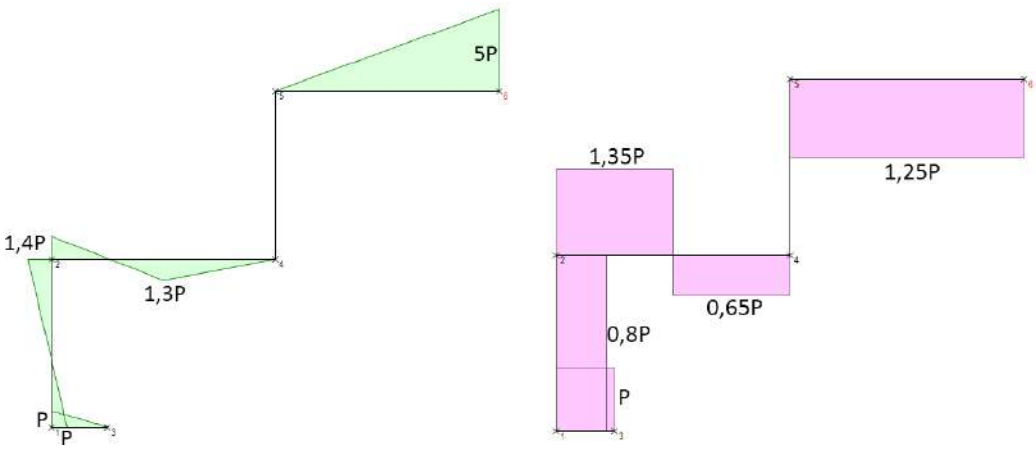
$d_3 = 7{,}75\ cm$

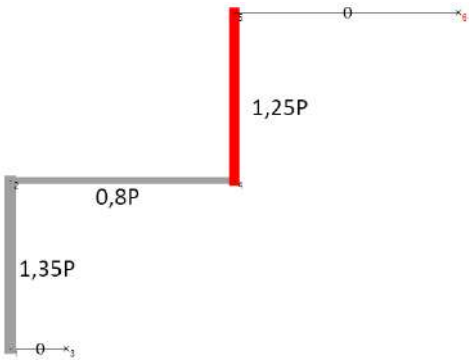
$$I_Y = \left(I_{Y_1} + A_1 \cdot {d_1}^2\right) + \left(I_{Y_2} + A_2 \cdot {d_2}^2\right) + \left(I_{Y_3} + A_3 \cdot {d_3}^2\right)$$

$$I_Y = \left(\frac{2{,}5 \cdot 10^3}{12} + (2{,}5 \cdot 10) \cdot 11{,}75^2\right) + \left(\frac{2{,}5 \cdot 2^3}{12} + (2{,}5 \cdot 2) \cdot 5{,}75^2\right)$$
$$+ \left(\frac{3 \cdot 25^3}{12} + (3 \cdot 25) \cdot 7{,}75^2\right) = 3659{,}9 + 1669{,}79 + 8410{,}94$$

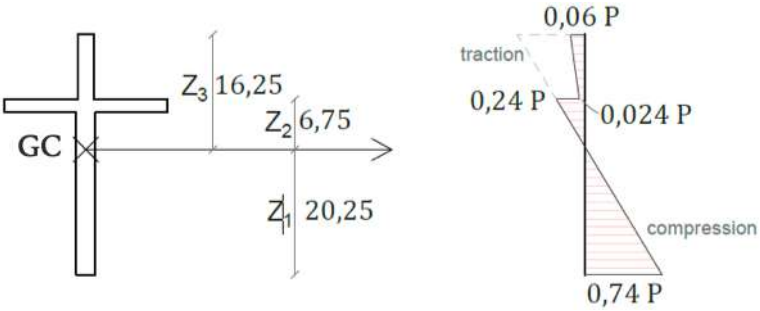
$$\boxed{I_Y = 13740{,}63\ cm^4}$$

<u>Diagrams</u>





Verification through the maximum negative bending moment's section: $M_{ED} = -5P$



$$\sigma_1 = \frac{M_{ED'}\, Z_1}{I_y} = \frac{-5P \cdot 100 \cdot (-20{,}25)}{13740{,}63} = 0{,}737\, P \leq f_{yD}; P \leq \frac{2750}{1{,}05 \cdot 0{,}737} = 3554\, kg$$

$$\sigma_2 = \frac{M_{ED'}\, Z_2}{I_y} = \frac{-5P \cdot 100 \cdot (6{,}75)}{13740{,}63} = -0{,}246\, P \begin{cases} -0{,}246\, P\ (in\ steel) \\ -0{,}0246\ (in\ concrete) \end{cases}$$

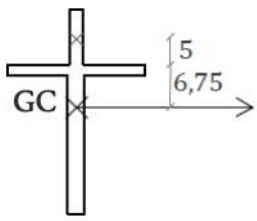
$$\sigma_3 = \frac{M_{ED'}\, Z_3}{I_y} = \frac{-5P \cdot 100 \cdot (16{,}75)}{13740{,}63} = -0{,}61\, P \rightarrow -0{,}061\, P \leq f_{CD}; P \leq \frac{-105}{1{,}5 \cdot 0{,}061}$$

$$= 1147\, kg \ \rightarrow we\ continue\ with\ this\ value$$

<u>Connectors calculation</u>

$$V_{X,ED} \leq V_{Con} \cdot n; 12260{,}4 \leq n \cdot 1270\, ; n \geq 9{,}65 \ \rightarrow \boxed{n = 10}$$

$$V_{Con} = A_{Con} \frac{f_{yD}}{\sqrt{3}} = 1 \frac{2750}{1{,}25 \cdot \sqrt{3}} = 1270\, kg$$

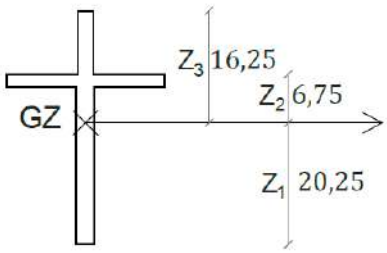


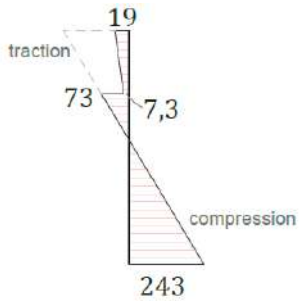
$$V_{X,ED} = \Delta V_{3C} \frac{S_y{}^{z_0}}{I_Y} = 573500 \frac{293{,}75}{13740{,}63} = 12260{,}4\, kg$$

$\Delta V_{3C} = 1{,}25 \cdot P \cdot 400 = 573500$; $S_y{}^{z_0} = (2{,}5 \cdot 10) \cdot (6{,}75 + 5) = 293{,}75\ cm^3$

Bar 2-4 Normal stress diagram

$M_{ED} = -1{,}4\ P = -1{,}4 \cdot 1147 = -1606\ kg \cdot m$; $N_{ED} = 0{,}8\ P = 917{,}6\ kg$





$$\sigma_1 = \frac{N_{ED}}{A} + \frac{M_{Y,ED}\ Z_1}{I_y} = \frac{917{,}6}{150} + \frac{-1606 \cdot 100 \cdot -20{,}25}{13740{,}63} = 242{,}8 \frac{kg}{cm^2} \leq f_{yD}; 243$$

$$< 2619\ \checkmark$$

$$\sigma_2 = \frac{N_{ED}}{A} + \frac{M_{Y,ED}\ Z_2}{I_y} = \frac{917{,}6}{150} + \frac{-1606 \cdot 100 \cdot 6{,}75}{13740{,}63}$$

$$= -72{,}7 \begin{cases} -73 < -2619\ \checkmark \\ -7{,}3 < f_{CD};\ -7{,}3 < -70\ \checkmark \end{cases}$$

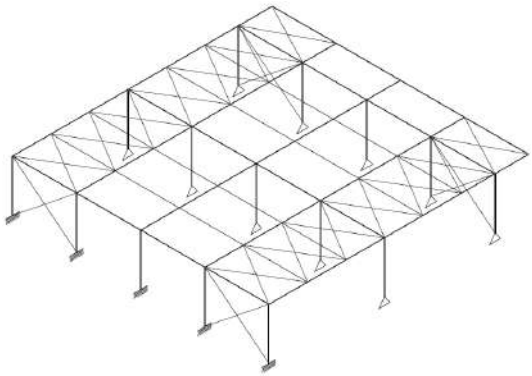
$$\sigma_3 = \frac{N_{ED}}{A} + \frac{M_{Y,ED}\ Z_3}{I_y} = \frac{917{,}6}{150} + \frac{-1606 \cdot 100 \cdot 16{,}75}{13740{,}63} = -189;\ -18{,}9 \leq \frac{-105}{1{,}5}; 18{,}9$$

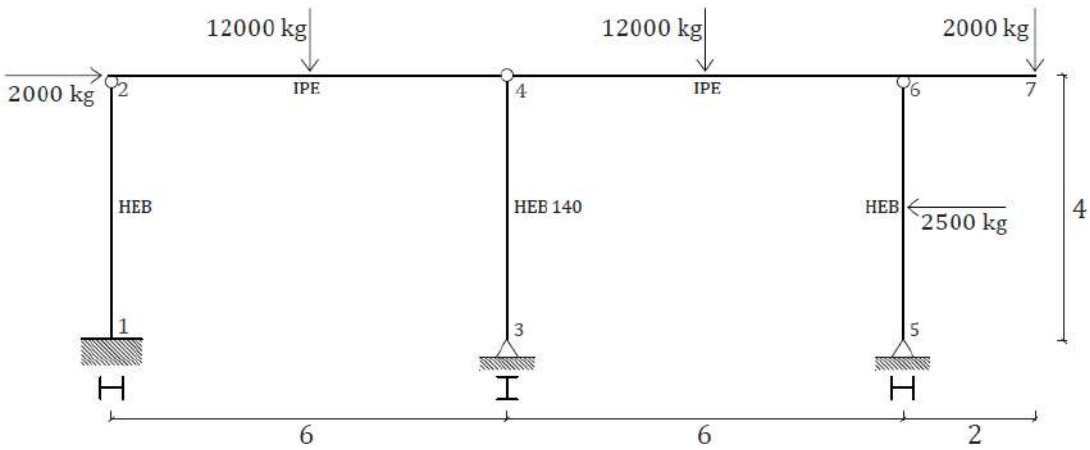
$$< 70 \checkmark$$

8.2. LIMIT STATE VERIFICATIONS

Exercise. Draw the frame's diagrams, and:

1. Dimension the frame's beams and the column 1-2 according to Serviceability limit states (SLS).
2. Verify the beam 2-4 Ultimate limit state (ULS) referring to the section's resistance and bar`s stability.
3. Dimension and verify the column 5-6 according to Ultimate limit state (ULS) section's resistance.
4. Verify the column 3-4 Ultimate limit state (ULS) according to the bar's stability.

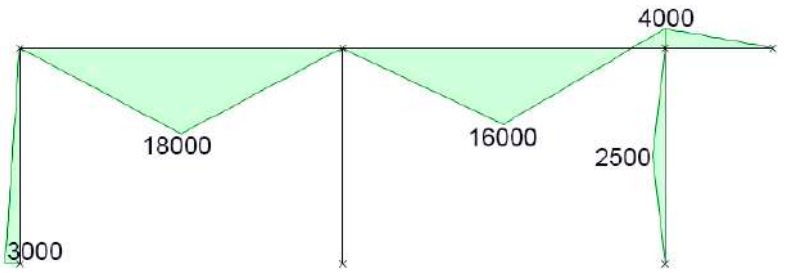


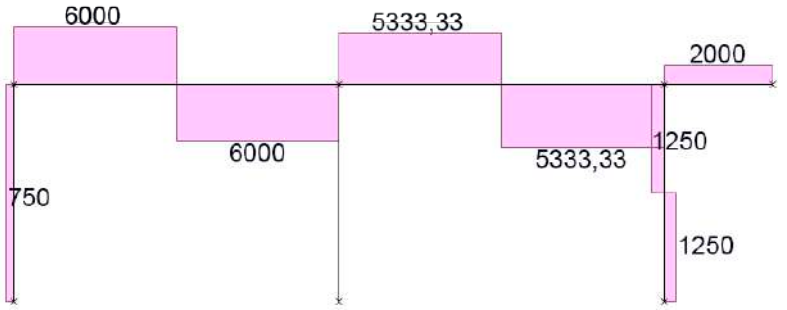


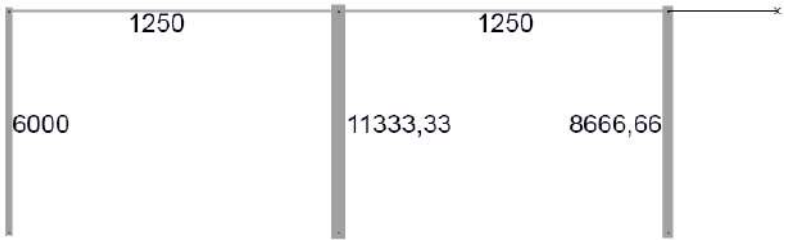
Data:

- S275 Steel
- Actions' magnification factor is 1,5.
- Material's reduction factor is 1,05.
- Don´t use simultaneity factors, only one load combination.
- Beam's deflection in the middle of the beam $\delta = L/500$; L being the beam length.
- Column's horizontal deflection $\delta = H/250$; H being the column height

Diagrams



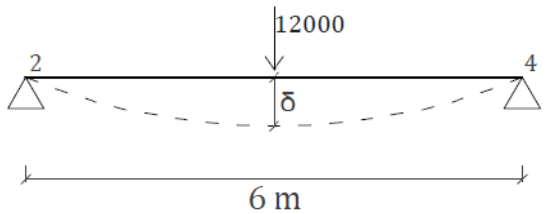




Beams

Serviceability limit state

Beam 2-4:

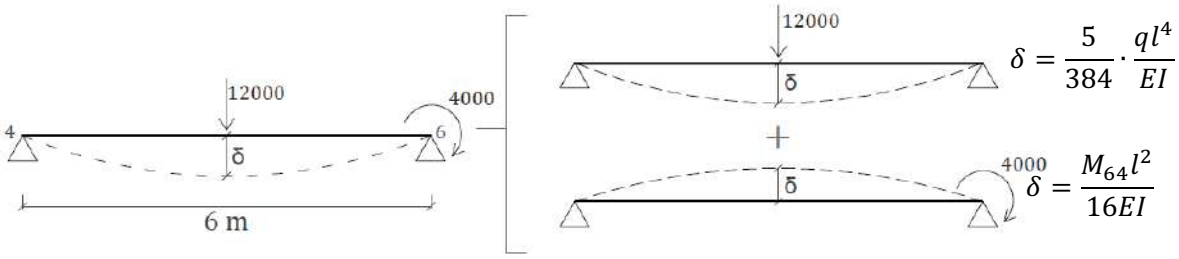


$$\delta = \frac{Pl^3}{48EI}$$

$$\delta_{MAX} = {}^{L}/_{500} = {}^{600}/_{500} = 1{,}2\ cm$$

$$\delta = \frac{Pl^3}{48EI} = \frac{12000 \cdot 600^3}{48 \cdot 2{,}1 \cdot 10^6 \cdot I} = \frac{25714{,}29}{I} \rightarrow I_{24} = \frac{25714{,}29}{1{,}2} = 21428{,}57\ cm^4 \rightarrow \boxed{IPE\ 400}$$

Beam 4-6:



$$\delta_{MAX} = {}^{L}/_{500} = {}^{600}/_{500} = 1{,}2\ cm$$

$$\delta = \frac{Pl^3}{48EI} - \frac{M_{65} l^2}{16EI} = \frac{12000 \cdot 600^3}{48 \cdot 2{,}1 \cdot 10^6 \cdot I} - \frac{400000 \cdot 600^2}{16 \cdot 2{,}1 \cdot 10^6 \cdot I}$$

$$\delta = \frac{21428{,}57}{I} \rightarrow I_{56} = \frac{21428{,}57}{1{,}2} = 17857{,}14\ cm^4 \rightarrow \boxed{IPE\ 400}$$

<u>Ultimate limit state ($M_{max} = 18000\ kg \cdot m$)</u>

$$\frac{N_{ED}}{A} + \frac{M_{y,ED}}{W_y} \leq f_{yD}\ ;\ \frac{1250 \cdot 1{,}5}{84{,}5} + \frac{18000 \cdot 100 \cdot 1{,}5}{1160} \leq \frac{2750}{1{,}05}\ ; 2350 < 2619\ \checkmark$$

When, along the bar, the bending moment is maximum the shear strength is null.

<u>Bar's stability</u>

$$\frac{M_{y,ED}}{\chi_{LT} \cdot W_y} \leq f_{yD}\ ;\ \frac{18000 \cdot 100 \cdot 1{,}5}{0{,}90 \cdot 1160} \leq \frac{2750}{1{,}05}\ ; 2586 < 2619\ \checkmark$$

- $M_{LT,v} = b_{LT,v} \cdot \frac{C}{l_c} = 1{,}06 \cdot 10^{12}\ {}^{1{,}14}/_{2000} = 6{,}04 \cdot 10^8 N \cdot mm = 6{,}04 \cdot 10^6\ kg \cdot cm$
  - $b_{LT,v} = 1{,}06 \cdot 10^{12} N \cdot mm^2$
  - $C = 1{,}14$
  - $L_C = 2000\ mmm$

- $M_{LT,w} = b_{LT,w} \cdot \frac{C}{{l_c}^2} = 4{,}5 \cdot 10^{15}\, {}^{1{,}14}/_{2000^2} = 1{,}28 \cdot 10^9 N \cdot mm = 1{,}28 \cdot 10^7\ kg \cdot cm$
  - $b_{LT,w} = 4{,}5 \cdot 10^{15} N \cdot mm^3$
  - $C = 1{,}14$
  - $L_C\ = 2000\ mmm$

- $M_{CR} = \sqrt{{M_{LT,v}}^2 + {M_{LT,w}}^2} = \sqrt{(6{,}04 \cdot 10^6)^2 + (1{,}28 \cdot 10^7)^2} = 1{,}42 \cdot 10^7\ kg \cdot cm$

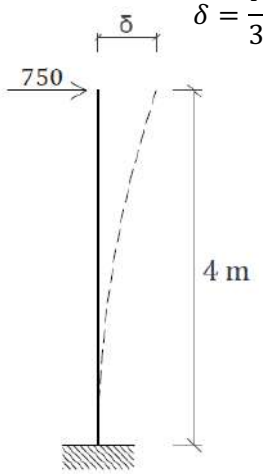
- $\overline{\lambda}_{LT} = \sqrt{{W_y \cdot f_y}/{M_{CR}}} = \sqrt{{1160 \cdot 2750}/{1{,}42 \cdot 10^7}} = 0{,}47 \rightarrow$ (b kur.) $\boxed{\chi_{LT} = 0{,}90}$

Columns

<u>Horizontal deflection</u>

Column 1-2:

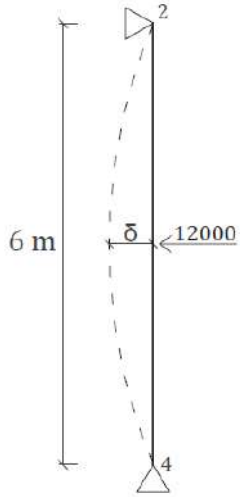
$$\delta = \frac{Pl^3}{3EI}$$



$$\delta_{MAX} = {H}/{250} = {400}/{250} = 1{,}6\ cm$$

$$\delta = \frac{Pl^3}{3EI} = \frac{750 \cdot 400^3}{3 \cdot 2{,}1 \cdot 10^6 \cdot I} = \frac{7619{,}2}{I} \rightarrow I_{12} = \frac{7619{,}2}{1{,}6}$$
$$= 4762\ cm^4 \rightarrow \boxed{HEB\ 200}$$

Section's resistance (column 5-6)



$$M_{max} = 2500\ kg \cdot m$$

$$\frac{M_{max}}{W_y} \leq f_{yD}\ ;\ W_y \geq \frac{M_{max}}{f_{yD}} = \frac{2500 \cdot 100 \cdot 1{,}5}{2750/1{,}05} = 14{,}32\ cm^3$$

$$\rightarrow\ HEB\ 120$$

Verification:

$$\frac{N}{A} + \frac{M_y}{W_y} \leq f_{yD}\ ;\ \frac{8333{,}\hat{3} \cdot 1{,}5}{34} + \frac{2500 \cdot 100 \cdot 1{,}5}{144} \leq \frac{2750}{1{,}05}\ ; 2971 \nless 2619\ \times$$

We are going to increase the profile and check it again (HEB 140):

$$\frac{N}{A} + \frac{M_y}{W_y} \leq f_{yD}\ ;\ \frac{8333{,}\hat{3} \cdot 1{,}5}{43} + \frac{2500 \cdot 100 \cdot 1{,}5}{216} \leq \frac{2750}{1{,}05}\ ; 2027 \nless 2619\ \checkmark$$

Stability (3-4 column)

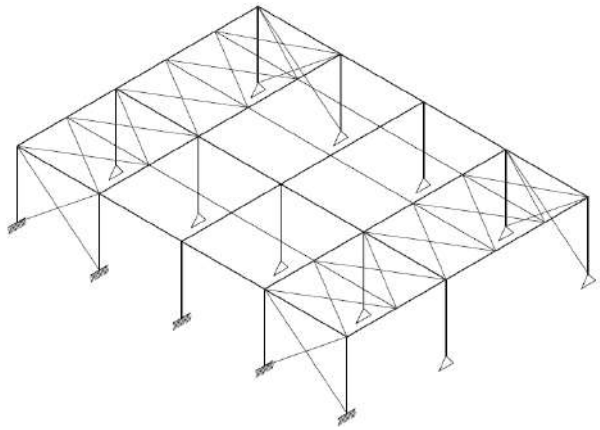
$$\frac{N}{\chi_z \cdot A} \leq f_{yD}\ ;\ \frac{11333{,}\hat{3}}{0{,}45 \cdot 54{,}3} \leq \frac{2750}{1{,}05}\ ; 696 < 2619\ \checkmark$$

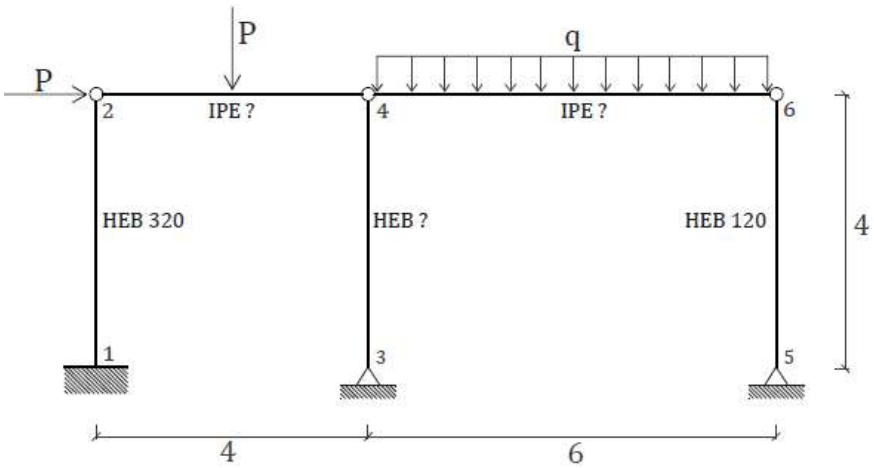
$$N_{CR,z} = \frac{\pi^2}{{l_{k,z}}^2} \cdot E \cdot I_z = \frac{\pi^2}{400^2} \cdot 2{,}1 \cdot 10^6 \cdot 889 = 115160\ kg$$

$$\left.\begin{array}{l} \bar{\lambda}_z = \sqrt{\dfrac{A \cdot f_y}{N_{CR,z}}} = \sqrt{\dfrac{54{,}3 \cdot 2750}{115160}} = 1{,}14 \\ Buckling\ curve\ \rightarrow C \end{array}\right\}\ \chi_z = 0{,}45$$

Exercise.

1. Through the Ultimate limit states (ULS), analyse 5-6 column and define the maximum **q** load for the structure.
2. Through the Serviceability limit states (SLS), analyse 1-2 column and define the maximum **P** load for the structure.
3. With **P** and **q** values:
    a. Draw strength's diagrams in characteristic value.
    b. Pre dimension 2-4 and 4-6 bars according to Serviceability limit states.
    c. Verify the lateral torsional buckling in 4-6 bar.
    d. Dimension 3-4 column according to Ultimate limit states.

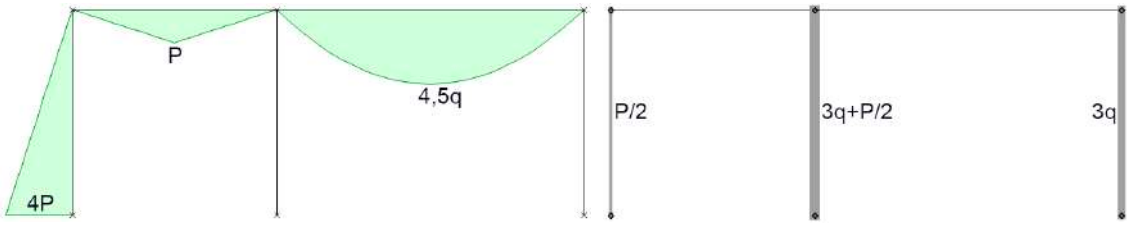


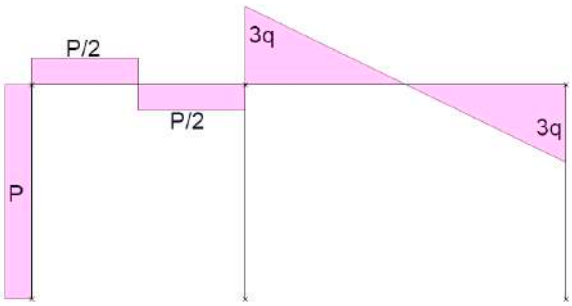


Data:

- S275 Steel
- Actions' magnification factor is 1,5.
- Material's reduction factor is 1,05.
- Don´t use simultaneity factors, only one load combination.
- Beam's deflection in the middle of the beam $\delta = L/500$; L being the beam length.
- Column's horizontal deflection $\delta = H/250$; H being the column height

Diagrams





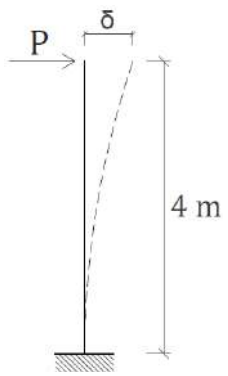
q load calculation

$$\frac{N \cdot 1{,}5}{\chi_z \cdot A} \leq f_{yD}\ ;\ \frac{3 \cdot q \cdot 1{,}5}{0{,}31 \cdot 34} \leq \frac{2750}{1{,}05}\ ;\ \ q = \frac{2750 \cdot 34 \cdot 0{,}31}{1{,}05 \cdot 1{,}5 \cdot 3} = 6134\ kg/m$$

$$N_{CR,z} = \frac{\pi^2}{{l_{k,z}}^2} \cdot E \cdot I_z = \frac{\pi^2}{400^2} \cdot 2{,}1 \cdot 10^6 \cdot 318 = 41193\ kg$$

$$\left.\begin{array}{l} \overline{\lambda}_z = \sqrt{\dfrac{A \cdot f_y}{N_{CR,z}}} = \sqrt{\dfrac{34 \cdot 2750}{41193}} = 1{,}5 \\ Buckling\ curve\ \rightarrow C \end{array}\right\}\ \chi_z = 0{,}31$$

P load calculation

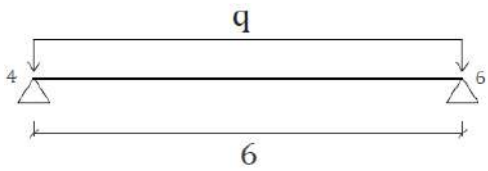


$$\delta_{H,MAX} = {}^{L}/_{250} = {}^{400}/_{250} = 1{,}6\ cm$$

$$\delta = \frac{Pl^3}{3EI}\ ; P = \frac{I \cdot 3 \cdot E \cdot \delta_H}{l^3} = \frac{30823 \cdot 3 \cdot 2{,}1 \cdot 10^6 \cdot 1{,}6}{400^3} = 4855\ kg$$

$$\boxed{q = 6134\ kg/m\ ;\ \ P = 4855\ kg}$$

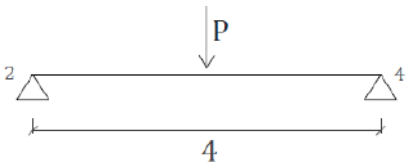
4-6 beam dimensioning



$$\delta_{MAX} = {}^{L}/_{500} = {}^{600}/_{500} = 1{,}2\ cm$$

$$\delta = \frac{5}{384}\frac{ql^4}{EI} = \frac{5}{384}\frac{61{,}34 \cdot 600^4}{2{,}1 \cdot 10^6 \cdot I} = \frac{49291{,}2}{I} \rightarrow I_{46} = \frac{49291{,}2}{1{,}2} = 41076\ cm^4 \rightarrow \boxed{IPE\ 500}$$

2-4 beam dimensioning



$$\delta_{MAX} = {}^{L}/_{500} = {}^{400}/_{500} = 0{,}8\ cm$$

$$\delta = \frac{Pl^3}{48EI} = \frac{4855 \cdot 400^3}{48 \cdot 2{,}1 \cdot 10^6 \cdot I} = \frac{3082{,}4}{I} \rightarrow I_{24} = \frac{3082{,}4}{0{,}8} = 3853\ cm^4 \rightarrow \boxed{IPE\ 240}$$

4-6 beam lateral torsional buckling

$$\frac{M_{ED}}{\chi_{LT} \cdot W_y} \leq f_{yD}\ ;\ \frac{27603 \cdot 100 \cdot 1{,}5}{0{,}91 \cdot 1930} \leq \frac{2750}{1{,}05}\ ; 2357 < 2619\ \checkmark$$

$$M_{ED} = \frac{q \cdot l^2}{8} = \frac{6134 \cdot 6^2}{8} = 27603\ kg \cdot m$$

- $M_{LT,v} = b_{LT,v} \cdot \frac{C}{l_c} = 1{,}8 \cdot 10^{12}\ {}^{1{,}14}/_{2000} = 1{,}03 \cdot 10^9 N \cdot mm = 1{,}03 \cdot 10^7\ kg \cdot cm$
  - $b_{LT,v} = 1{,}8 \cdot 10^{12} N \cdot mm^2$
  - $C = 1{,}14$
  - $L_C = 2000\ mmm$

- $M_{LT,w} = b_{LT,w} \cdot \frac{C}{{l_c}^2} = 8{,}9 \cdot 10^{15}\, {}^{1{,}14}/_{2000^2} = 2{,}54 \cdot 10^9 N \cdot mm = 2{,}54 \cdot 10^7\ kg \cdot cm$
  - $b_{LT,w} = 8{,}9 \cdot 10^{15} N \cdot mm^3$
  - $C = 1{,}14$
  - $L_C\ = 2000\ mmm$
- $M_{CR} = \sqrt{{M_{LT,v}}^2 + {M_{LT,w}}^2} = \sqrt{(1{,}03 \cdot 10^7)^2 + (2{,}54 \cdot 10^7)^2} = 2{,}74 \cdot 10^7\ kg \cdot cm$
- $\bar{\lambda}_{LT} = \sqrt{{W_y \cdot f_y}/{M_{CR}}} = \sqrt{{1930 \cdot 2750}/{2{,}74 \cdot 10^7}} = 0{,}44 \rightarrow$ (b kur.) $\boxed{\chi_{LT} = 0{,}91}$

<u>3-4 column dimensioning</u>

$$N_{ED} = 1{,}5 \cdot \left(3q + \frac{P}{2}\right) = 1{,}5 \cdot \left(3 \cdot 6134 + \frac{4855}{2}\right) = 34489\ kg$$

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z \rightarrow\ I_z = \frac{{l_k}^2 \cdot N_{CR,Z}}{\pi^2 \cdot E};\ I_z = \frac{{l_k}^2 \cdot N_{CR,Z}}{\pi^2 \cdot E} = \frac{(400)^2 \cdot 34489}{\pi^2 \cdot 2{,}1 \cdot 10^6} = 266{,}24\ cm^4$$

$$I_z = 267\ cm^4 \rightarrow \boxed{HEB\ 120}$$

- <u>Verification. Buckling:</u>

$$\frac{N_{ED}}{\chi_Z \cdot A} \leq f_{yD}\ ;\ \frac{34489}{0{,}31 \cdot 34} \leq \frac{2750}{1{,}05}\ ; 3272 < 2619\ \times\ \rightarrow not\ valid$$

- χ_Z calculation:

As β coefficient for both axes is the same, directly we are going to verify the column in z axis.

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z = \frac{\pi^2}{(400)^2} \cdot 2{,}1 \cdot 10^6 \cdot 318 = 41193\ kg$$

$l_{k,z} = l$

$$\left.\begin{array}{l} \bar{\lambda}_z = \sqrt{\frac{f_y \cdot A}{N_{CR,z}}} = \sqrt{\frac{2750 \cdot 34}{41193}} = 1{,}51 \\ C\ curve \end{array}\right\} \quad \chi_z = 0{,}31$$

- <u>Resize -> $HEB\ 140$</u>

$$\frac{N_{ED}}{\chi_Z \cdot A} \leq f_{yD}\ ;\ \frac{34489}{0{,}39 \cdot 43} \leq \frac{2750}{1{,}05}\ ; 2057 < 2619\ \checkmark\ \rightarrow\ valid \rightarrow \boxed{HEB\ 140}$$

- χ_z calculation:

$$N_{CR,Z} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I_z = \frac{\pi^2}{400^2} \cdot 2{,}1 \cdot 10^6 \cdot 550 = 71246\ kg$$

$$l_{k,z} = l$$

$$\left.\begin{array}{r} \overline{\lambda_z} = \sqrt{\frac{f_y \cdot A}{N_{CR,z}}} = \sqrt{\frac{2750 \cdot 43}{71246}} = 1{,}29 \\ C\ curve \end{array}\right\} \quad \chi_z = 0{,}39$$

9. BIBLIOGRAPHY

“Resistencia de materiales”
Manuel Vázquez
Ed: Noela

“Estructuras de Edificación”
Jose Zubiaur Bilbao
Ed: Euskal Herriko Unibertsitatea

“Razón y ser de los tipos estructurales”
E. Torroja Miret
Ed.:Consejo Superior de Investigaciones

“Análisis de estructuras. Métodos clásicos y matriciales”
Pascual Martí Montrull
Ed.: Horacio Escarabajal Editores

“Strutture. Morfología strutturale in architettura”
Mario Cedolini
Ed.: Arsenale Editoriale

“Números gordos en el proyecto de estructuras”
Juan Carlos Arroyo Portero
Ed.: Cinter Divulgación Técnica

“Proyecto y Cálculo de Estructuras de Hormigón Armado. Tomos I y II”
J. Calavera
Ed.: Intemac

“Construcción de Estructuras de Hormigón Armado”
Pascual Urbán Brotóns
Ed.: Editorial Club Universitario

“Construcción en acero. Sistemas estructurales y constructivos en edificación”
Constantino Hurtado / Ruth Vega
Ed.: Munilla-Lería

“Curso de estructuras metálicas”
Brufau, Arguijo
Ed.: E.T.S.A.V.

“Estructuras de Acero. Cálculo” (2. edizioa)
R. Argüelles Alvarez, R. eta J.Mª Argüelles Bustillo, F. Arriaga Martitegui eta J.R. Atienza Reales
Ediciones Bellisco

“Estructuras metálicas para edificación”
José Monfort Lleonart
Editorial Universidad Politécnica de Valencia

“Manual para cálculo de estructuras metálicas”
ENSIDESA (5 tomo)
Ed.: Empresa Nacional Siderúrgica, S.A., Oviedo, 1990

“CTE documento básico SE-AE acciones en la edificación”
Ed.: Ministerio de Vivienda

“CTE documento básico SE-A seguridad estructural en acero”
Ed.: Ministerio de Vivienda

ANNEX 1. PRE DIMENSIONING METHODS

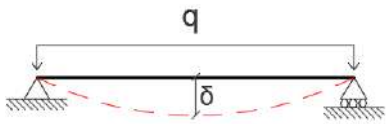
In these notes we have seen how to verify the frames, but often profile type and the strengths upon the structure were already known.

In this section, we will use the procedures we have already seen for pre-dimensioning structural elements. Some methods are going to be more effective than others, so we will start with those. When the more effective methods are not suitable, we will go with the next one in the following order:

a) Deformation limits: horizontal and vertical deflection
b) Normal stress due to the bending moment
c) Buckling: Critical tension limit
d) Normal stress due to the axial strength

Before starting to explain these methods, we have to underline that using any type of pre-dimensioning method could not be the final one. When verifying the profile type we have chosen, it could be that it is not valid and that we have to resize it. Therefore, the pre-dimensioning is going to be just the beginning. However, the more precise we do it, the less change we will have to do in the development of the calculation.

a) Deformation limits: vertical and horizontal deflection



$\delta_{MAX} = {}^{l}/_{500}$ (The regulations defines the maximum acceptable deflection)

To define the minimum inertia that a beam needs, and also to be less than the allowed deflection, we will use the following expression:

$$\delta = \frac{5}{384} \cdot \frac{qL^4}{EI} \; ; I = \frac{5}{384} \cdot \frac{qL^4}{E(L/500)}$$

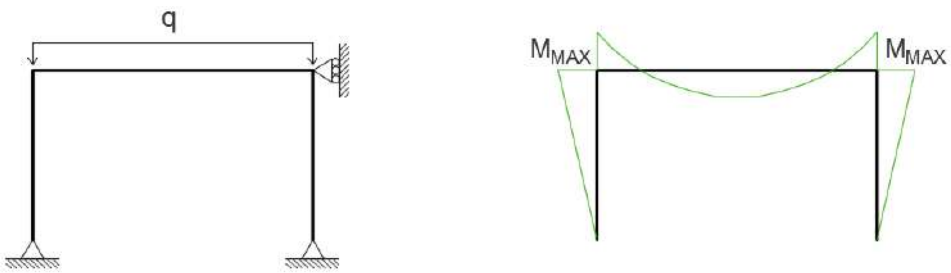
$\delta_{H,MAX} = {}^{l}/_{250}$(The regulations defines the maximum acceptable deflection)

To define the minimum inertia that a column needs, and also to be less than the allowed horizontal deflection, we will use the following expression:

$$\delta_H = \frac{PL^3}{3EI}\ ; I = \frac{PL^3}{3E(h/250)}$$

b) The limit of Normal stress created by flection

It could also be that in some situations we can´t apply the conditions of the deformation limits. That is what happens in the following example, where the columns don´t have horizontal deflection.



In these cases, the bar that we are dimensioning will receive a bending moment, and will have to fulfil the following condition.

$$\frac{M_{MAX}}{W} \leq f_{yD} \quad ,then \ \rightarrow \frac{M_{MAX}}{f_{YD}} \leq w$$

With this condition we define the minimum resistant module of the bar, so that we respect the yield strength of the material.

c) Buckling: Critical tension limit

Usually, the two previous methods are enough to pre-dimension the beams. Nevertheless, it often happens that the limit of horizontal deflection is not suitable (because the edges of the bar can´t be displaced, or because it's a double-hinged bar) and works just on an axial strength (with no bending moment). In these cases we will use the third or the fourth method, even if its effectiveness is lower.

By using Euler's critical tension to buckling, we can define the minimum moment of inertia we need in our bar:

$$N_{CR} = \frac{\pi^2}{{l_k}^2} \cdot E \cdot I \rightarrow \; I = \frac{{l_k}^2 \cdot N_{CR}}{\pi^2 \cdot E}$$

Where:

$N_{CR} = Critical\ tension\ for\ buckling$

$l_k = buckling\ lenth$

$I = Moment\ of\ inertia$

With this method we define the minimum value of moment of inertia, and through this factor we can choose the profile. However, this is not a very efficient method, because the N_{CR} critical tension to buckling doesn't take into account material imperfections. That means that when applying the buckling curves and calculating the reduction factor for buckling, the verification is often not valid, and we will have to choose a bigger one.

d) The limit of Normal stress generated by axial strength

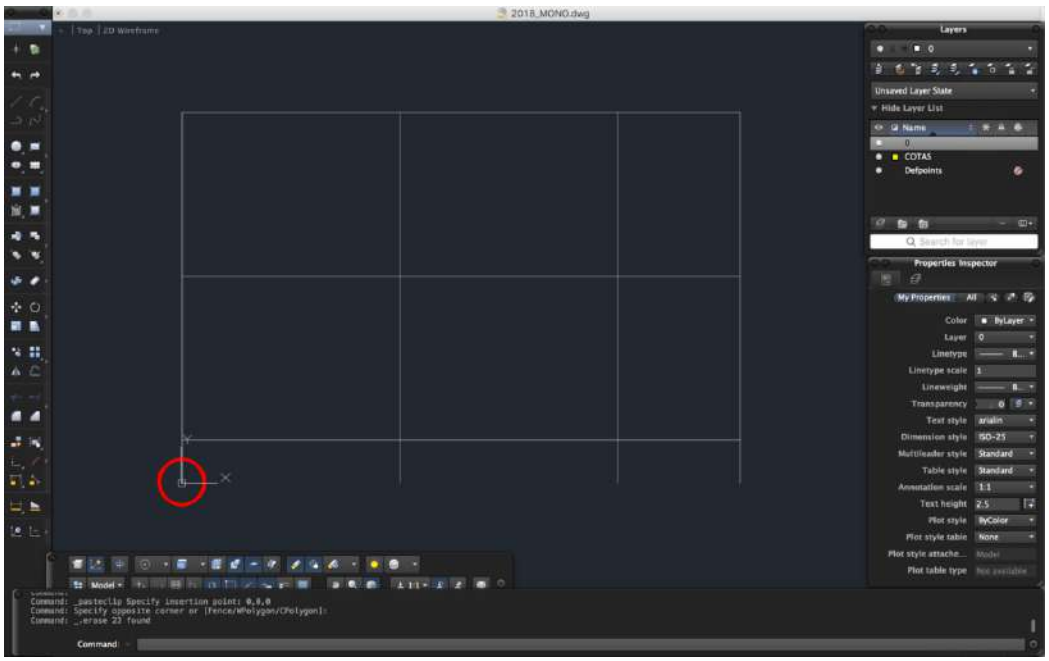
In common situations, the pieces working under compression have the buckling as the most restrictive condition. However, the pieces working under tensile strength will have to follow this method and the following expression:

$$\frac{N}{A} \leq f_{yD} \quad , then \quad \rightarrow \quad \frac{N}{f_{yD}} \leq A$$

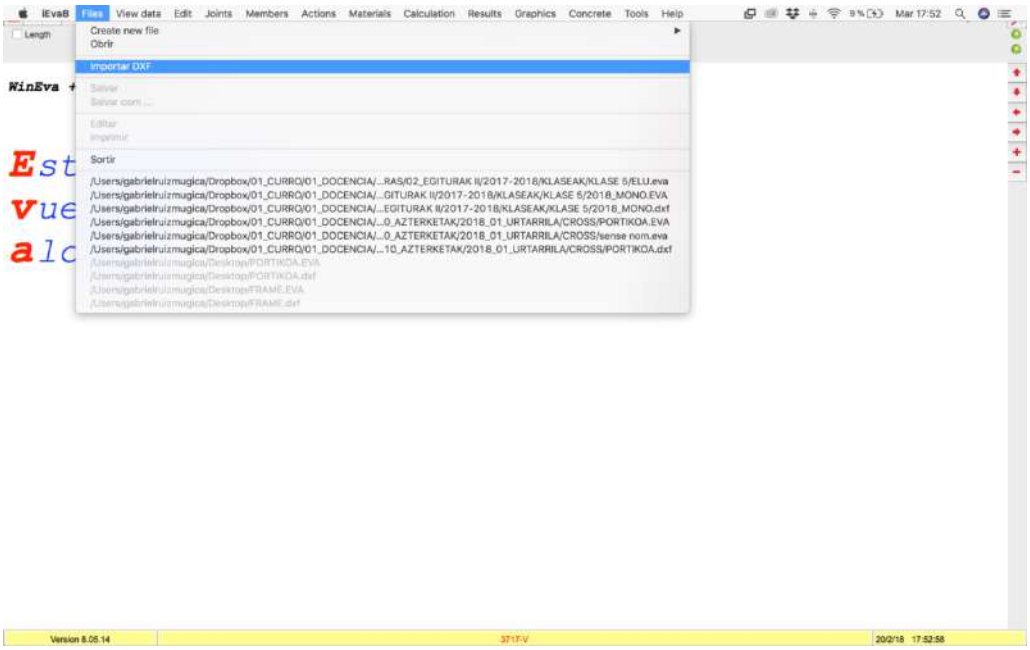
With the previous expression we define the minimum surface that the cross section will need.

ANNEX 2. COMPUTER CALCULATUION OF STRUCTURES. WINEVA

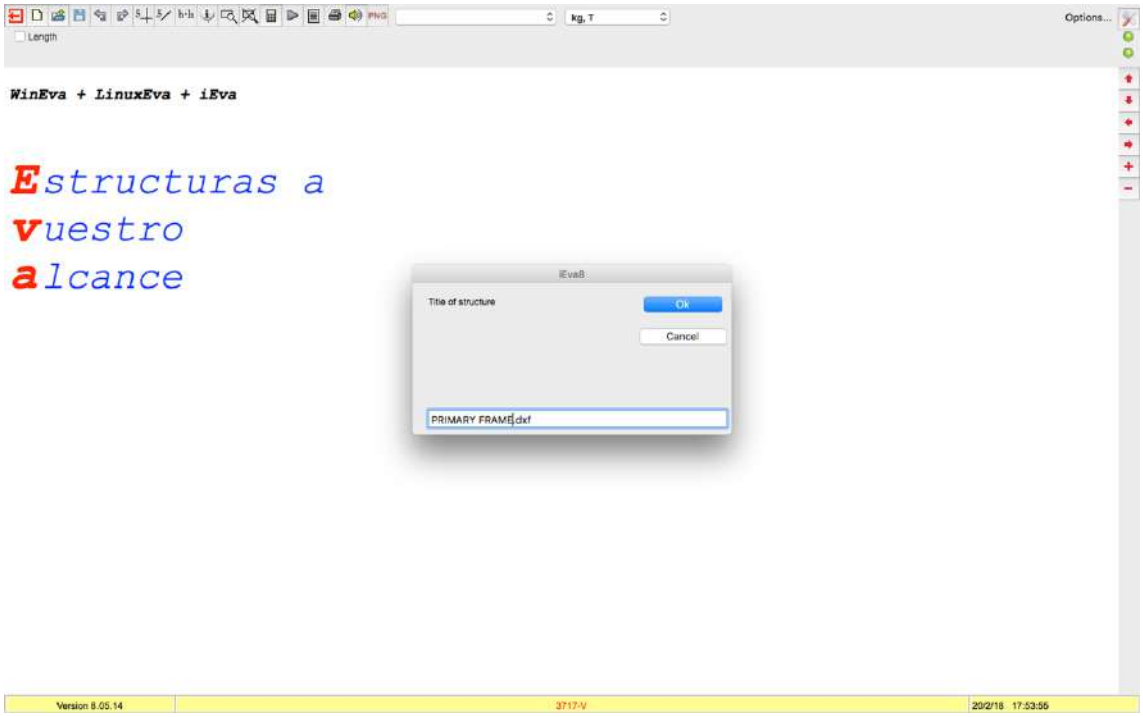
We first draw the geometry of the frame we want to calculate in Autocad. It is better if we locate one of the corners of our frame in the origin of the coordinate system of the Autocad's model space. When drawing the frame, we need to focus on making an independent line with every bar length between nodes. Once the frame is drawn, save it like a ".dxf" file.



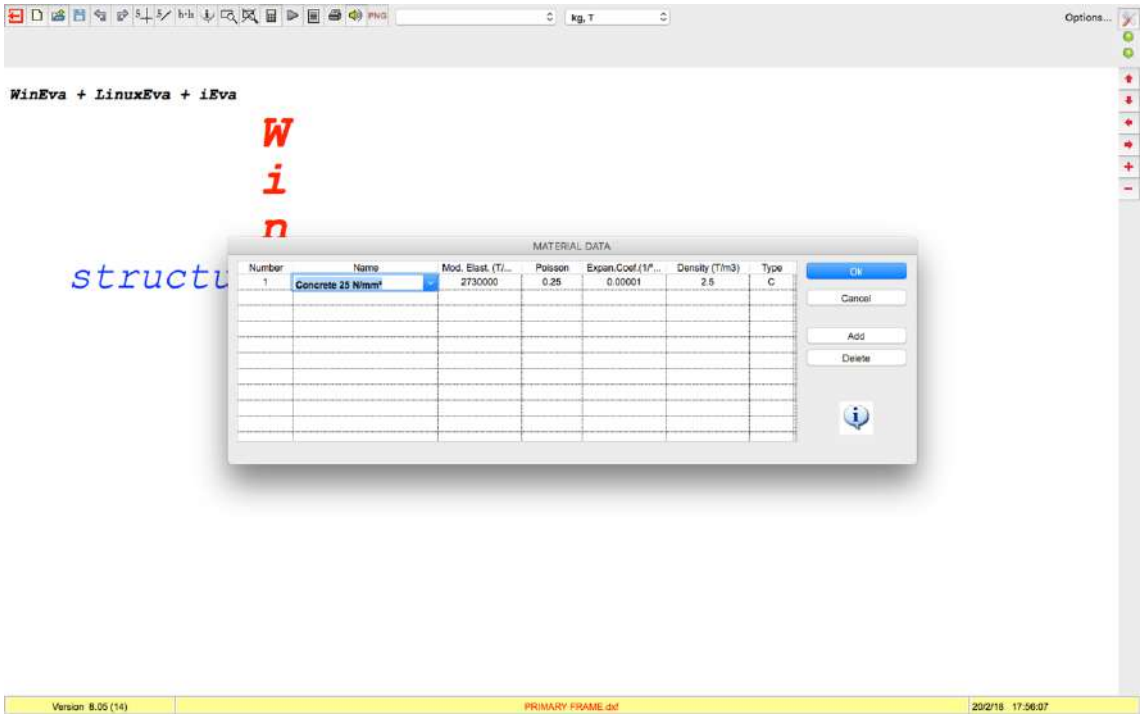
Now open Wineva. Open the "file" menu and click on "Import dxf".



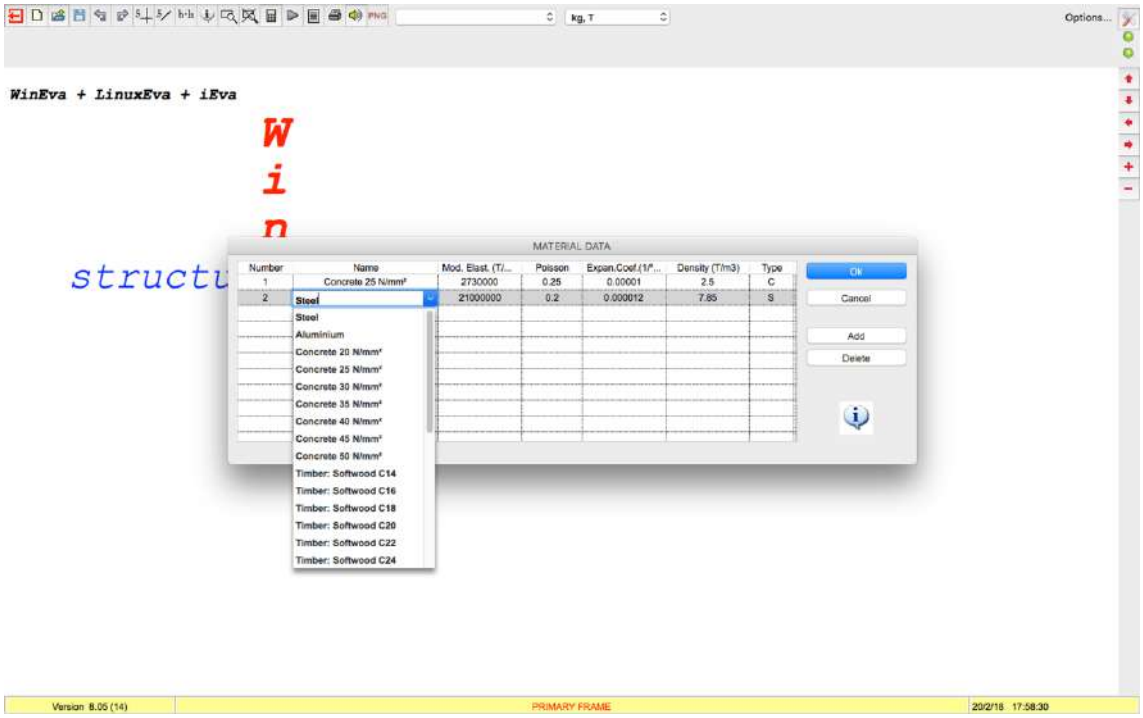
When opening the frame's file you have to name it.



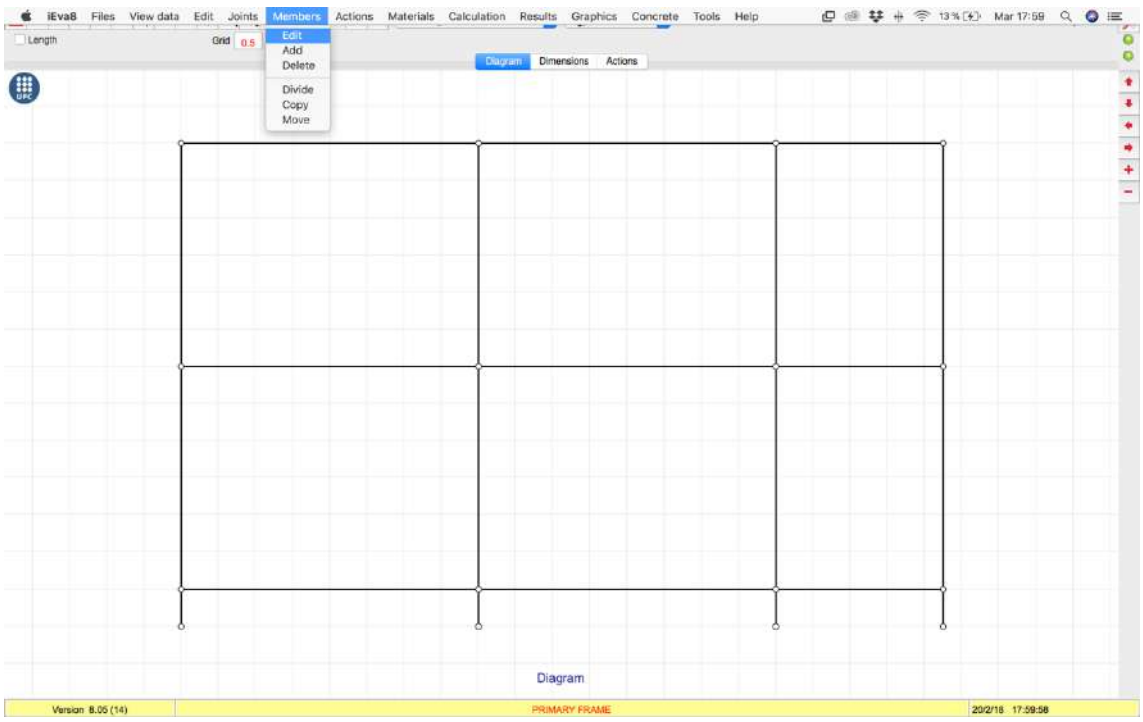
When opening the frame, the program asks about the material the structure is made of. In this case, we have to choose "HA-25 concrete".



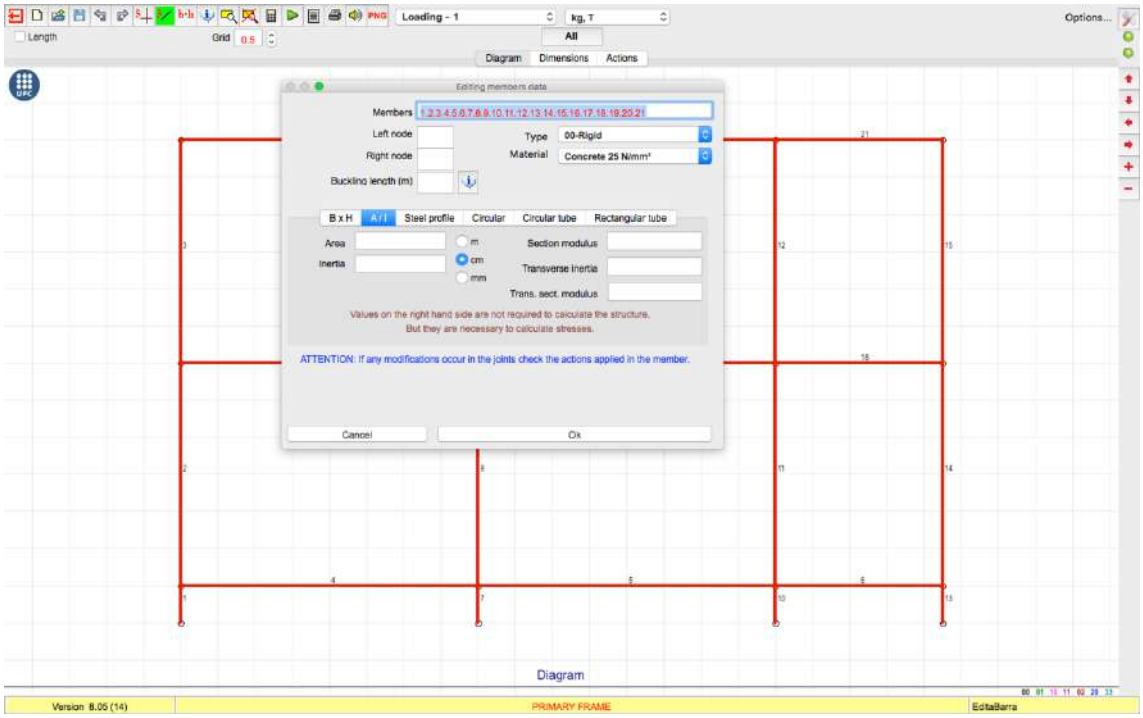
If we are not sure about the material we are going to use to build up the frame we can add more materials.



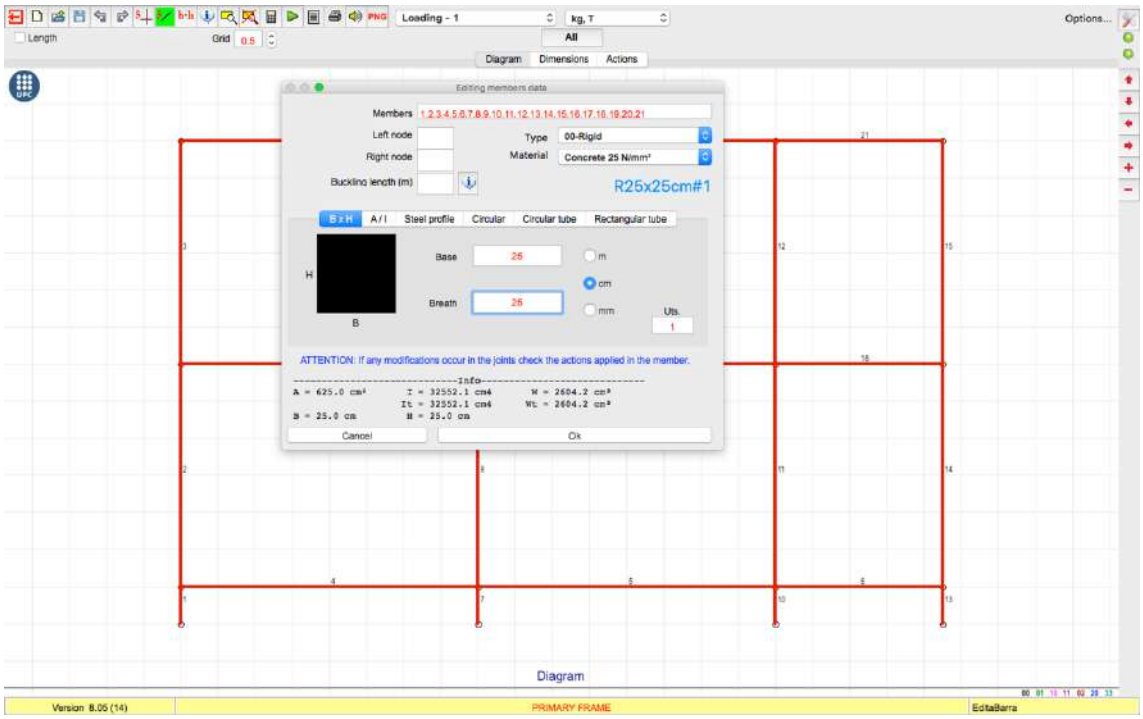
After opening the frame we have to first describe the bar's features. We can do it by clicking on "members" menu, and going to "edit".



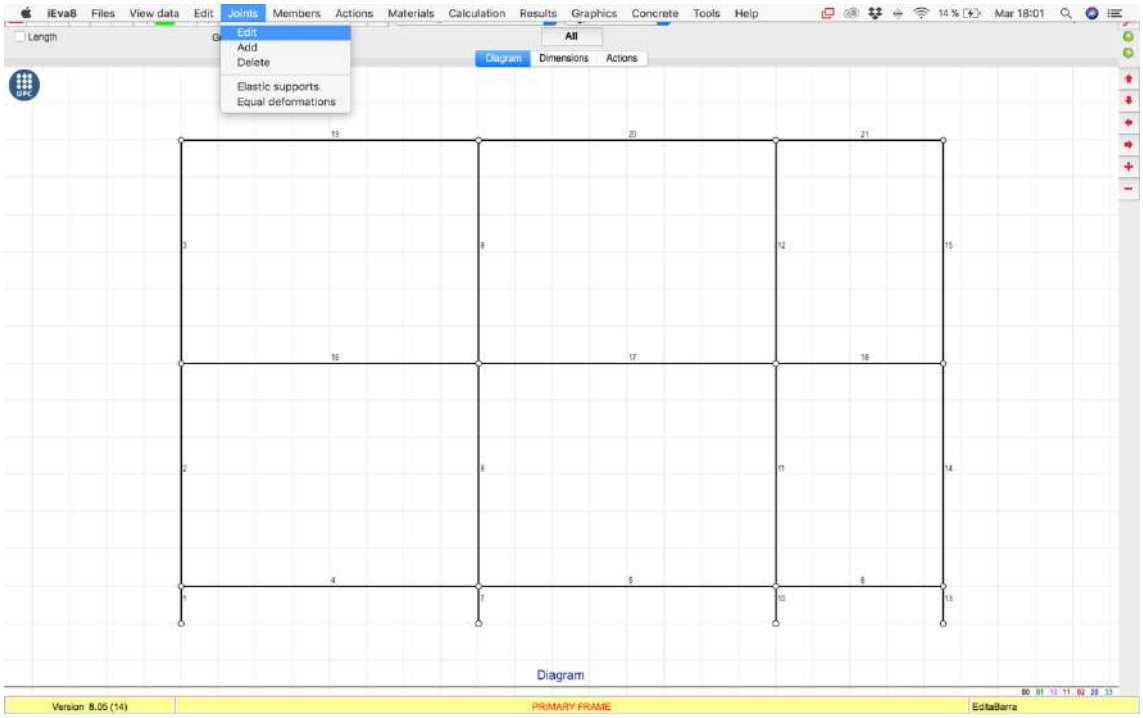
Choose the bars with the same type of section and press the right button of the mouse to activate the edition.



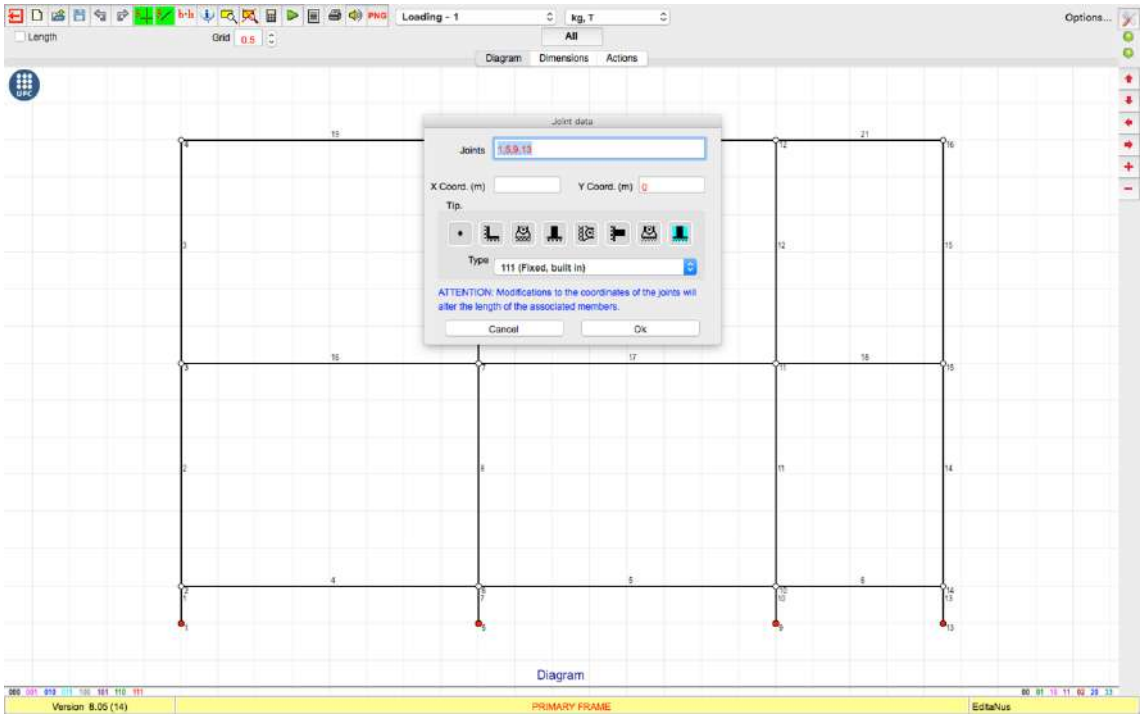
In the “b*h” section, we introduce the dimensions of the section in cm.



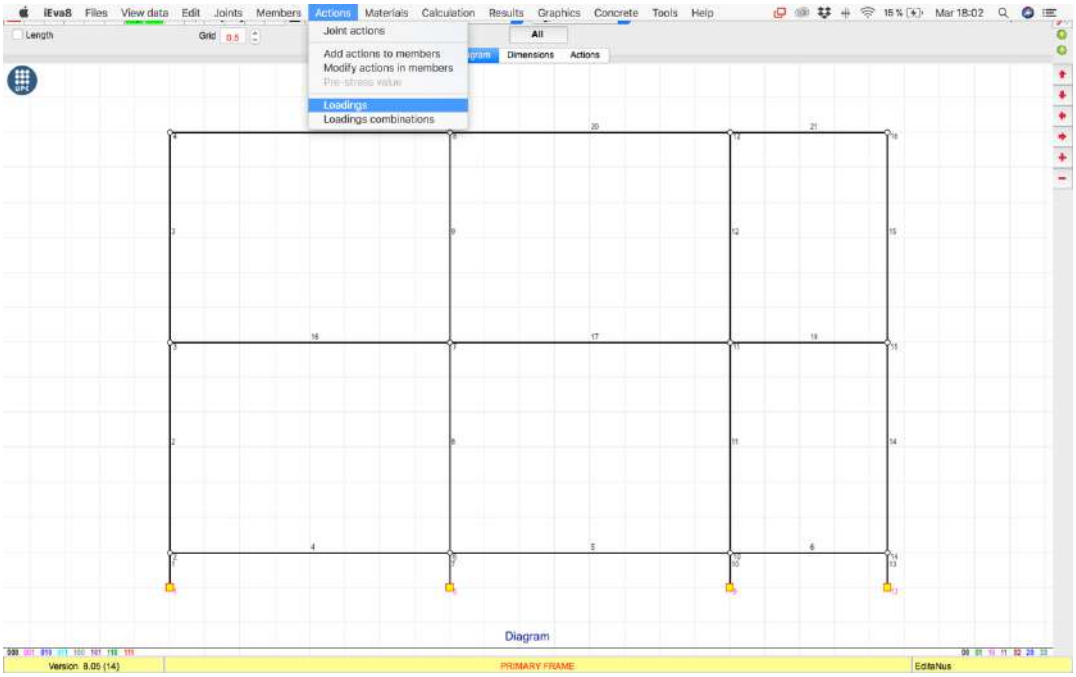
After defining the bars we have to edit the joints. Click on “joints” and then on “edit”.



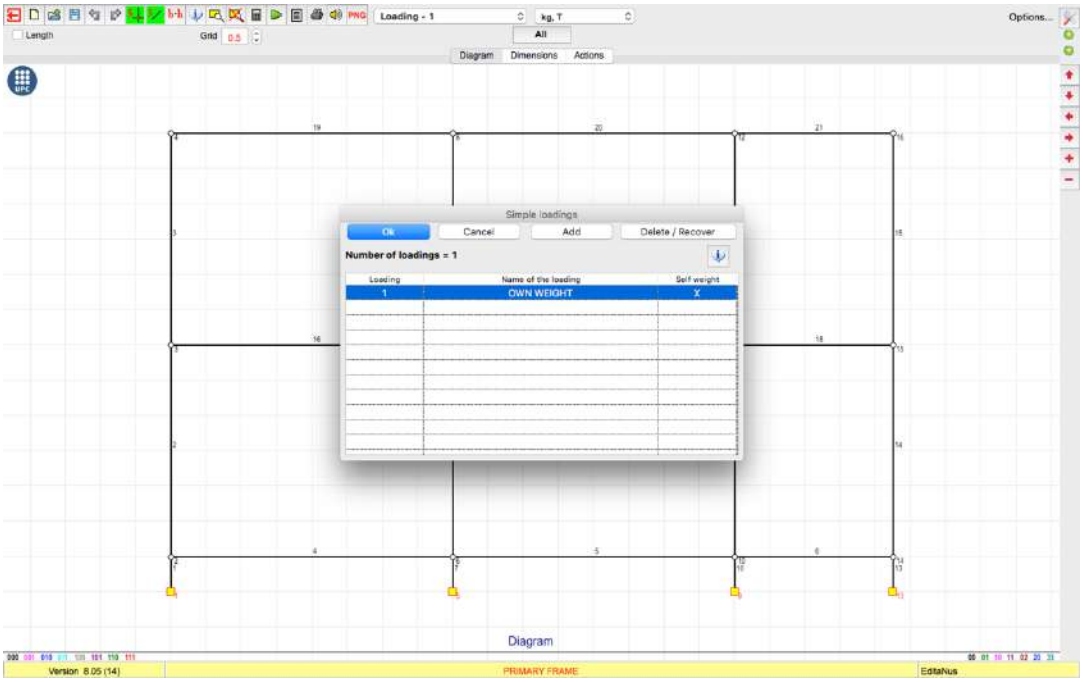
Here we define the characteristics of the supports (freedom degrees).



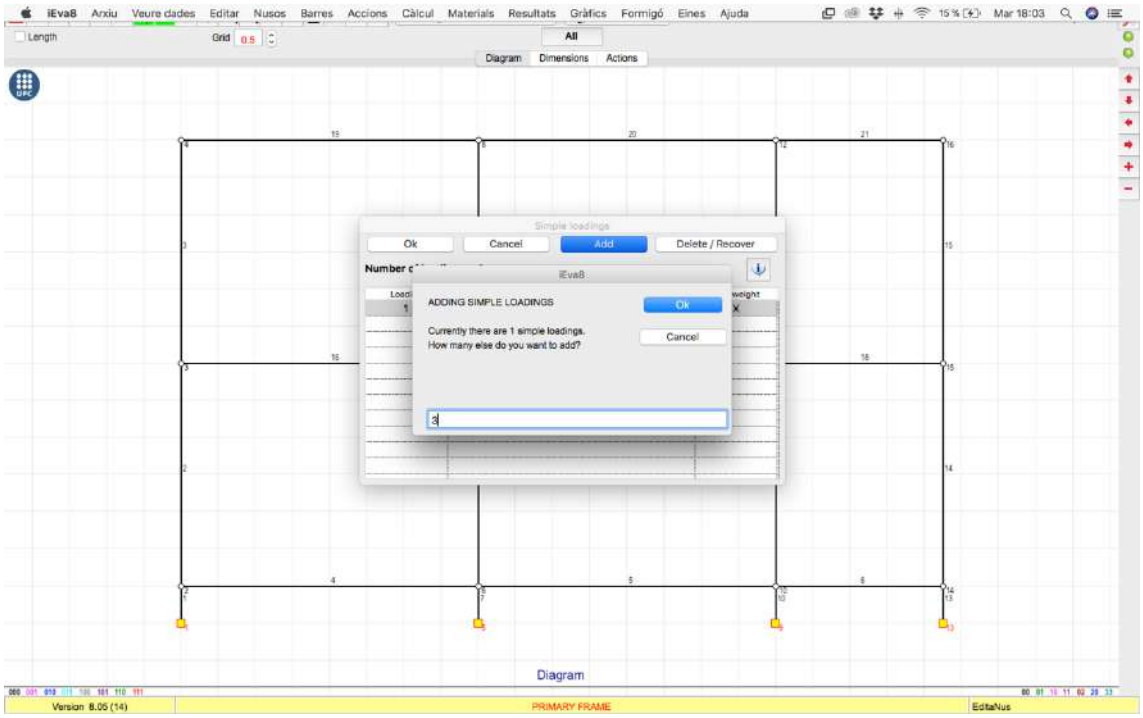
Once that the geometry and its characteristics are defined we are going to add the actions. First of all in the "actions" section click on "Loadings" to set the hypotheses.



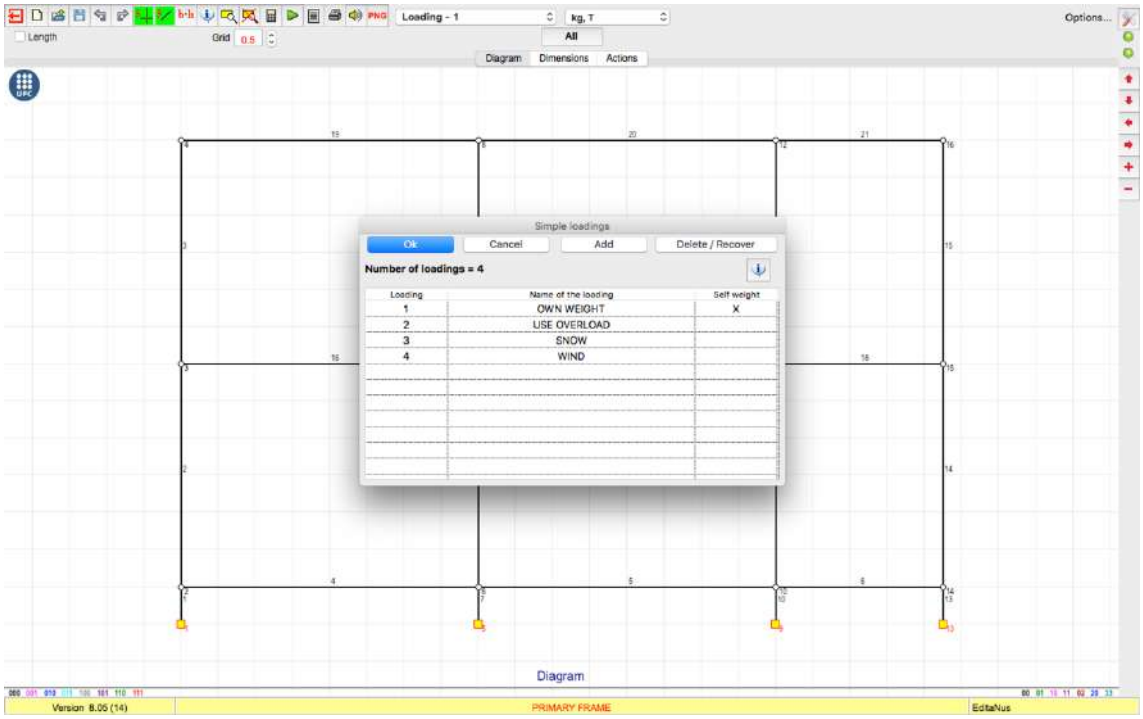
Name "own weight" to the only hypothesis shown here.



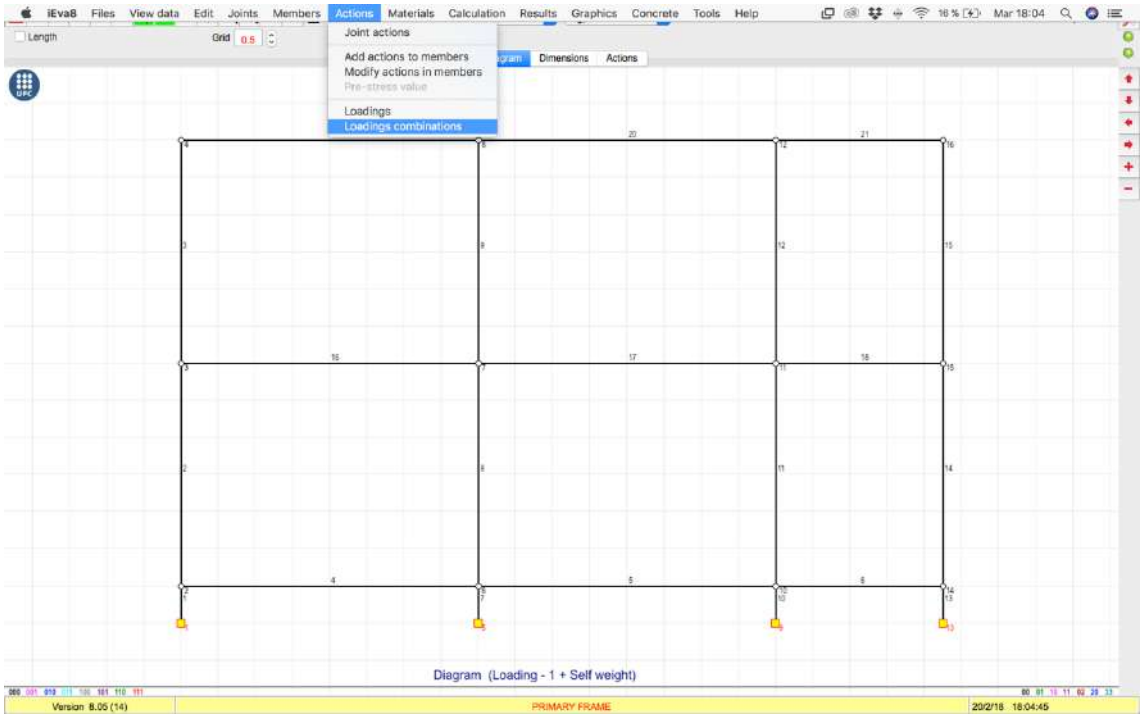
Click on "add" and add three more hypotheses.



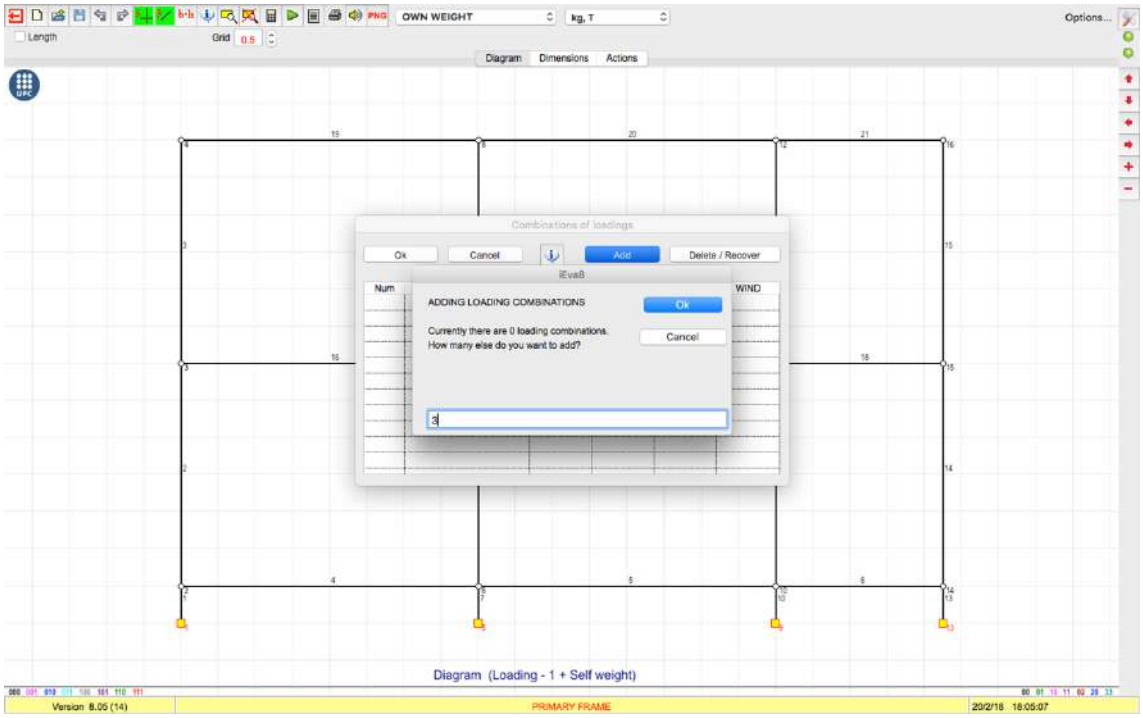
We are going to name these new hypotheses as “use overload”, “snow” and “wind”. These four loadings are just simple hypotheses, which have to be combined.



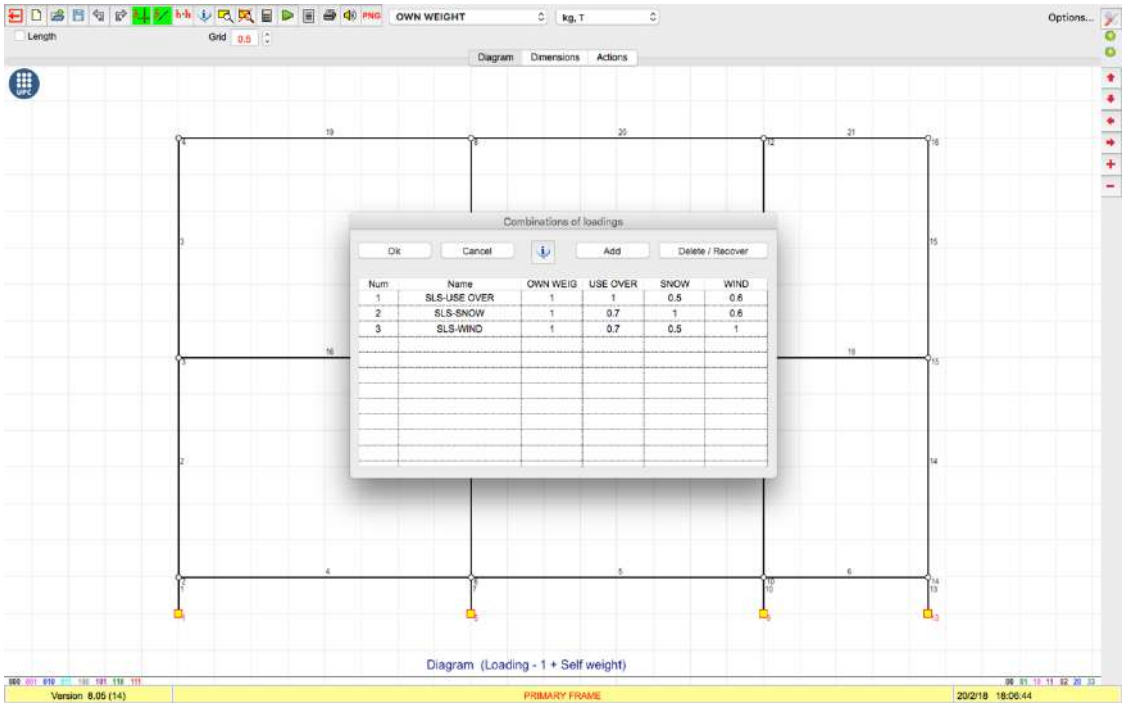
To combine them, in “actions” click on “loadings combinations”.



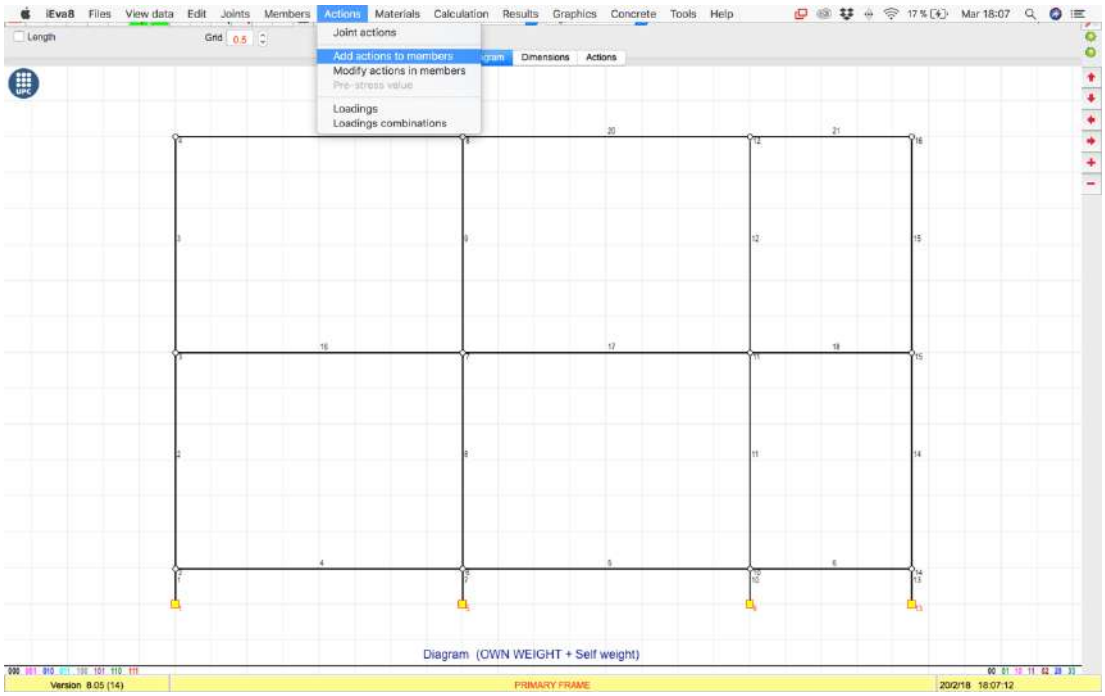
Create now 3 combinations, one for each variable load.



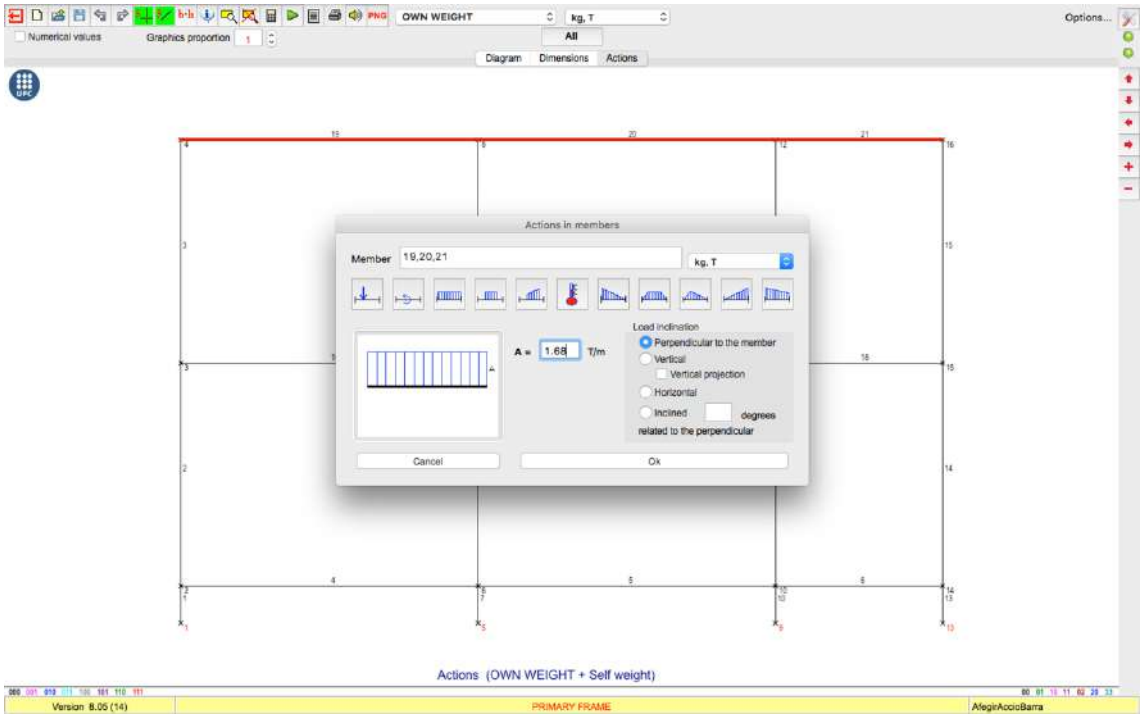
In this chart we have to add the simultaneity factors for each variable load.



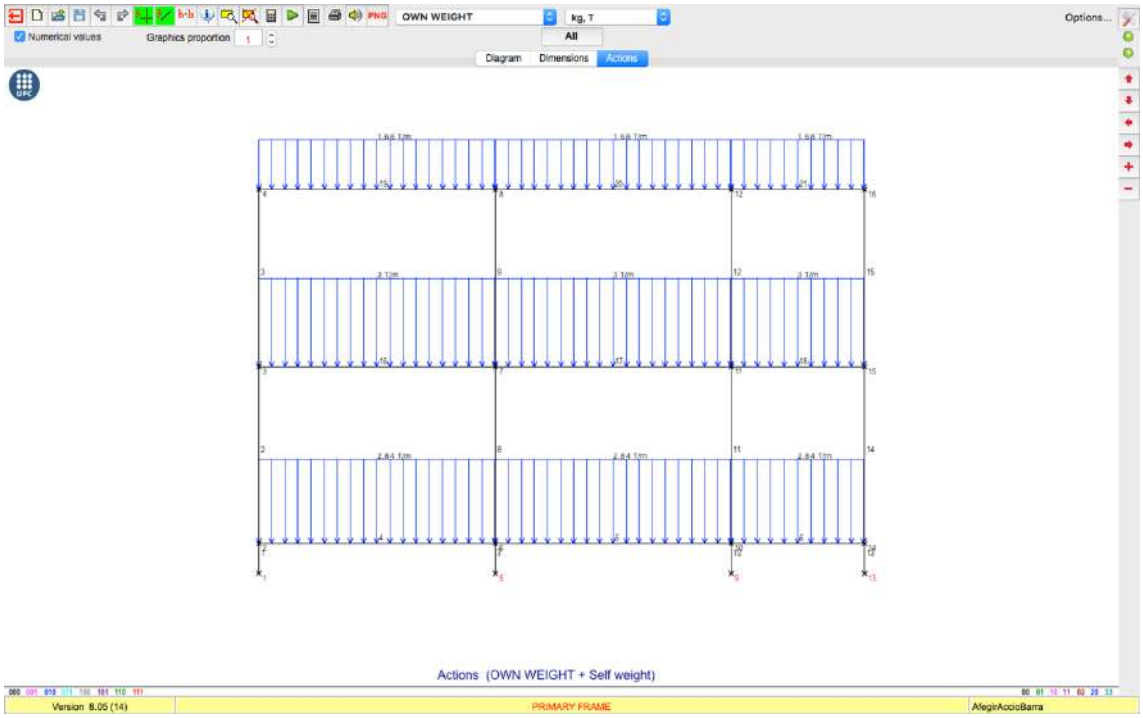
Finally, to add the actions that affect the frame, in “actions” section click on “add actions to members”.

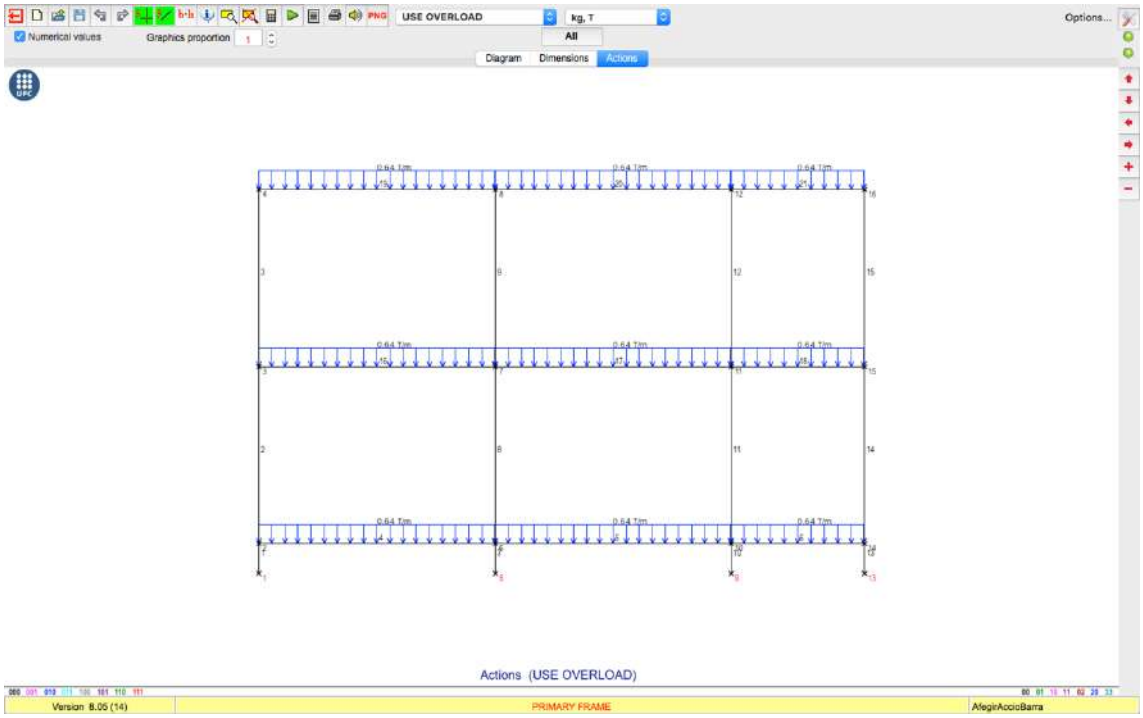


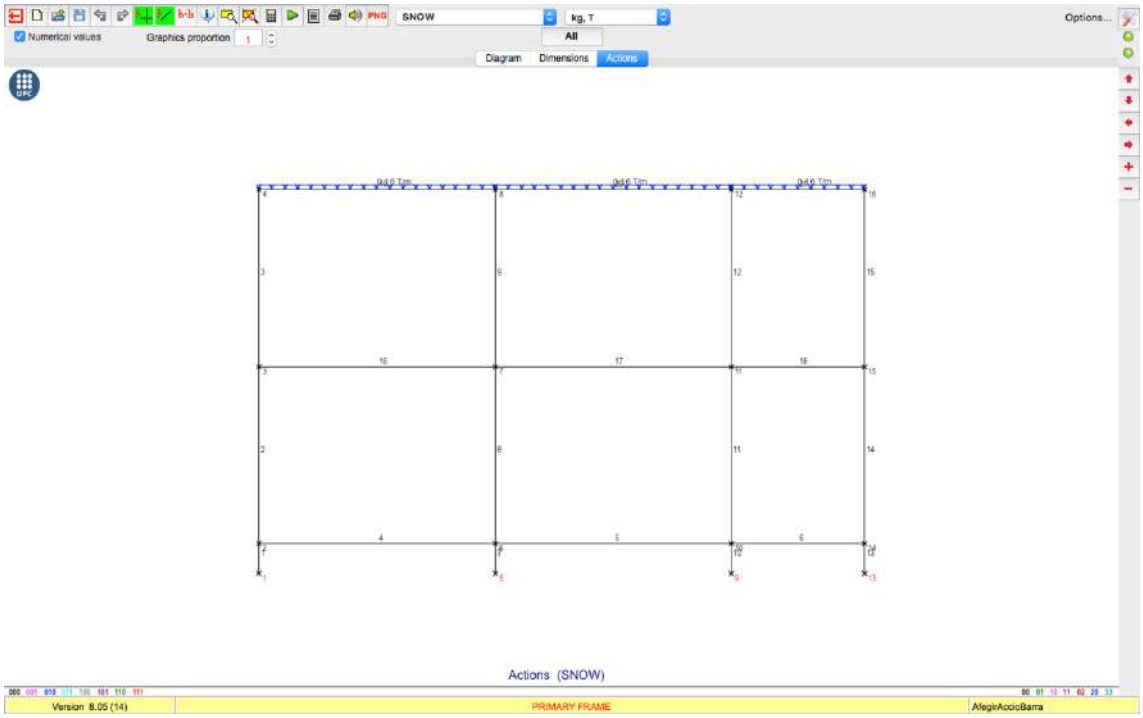
Choose the bars bearing the same load and click on the right button.



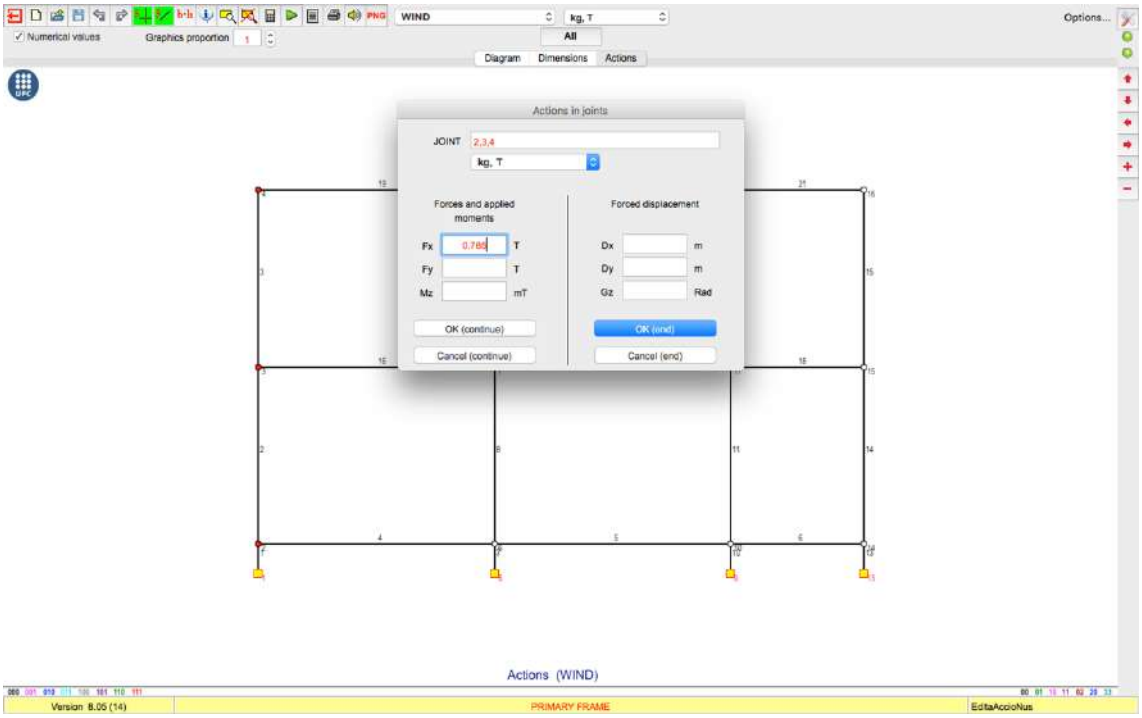
We have to follow the same procedure with the distributed loads and the simple hypotheses.



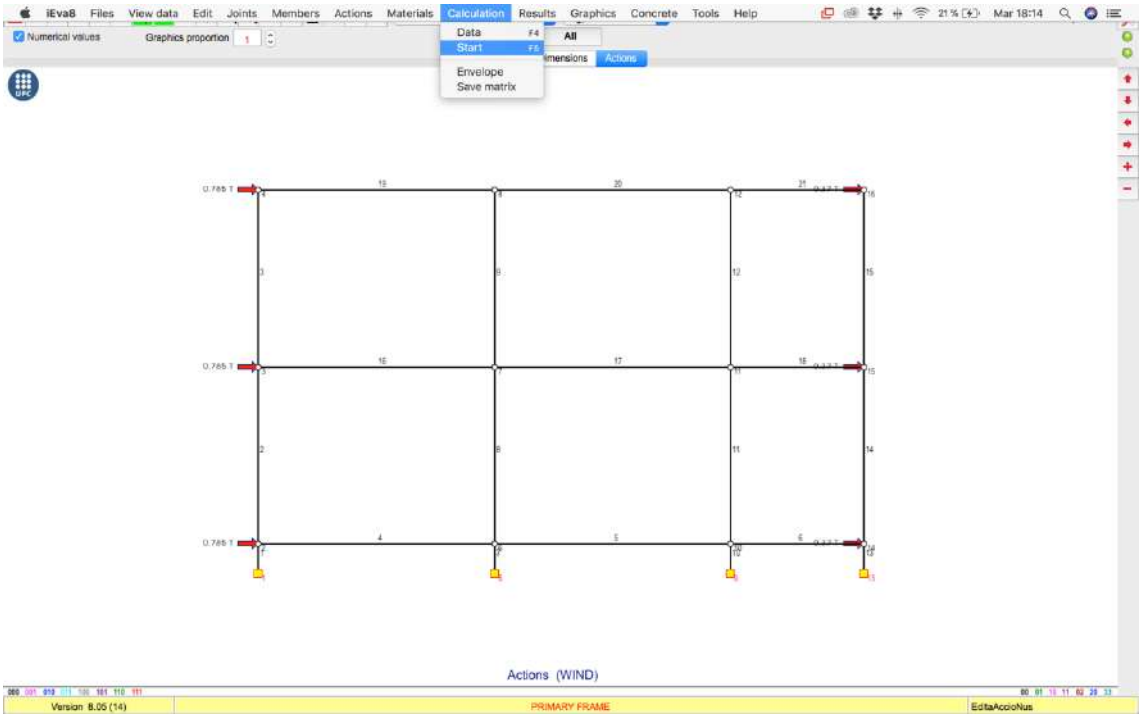




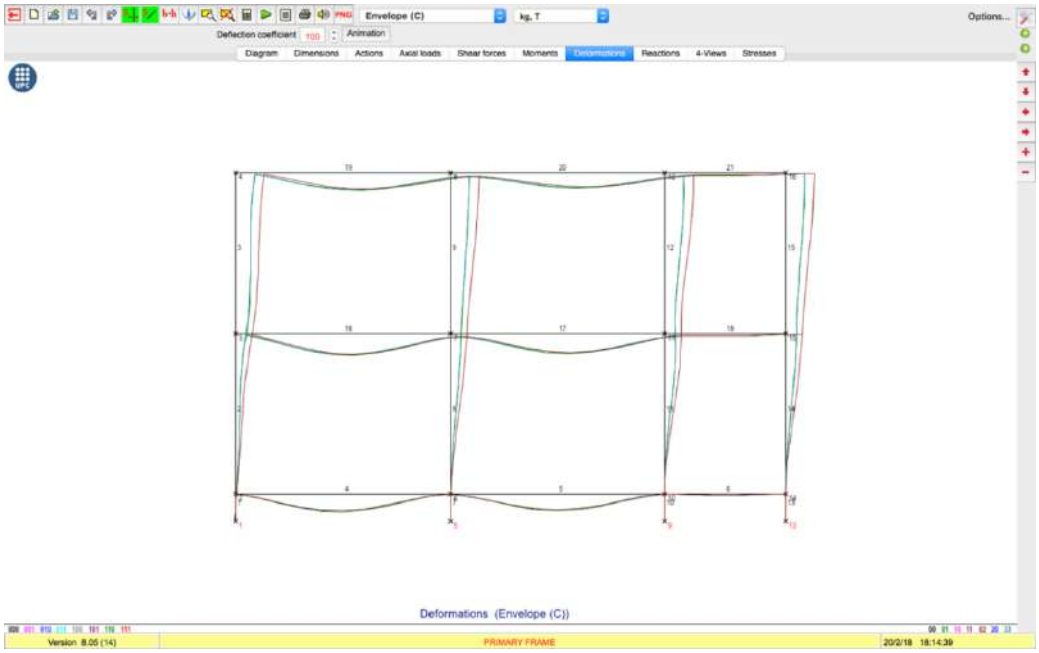
In order to add the wind load affecting the joints, in “actions” section click on “joint actions” and introduce a value.



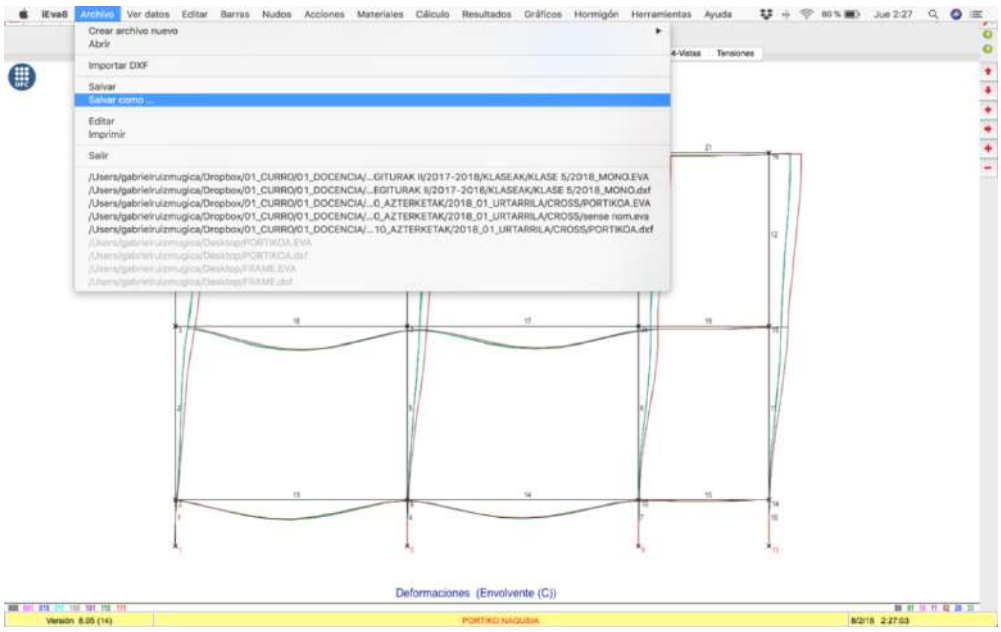
Now the frame is ready to be calculated. Therefore, open the “calculation” menu and click on “start”.



Among all the hypotheses choose “envelope” to see the result of all of them together. Clicking on any bar or joint we can see the maximum deflections. This way we can check with the serviceability limit state.

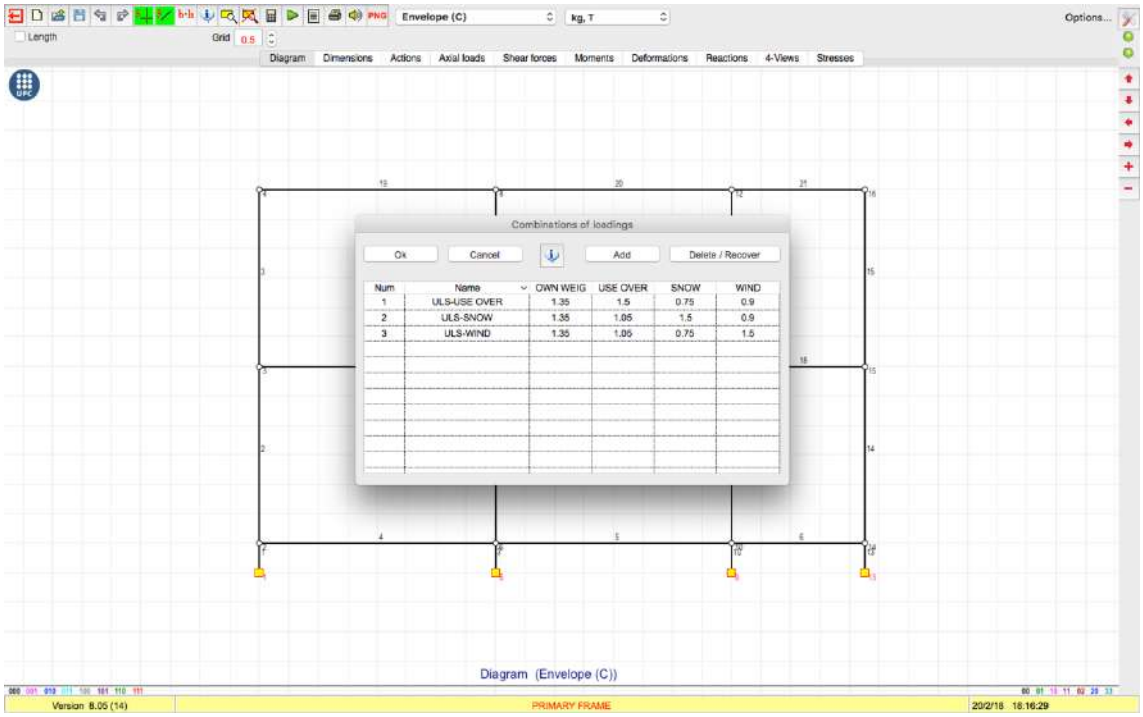


Before verifying the ultimate limit states we need to save the frame, in the “file” menu click on “salvar como” and create a new file to change the hypotheses combination.



In “actions” menu click on “loadings combinations” again and change the factors of the simple hypotheses, this time adding the magnification factors. Then calculate the frame again

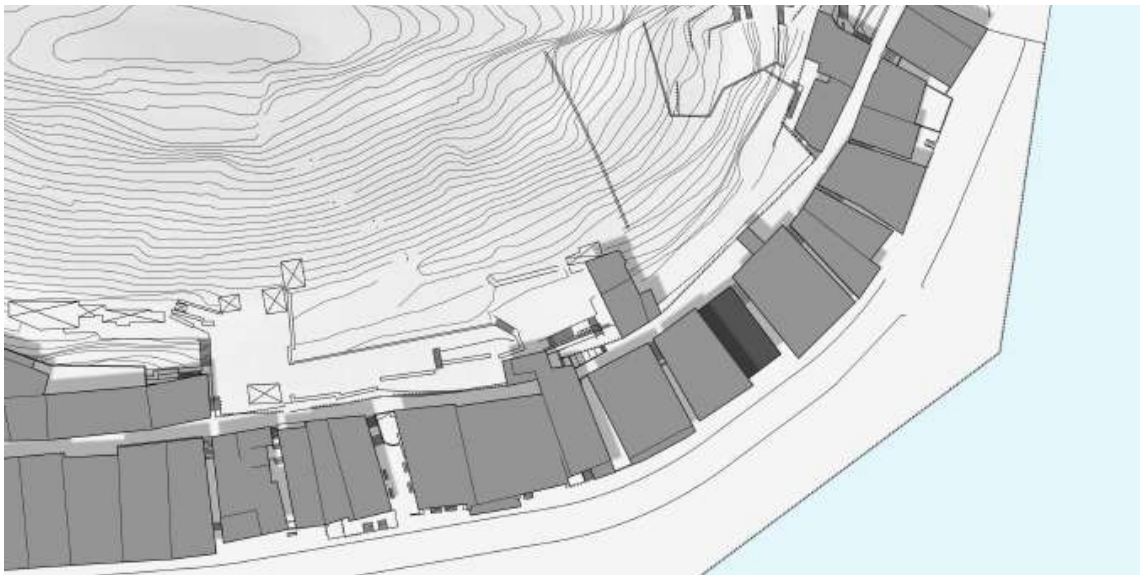
and check the diagrams with the ultimate limit states.



ANNEX 3. DESIGN AND CALCULATION OF A COMPLETE STRUCTURE

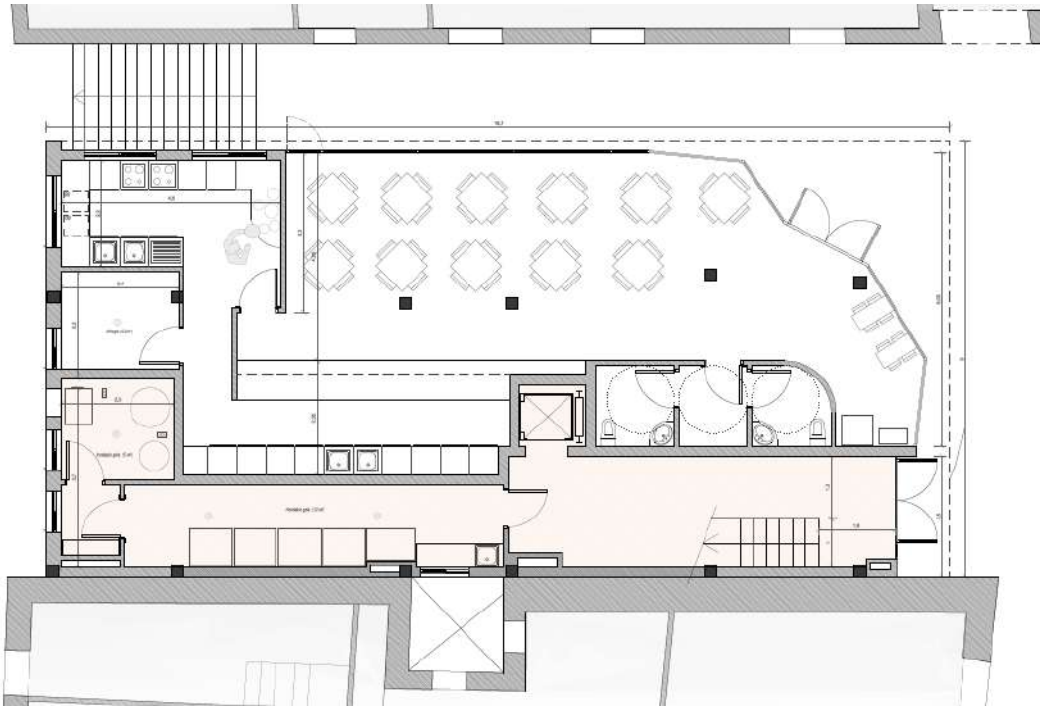
PROJECT DESCRIPTION

Our site is located in the historical centre of Pasajes San Pedro village, between a hill and the sea. San Pedro Street, which used to be the main street of the village, goes along the hillside, with this hill in one side and residential houses in contact with the sea at the other. However, when they built the port of Pasajes they also built a public street that was going along the bay (Pescadores Street). This one is now the main street, and while walking here seems that the buildings work as buttress containing the hill.



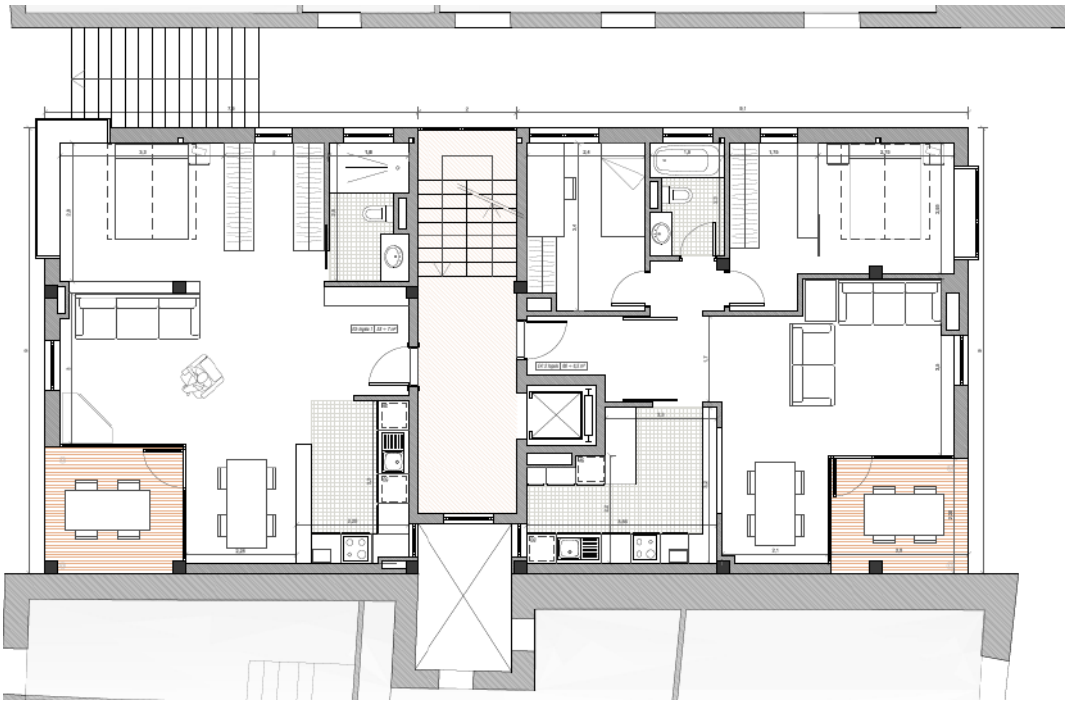
Our site is part of this main building line. Along this line there are some small passages connecting both streets (this passages are about 2 meters wide). Our specific site has a party wall on one side and one of these passages on the other.

On the ground floor of the building there is a bar-restaurant. On the side looking to the bay there is an irregular glass wall which recesses from the main building alignment. On the opposite side is located the storage and the kitchen, as there is a two meter gradient and there is not much light. The entrance hall for the dwellings is attached to the party wall.



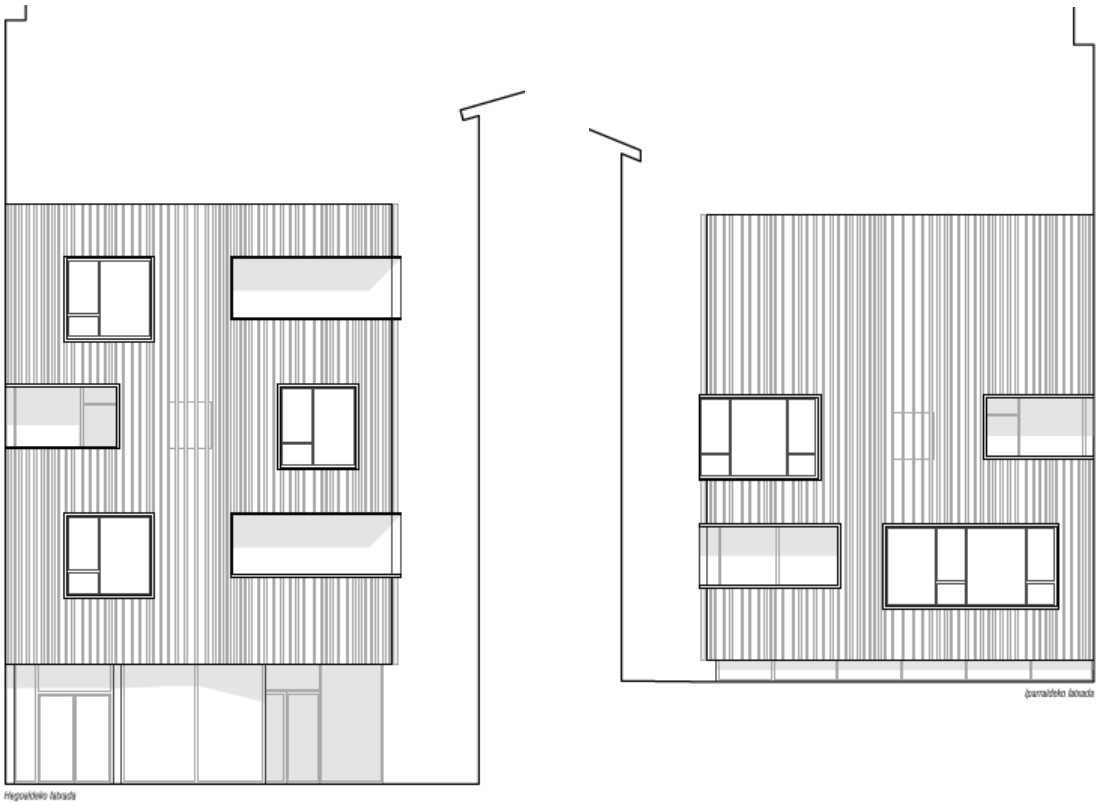
The main building structure is divided in two “lines”. The first one is against the party wall and the other is separated between 5 and 6 meters, leaving a distance of three meters until the passage. In order to construct this structure and its distances is going to be made of steel. First floor (dwellings):

Second floor (dwellings):

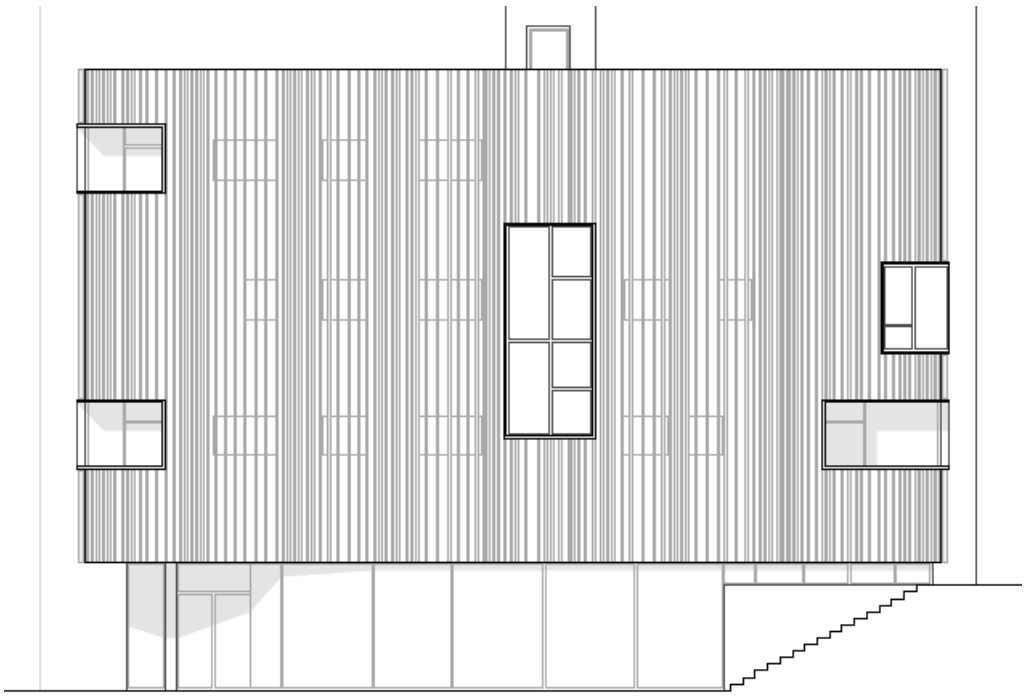


Third floor (dwellings and storage rooms):

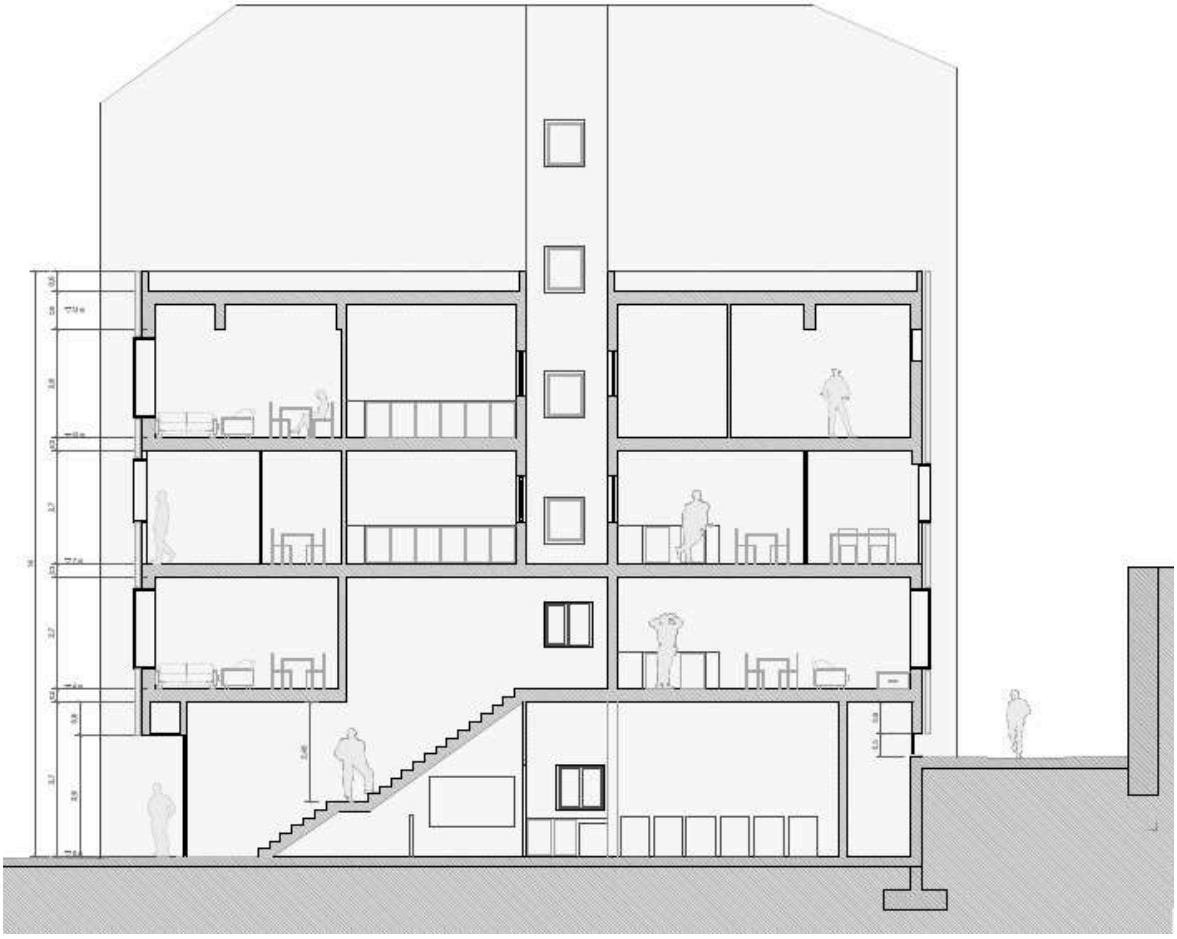
Elevations from San Pedro and “Pescadores” streets:



Elevation from the passage:



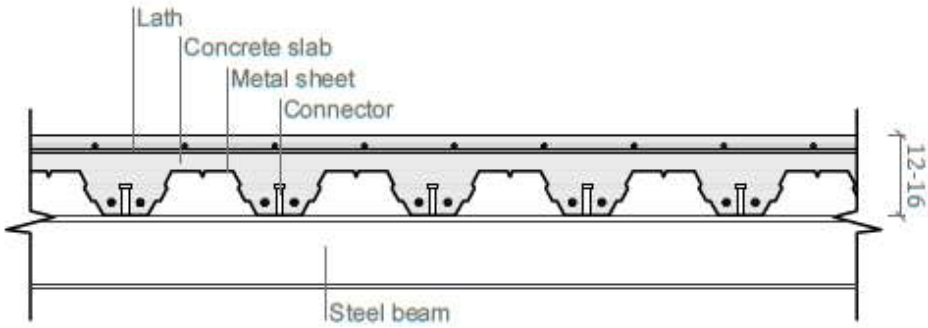
Longitudinal section:



STRCUTURAL DIMENSIONING

Structural description

The main structure of the building is going to be made of steel (columns, beams and joists). The floors slabs are going to be carrying the loads in one way using a Steel deck floor system. This means that on top of the Steel rib floor we will put a metal sheet, and then to ensure the continuity of the floor we will pure a concrete slab.

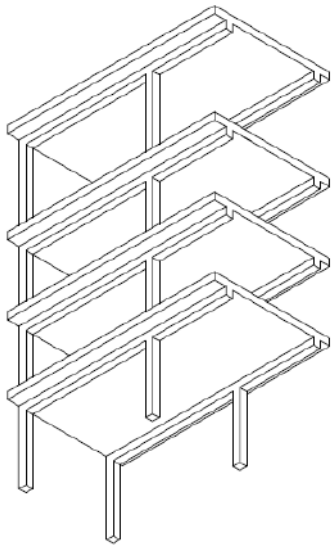


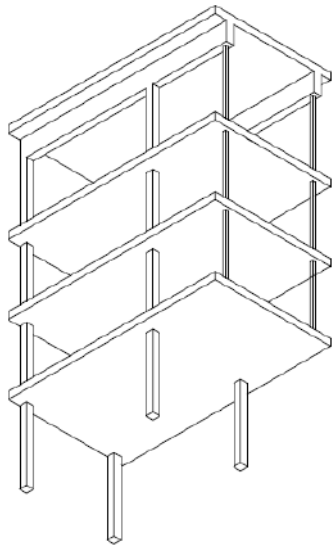
TYPE	CHARACTERISTICS	INTER-AXIS (m)	SPAN L (m)	DEPTH H (m)	WEIGHT P $[kN/m^2]$	COST K $[€/m^2]$
Steel deck floor system	Possible values	-	< 5.00	0.10 – 0.20	1.75 – 3.50	40 - 70
ONE WAY	Usual values (recommended)	-	3.00 – 4.00	0.12 – 0.16	2.25 – 2.75	45 - 55
	Is the lightest floor slab. It has an easy and effective assembly. It can only be used with steel beams and joists. The continuation of the floor is achieved by the concrete slab on top of the steel sheet. The cantilevers are about 0,5 – 1 meter.			$H = \frac{L}{(23-27)}$	$H = P \cdot (16-18)$	$C = H \cdot (300-400)$

The main frames of the structure will go across the building, perpendicular to the party wall. This means that on this direction the two columns that set up the frame must be aligned and parallel between them. However, on the other direction they don't have to be aligned, because they are not creating any frame.

On the side of the passage there is a 3 meter long cantilever all along this depth. As is a great cantilever we need an extraordinary method to solve this. We can do it in two different ways: On the one hand, we can solve it separately on each floor, simply increasing the depth of the beams to bear the cantilever. The main problem of this solution is that as the frames go

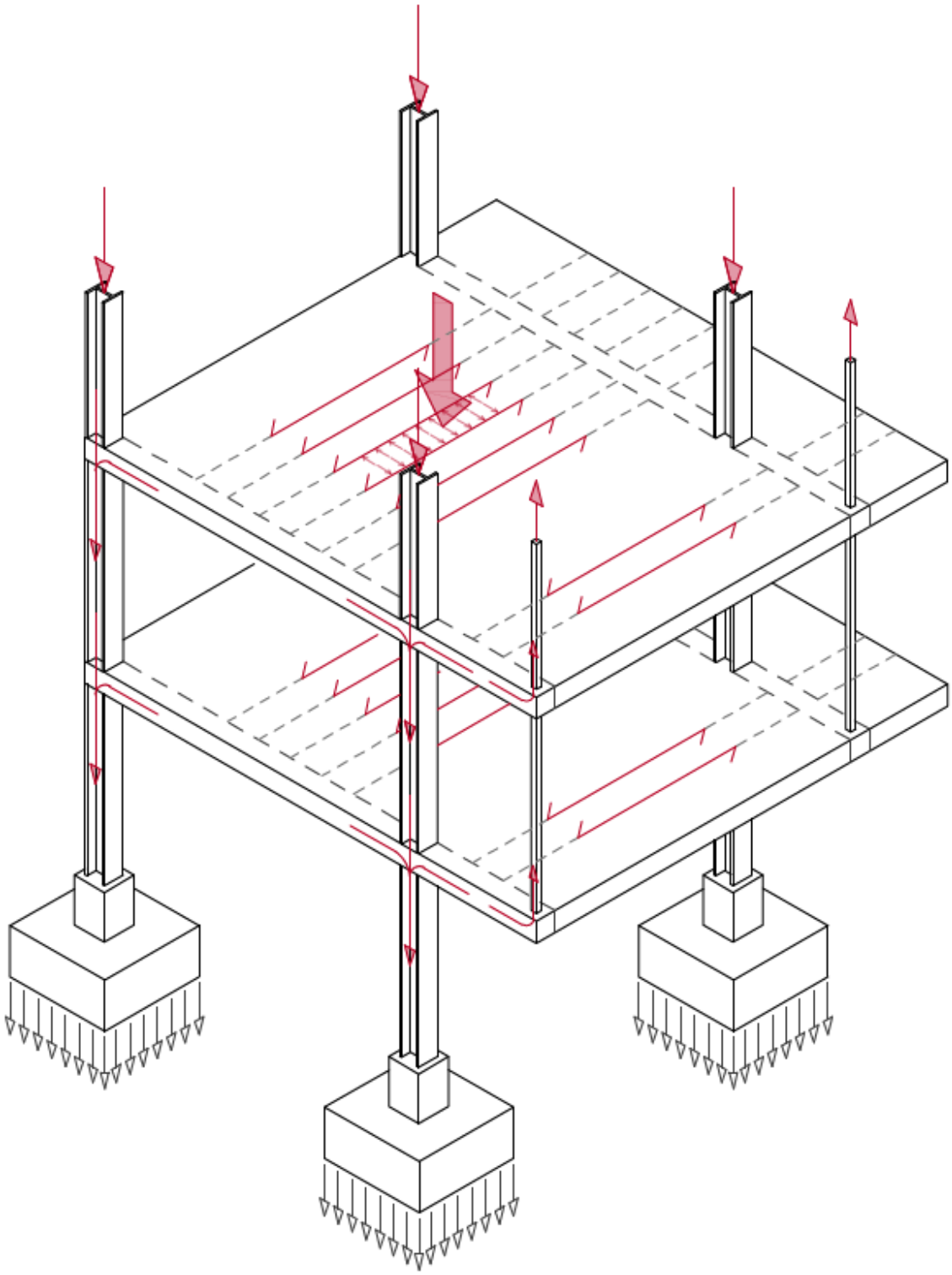
across the building this depth beams will be interrupting the False ceiling every 3-5 meters. On the other hand, we also can solve this problem by using an overall solution. We can just increase the last floor beams and use some steel columns bearing just traction and holding the cantilevers from the depth. This way, the last floor beam is going to have a great depth but the rest floor beams are going to bet flat ones.





These columns holding the cantilever are going to be bearing just traction strength, therefore they work as braces. They are not going to rely on the ground, they are going to be hanging from the top depth beam. Moreover, as they only work under traction strength they don´t suffer buckling and therefore, they can be super thin.

On the main frames, the maximum distance between columns is 6 meters, so this is going to be the bigger beam. They will have rigid joints between them, transmitting the moments to the columns and therefore decreasing their own ones. The biggest distance between frames is 4,7 meters, so this is going to be the length of the biggest joist. The joint between beam and joist is going to be hinged.



Load Quantification

The first step of the dimensioning is to identify and quantify the loads affecting our building structure:

Permanent loads (own weight):		
One way floor slab (Steel deck floor system + joists)	g_1	$2{,}5\ kN/m^2$
Floor and inner partitions	g_2	$1{,}5\ kN/m^2$
False ceiling	g_3	$0{,}5\ kN/m^2$
Flat roof (slope concrete, gravel…)	g_4	$1{,}5\ kN/m^2$
Façade (façade itself + wooden slats)	g_5	$8\ kN/ml$
Variable loads:		
Use overload	g_6	$2\ kN/m^2$
Snow	g_7	$0{,}5\ kN/m^2$
Wind	g_8	$-0{,}5/0{,}8\ kN/m^2$

Wind load calculation:

$$q_e = q_b \cdot C_e \cdot C_p$$

q_b : $dynamic\ wind\ load = 0{,}5\ kN/m^2$
C_e : $exposition\ coefficient = 2$
C_p : $wind\ coefficient;\qquad pressure = 0{,}8\ kN/m^2$
$\qquad\qquad\qquad\qquad\qquad\qquad suction = -0{,}5\ kN/m^2$

Tabla 3.5. Coeficiente eólico en edificios de pisos

	Esbeltez en el plano paralelo al viento					
	< 0,25	0,50	0,75	1,00	1,25	≥ 5,00
Coeficiente eólico de presión, c_p	0,7	0,7	0,8	0,8	0,8	0,8
Coeficiente eólico de succión, c_s	-0,3	-0,4	-0,4	-0,5	-0,6	-0,7

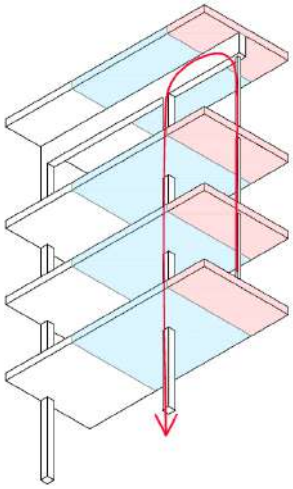
$q_e = q_b \cdot C_e \cdot C_p = 0{,}5 \cdot 2 \cdot 0{,}8 = 0{,}80\ kN/m^2$ Pressure side

$q_e = q_b \cdot C_e \cdot C_p = 0{,}5 \cdot 2 \cdot (-0{,}5) = 0{,}50\ kN/m^2$ Suction side

Main columns dimensioning

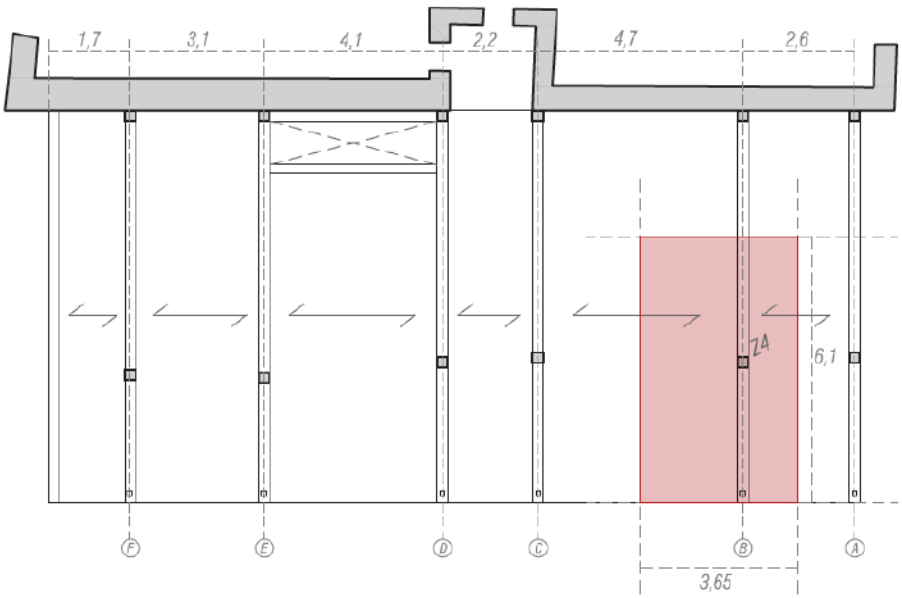
To define the dimensions of the columns first we have to analyse how much load is going to bear. As a result of the own weight and the use overload loads this mains frames are going to work under axial strengths (compression). However, on the opposite direction, as a result of the wind load there is a risk of overturning (horizontal structure collapse). Therefore we will do both verifications and see which one is worse.

Axial strength (Compression)



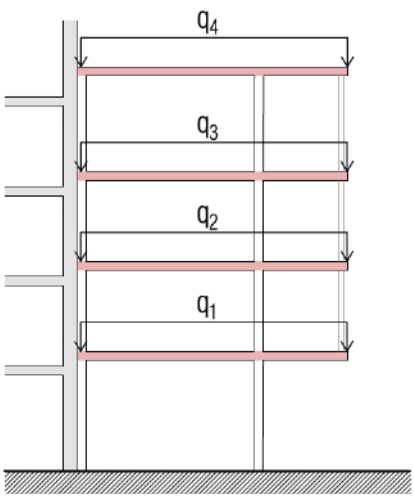
To dimension the column according to the axial strength we have to select the column bearing the most load. As we have the weight of the façade and the floors, we will select the column holding the most are of floor slab and the most meters of façade. This is the z4 column; the area that is holding has 22,2 m^2 of floor (6,1 x 3,65m) and 3,65 meters of façade.

To the area held by the column we have also added the one held by the brace. The brace will take his load and transmit it to the top beam, which then from here will be transmitted to the column. Therefore the column will receive at the last step all the load.



The bottom part of this column is going to bear the loads of all the top floors and façade parts. As this main frame is between the party wall and the passage doesn't receive wind load, therefore it only bears vertical loads. To pre dimensions our columns we will add all the loads to get just one general value.

1. floor	$g_1 + g_2 + g_3 + g_5 + g_6$	$6{,}5\ kN/m^2$	$8\ kN/ml$
2. floor	$g_1 + g_2 + g_3 + g_5 + g_6$	$6{,}5\ kN/m^2$	$8\ kN/ml$
3. floor	$g_1 + g_2 + g_3 + g_5 + g_6$	$6{,}5\ kN/m^2$	$8\ kN/ml$
Roof	$g_1 + g_3 + g_4 + g_5 + g_6$	$7\ kN/m^2$	$8\ kN/ml$



Then, the increased load value:

$$N_{Z4} = Q \cdot A \cdot 1{,}5 = (6{,}5 + 6{,}5 + 6{,}5 + 7) \cdot 22{,}2 \cdot 1{,}5 + (8 + 8 + 8) \cdot 3{,}65 \cdot 1{,}5$$
$$= 588{,}3 \cdot 1{,}5 + 87{,}6 \cdot 1{,}5 = 1013{,}85\ kN = 1013850\ N$$

The type of steel is S275. This means the yield strength of the steel is 275 N/mm^2. The steel reduction factor is $\gamma = 1{,}05$, and the eccentricity factor 1,2:

$$\vartheta = \frac{N \cdot 1{,}2}{A} < f_{yD}\ ;\ f_{yD} = \frac{275}{1{,}05} = 261{,}9\ N/mm^2$$

$$A = \frac{N \cdot 1{,}2}{f_{yD}} = \frac{1013850 \cdot 1{,}2}{261{,}9} = 4645{,}36\ mm^2 = 46{,}45\ cm^2\ \rightarrow \boxed{HEB\ 160}$$

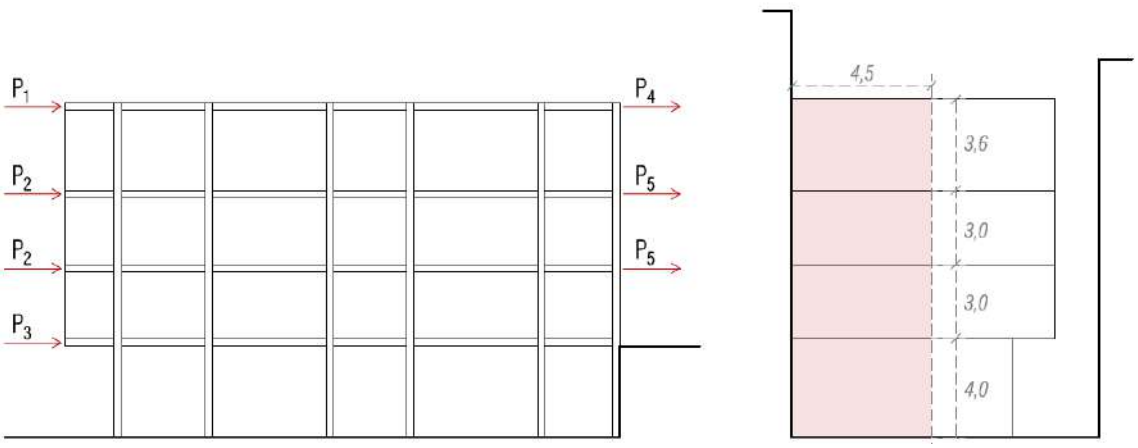
That means that we need a profile with just 46,45 cm^2 of area to hold the axial strength. As the profile is going to work as a column we will choose from the HEB profiles: **HEB-160**, with 54,3 cm^2 of area.

This dimensioning method works better with concrete structures, as in this type of structure the axial strength has a bigger impact. Steel structures have a lot of resistance against axial strengths, but unless this is traction, we have to check the buckling in the columns, which has a lot less resistance.

Horizontal deflection

The horizontal deflection is originated by the wind, in the opposite direction of the main frames. In this direction we don't have a frame as itself, but we do have the joists connecting the main frames, so we can somehow say that these joists create a "second degree frame". The columns located against the party wall are aligned, and the others they have a eccentricity of just half a meter, therefore this second degree frame is going to be the one working against the wind load.

The wind load on pressure side is 0,8 kN/m^2 and on suction side -0,5 kN/m^2. Taking into account the dimensions of the façade:



$P_1 = 0{,}8 \cdot (4{,}5 \cdot 3{,}6) \cdot 1{,}5 = 19{,}5\ kN$

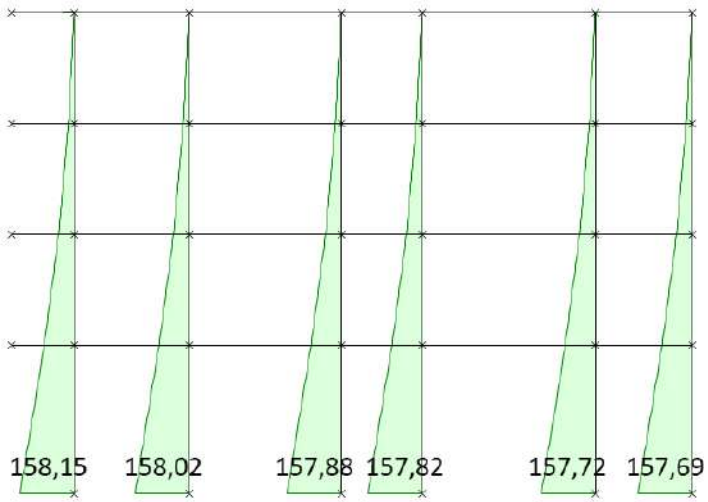
$P_4 = 0{,}5 \cdot (4{,}5 \cdot 3{,}6) \cdot 1{,}5 = 12{,}2\ kN$

$P_2 = 0{,}8 \cdot (4{,}5 \cdot 3{,}0) \cdot 1{,}5 = 16{,}2\ kN$

$P_5 = 0{,}5 \cdot (4{,}5 \cdot 3{,}0) \cdot 1{,}5 = 10{,}2\ kN$

$P_3 = 0{,}8 \cdot (4{,}5 \cdot 4{,}0) \cdot 1{,}5 = 21{,}6\ kN$

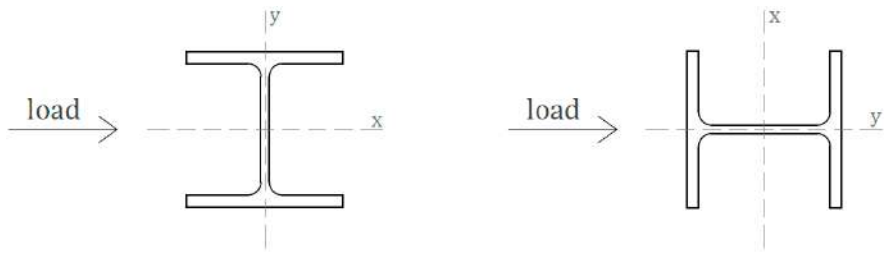
Once we have defined these loads we will calculate the frame using a computer base program calculation to discover which the strengths in the base of the columns are. This way we will know which one is worst situation and whit steel profile do we need to solve it. As we are dimensioning the profile against horizontal deflection we will have to take into account the flection moment, the shear strength is going to be negligible comparing to the flection. We have to mention that before calculating it with the computer program we should define the profile size. However, the frame we are analysing is a hyperstatic frame and therefore there is no easy way of doing it. But since all the columns will be designed with the same profile type, all of them will receive the same proportion of moment in the bottom of the columns, in spite of calculating with a different size of profile. This way we will get the value of the moment and depending on this value we can define the size of our column profile not to overcome the yield strength of the material.



Almost all of the columns receive the same amount of moment of their base because the structure is symmetric. The biggest value is in their base because they have an embedment. Therefore, our columns need to have enough resistance to receive that moment:

$$\boxed{\sigma_N = \frac{M \cdot z}{I} = \frac{M}{W}} \qquad \begin{cases} M = 158{,}15\ kN \cdot m \\ W = \begin{cases} W_x \\ W_y \end{cases} \end{cases}$$

The same steel profile has two different resistant modules $(\mathrm{W_x}\ \mathrm{and}\ \mathrm{W_y})$, depending on the axis direction. If the strong axis direction of the profile is at the same direction of the load, is going to work better and therefore we will need to use a smaller profile:



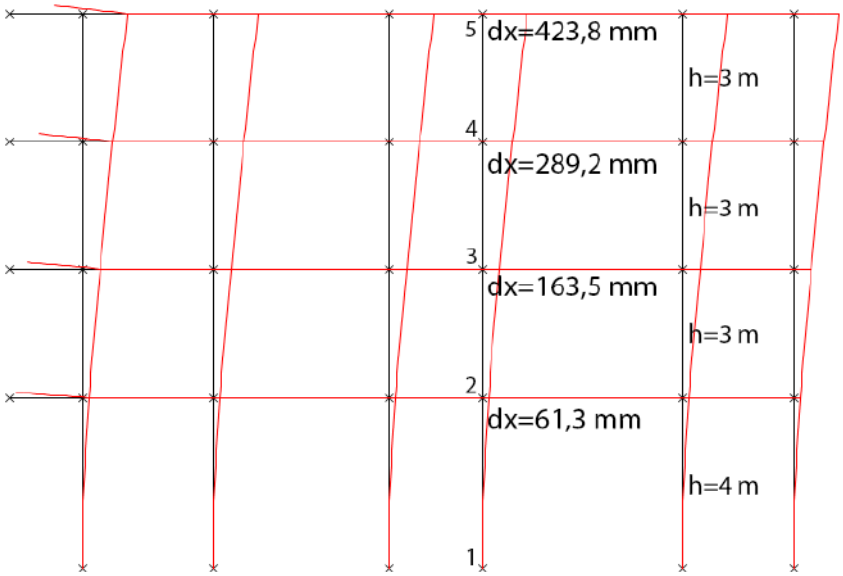
We are going to use the second option, directing profiles strong axis to the load. So we have to take W_x resistant module:

$$\sigma_N = \frac{M}{W_x} \rightarrow W_x = \frac{M}{\sigma_N} = \frac{158{,}15 \cdot 10^6}{261{,}9} = 603856\ mm^3 = \boxed{603{,}9\ cm^3}$$

Perfil	Dimensiones							Términos de sección								
	h mm	b mm	e mm	e_1 mm	r_1 mm	h_1 mm	u mm	A cm²	S_x cm³	I_x cm⁴	W_x cm³	i_x cm	I_y cm⁴	W_y cm³	i_y cm	I_t cm⁴
HEB 100	100	100	6,0	10,0	12	56	567	26,0	52,1	450	90	4,16	167	33	2,53	9,34
HEB 120	120	120	6,5	11,0	12	74	686	34,0	82,6	864	144	5,04	318	53	3,06	14,90
HEB 140	140	140	7,0	12,0	12	92	805	43,0	123,0	1.509	216	5,93	550	79	3,58	22,50
HEB 160	160	160	8,0	13,0	15	104	918	54,3	177,0	2.492	311	6,78	889	111	4,05	33,20
HEB 180	180	180	8,5	14,0	15	122	1.040	65,3	241,0	3.831	426	7,66	1.363	151	4,57	46,50
HEB 200	200	200	9,0	15,0	18	134	1.150	78,1	321,0	5.696	570	8,54	2.003	200	5,07	63,40
HEB 220	220	220	9,5	16,0	18	152	1.270	91,0	414,0	8.091	736	9,43	2.843	258	5,59	84,40
HEB 240	240	240	10,0	17,0	21	164	1.380	106,0	527,0	11.259	938	10,30	3.923	327	6,08	110,00
HEB 260	260	260	10,0	17,5	24	177	1.500	118,4	641,0	14.919	1.150	11,20	5.135	395	6,58	130,00
HEB 280	280	280	10,5	18,0	24	196	1.620	131,4	767,0	19.270	1.380	12,10	6.595	471	7,09	153,00
HEB 300	300	300	11,0	19,0	27	208	1.730	149,1	934,0	25.166	1.680	13,00	8.563	571	7,58	192,00
HEB 320	320	300	11,5	20,5	27	225	1.770	161,3	1.070,0	30.823	1.930	13,80	9.239	616	7,57	241,00
HEB 340	340	300	12,0	21,5	27	243	1.810	170,9	1.200,0	36.656	2.160	14,60	9.690	646	7,53	278,00

Therefore, we will use **HEB 220** profile, with $W_x = 736\ cm^3$. If we use a HBE profile in the opposite direction (using his weak axis) we would need a HEB 340, with $W_y = 646\ cm^3$.

Once we have defined the profile we will introduce it in WinEva to verify again the horizontal deflection of the columns. We will check the local and the total collapse, which can´t be bigger than $\delta = H/250$.



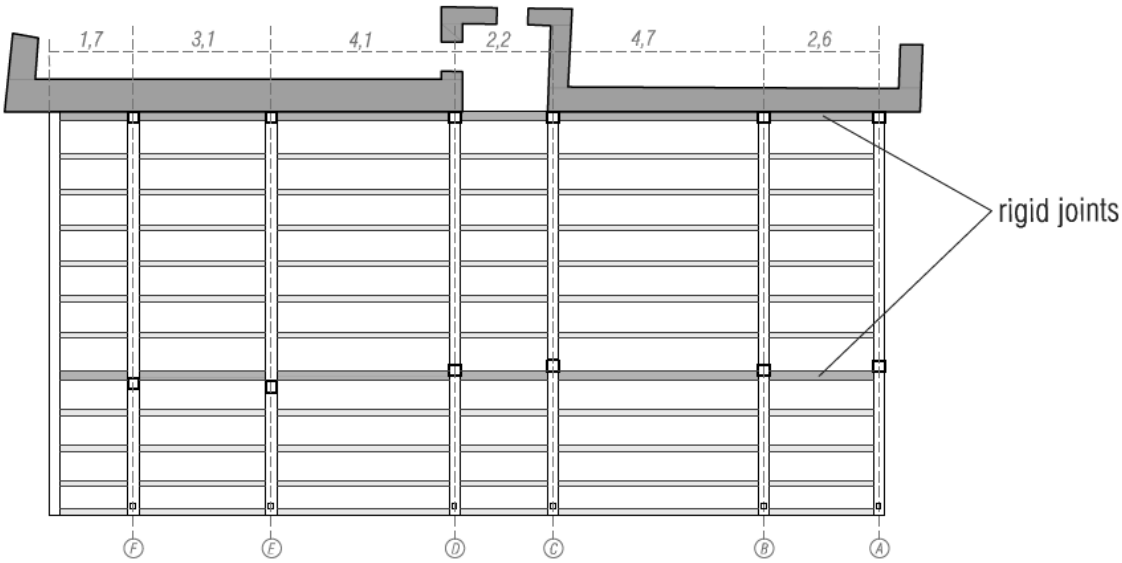
Total collapse:

$$H = 13\ m = 13000\ mm \ \rightarrow\ \delta_{MAX} = {}^{13000}/_{250} = 52\ mm \ \rightarrow\ \underline{423{,}8 \nless 52\ \times \rightarrow 815\%}$$

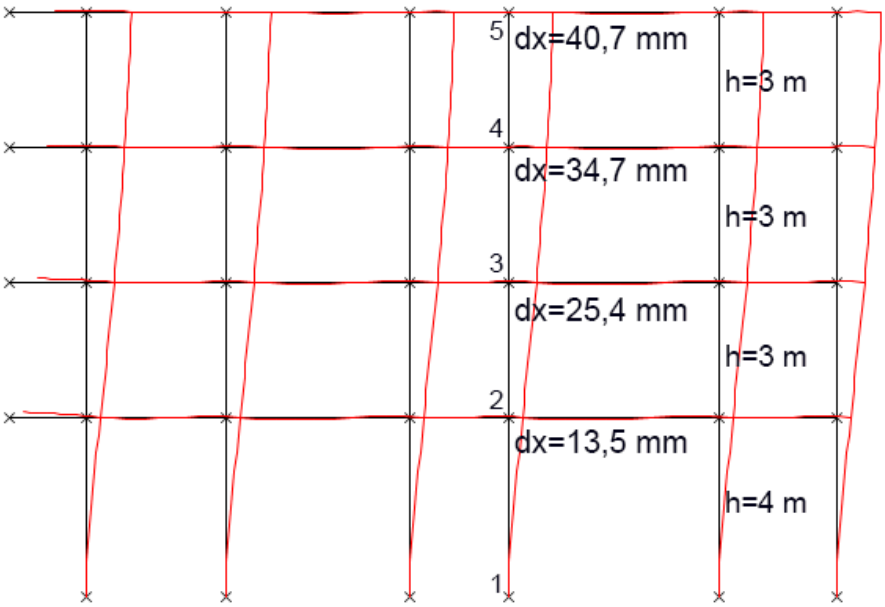
The total collapse of the building structure is 8 times bigger than what the regulation allows. This happens because steel profiles have a lot of resistance against tensions until they get to their classification state, however, in this process it gets deformed. Not to overcome the maximum collapse we would have to increase a lot the profiles, from a HEB 220 to a HEB 450. This wouldn´t be an effective solution, as only a really small part (17%) of the profile would be in tension.

Therefore we should change the general structural scheme. Until now, just the embedment of the columns was working against the collapse produced by the wind load. To avoid this we have to create a frame using the structural elements: triangulations, using elevator´s box, using the stairs…

On this case we are going to use the structure that we already have inside the floor slabs. We are going to use some of the joists creating a “second degree frame”. These joists instead of being hinged will have rigid joints with the main beams. This way, the moment created by the flange horizontal load is going to be transmitted through these joists. These rigid joists are meant to connect the columns creating the second degree frames, so they will only be two lines of these joists.



We will introduce these changes into the virtual frame in the computer programme to verify again the horizontal deflection:



Desplome totala:

$$H = 13\ m = 13000\ mm \ \rightarrow\ \delta_{MAX} = {}^{13000}/_{250} = 52\ mm \ \rightarrow\ \underline{40{,}7 < 52\ \checkmark}$$

Desplome lokalak:

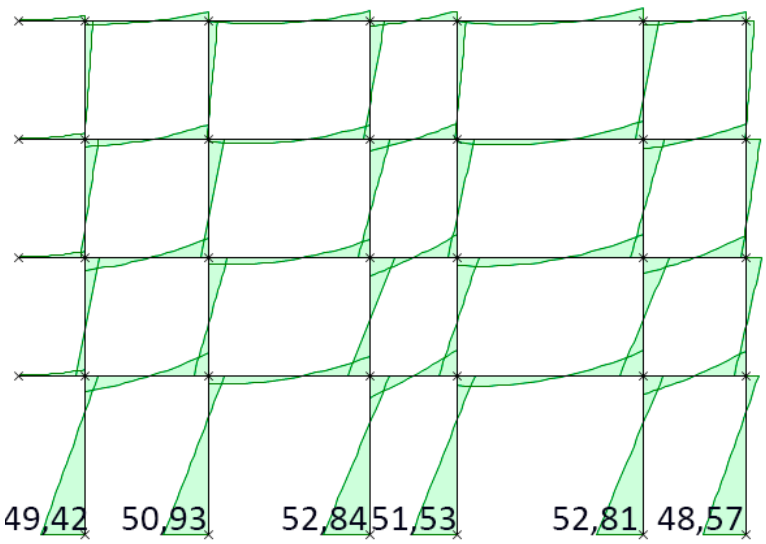
$$h_{12} = 4\ m = 4000\ mm \ \rightarrow\ \delta_{max} = {}^{4000}/_{250} = 16\ mm \ \rightarrow\ \underline{13{,}5 < 16\ \checkmark}$$

$h_{23} = 3\ m = 3000\ mm\ \rightarrow\ \delta_{max} = {}^{3000}/_{250} = 12\ mm\ \rightarrow (25{,}4 - 13{,}5 =)\ \underline{11{,}9 < 12\ \checkmark}$

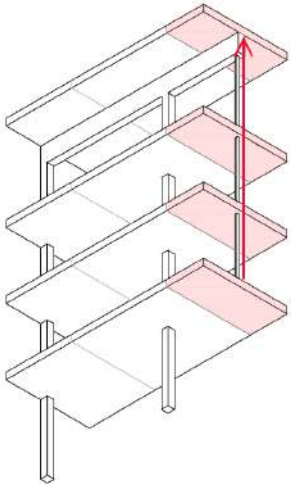
$h_{34} = 3\ m = 3000\ mm\ \rightarrow\ \delta_{max} = {}^{3000}/_{250} = 12\ mm\ \rightarrow (34{,}7 - 25{,}4 =)\ \underline{9{,}3 < 12\ \checkmark}$

$h_{45} = 3\ m = 3000\ mm\ \rightarrow\ \delta_{max} = {}^{3000}/_{250} = 12\ mm\ \rightarrow (40{,}7 - 34{,}7 =)\ \underline{6 < 12\ \checkmark}$

However, when introducing this change in the building structure we will suffer a complete change of the strengths and tensions. We are increasing now the moment in the joists, so maybe the joists with rigid joints are bigger than the others.



Dimensioning of the axial braces

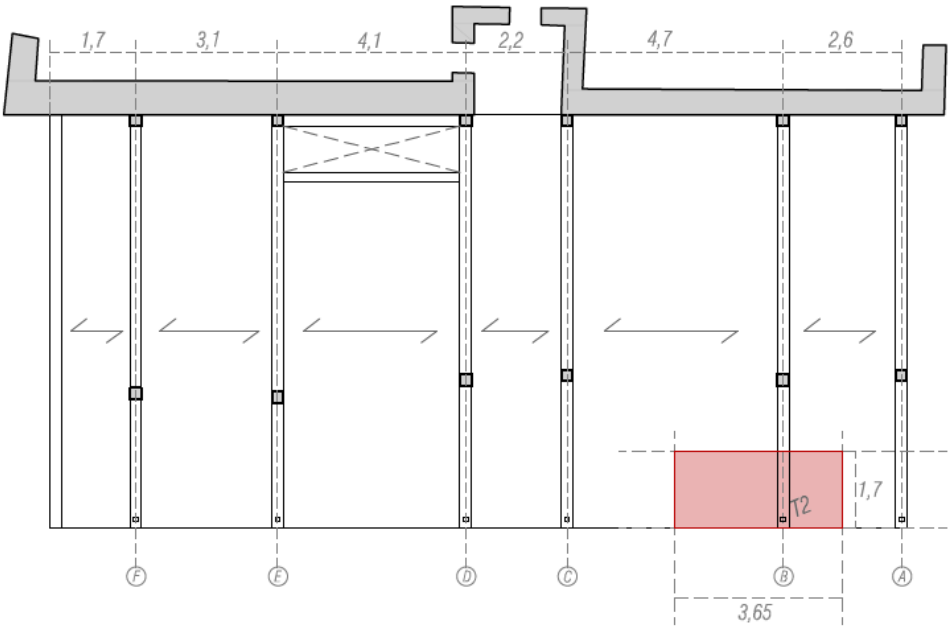


These braces are inside the main frame, sustaining the 3 meter long cantilever. Theses braces do not reach the ground level, they are hanging from the top depth beam. This is why they are working under traction strength.

Se to dimension the brace we only have to quantify this axial tension. Moreover, as this tension is going to be always traction we are not going to suffer buckling problems, been able to use very thin profiles.

Axial strength (traction)

We are going to repeat the same process of the column, we choose the brace bearing the most amount of load. This is T2 brace: his floor area is 6,2 m² (1,7x3,65m) and 3,65 façade meters.



The own weight of its floor has been calculated in the previous part. Therefore, with this loads and with the floor area we will calculate the loads bearing the brace.

1. floor	$g_1 + g_2 + g_3 + g_5 + g_6$	$6{,}5\ kN/m^2$	$8\ kN/ml$
2. floor	$g_1 + g_2 + g_3 + g_5 + g_6$	$6{,}5\ kN/m^2$	$8\ kN/ml$
3. floor	$g_1 + g_2 + g_3 + g_5 + g_6$	$6{,}5\ kN/m^2$	$8\ kN/ml$
Roof	$g_1 + g_3 + g_4 + g_5 + g_6$	$7\ kN/m^2$	$8\ kN/ml$

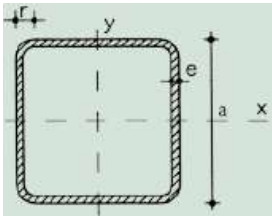
Column increased load:

$$N_{T2} = Q \cdot A \cdot 1{,}5 = (6{,}5 + 6{,}5 + 6{,}5 + 7) \cdot 6{,}2 \cdot 1{,}5 + (8 + 8 + 8) \cdot 3{,}65 \cdot 1{,}5$$
$$= 164{,}3 \cdot 1{,}5 + 87{,}6 \cdot 1{,}5 = 377{,}85\ kN = 377850\ N$$

We are using the same type of steel (S275). This means the yield strength of the steel is 275 N/mm^2. The reduction factor is $\gamma = 1{,}05$, and the eccentricity factor 1,2:

$$\vartheta = \frac{N \cdot 1{,}2}{A} < f_{yD}\ ;\ f_{yD} = \frac{275}{1{,}05} = 261{,}9\ N/mm^2$$

$$A = \frac{N \cdot 1{,}2}{f_{yD}} = \frac{377850 \cdot 1{,}2}{261{,}9} = 1731{,}27\ mm^2 = 17{,}31\ cm^2$$



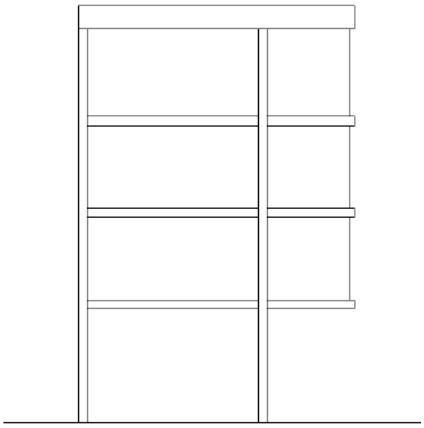
This time we are going to use an empty profile, with its area spread around the perimeter.

Perfil	Dimensiones				Términos de sección				
	a mm	e mm	r mm	u mm	A cm²	S cm³	I cm⁴	W cm³	i cm
# 90.4	90	4	10	343	13,20	21,10	159,00	35,40	3,47
~~# 90.5~~	~~90~~	~~5~~	~~13~~	~~339~~	~~16,10~~	~~25,30~~	~~189,00~~	~~41,90~~	~~3,42~~
~~# 90.6~~	~~90~~	~~6~~	~~15~~	~~334~~	~~18,90~~	~~29,20~~	~~214,00~~	~~47,60~~	~~3,36~~
# 100.3	100	3	8	387	11,30	20,10	175,00	35,00	3,93
# 100.4	100	4	10	383	14,80	26,40	223,00	44,60	3,88

Dimensioning of the beams

The dimensioning of beams and joists is different depending on the material we are using, according to its properties. When we use concrete we can choose the dimensions from a standardized chart. These charts relate the span they need to cover with the beam depth, the longer the span the bigger the depth.

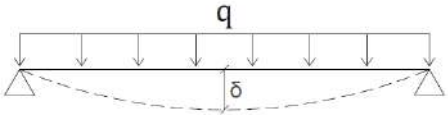
Nevertheless, in Steel beams the bigger problem is not their resistance, but the deflection. Therefore we will use the serviceability limit state (SLS) to dimension the profile (in the SLS state we don´t increase the loads):



Depth beam (top floor):

$$\delta = \frac{l}{300}$$

Two side supported beams (rest of floors):



$$\delta = \frac{l}{500}$$

Depth beam:

$$\delta_{MAX} = {}^{L}/_{300} = {}^{280}/_{300} = 0{,}9\hat{3}\ cm\ \ ;\ \ P = 251900\ N \approx 25190\ kg$$

$$\delta = \frac{Pl^3}{3EI} = \frac{25190 \cdot 280^3}{3 \cdot 2{,}1 \cdot 10^6 \cdot I} = \frac{87773}{I} \rightarrow I = \frac{87773}{0{,}9\hat{3}} = 94043\ cm^4 \ \rightarrow\ \boxed{IPN\ 550}$$

Two side supported beam:

$$\delta_{MAX} = {}^{L}/_{500} = {}^{560}/_{500} = 1{,}12\ cm\ \ ;\ \ q = 6{,}5 \cdot 3{,}5 = 22{,}75\ kN/m = 2275\ kg/m$$

$$\delta = \frac{5}{384} \cdot \frac{ql^4}{EI} = \frac{5 \cdot 22{,}75 \cdot 560^4}{384 \cdot 2{,}1 \cdot 10^6 \cdot I} = \frac{13872}{I} \rightarrow I = \frac{13872}{1{,}12} = 12385\ cm^4 \ \rightarrow\ \boxed{IPN\ 320}$$

Joists (worst case):

$$\delta_{MAX} = {}^{L}/_{500} = {}^{440}/_{500} = 0{,}88\ cm\ \ ;\ \ q = 6{,}5 \cdot 0{,}8 = 5{,}2\ kN/m = 520\ kg/m$$

$$\delta = \frac{5}{384} \cdot \frac{ql^4}{EI} = \frac{5 \cdot 5{,}2 \cdot 440^4}{384 \cdot 2{,}1 \cdot 10^6 \cdot I} = \frac{1208}{I} \rightarrow I = \frac{1208}{0{,}88} = 1372\ cm^4 \ \rightarrow \ \boxed{IPN\ 180}$$

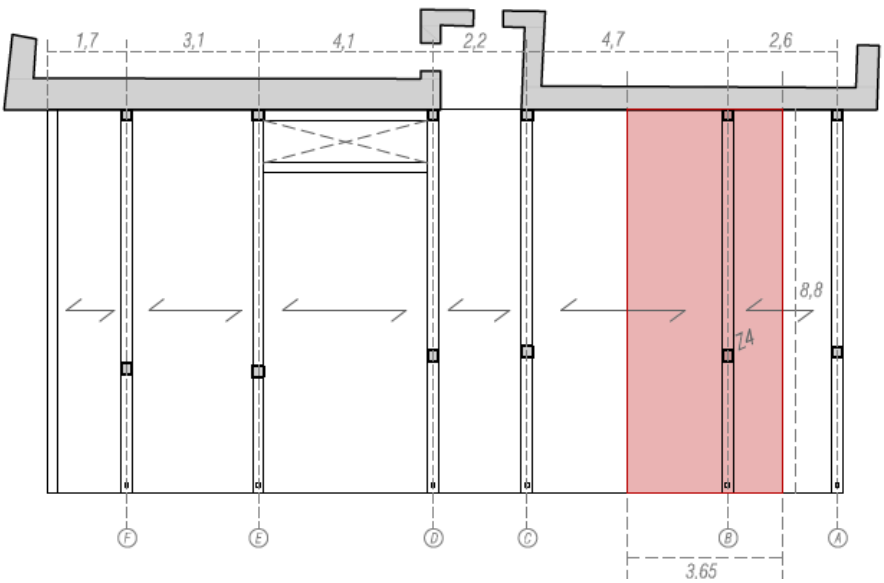
FRAME ORGANIZATION. LIMIT STATE VERIFYCATIONS

First degree frame

This frame is transmitting the loads from the building itself to the foundations in the ground. The main loads are the own weight, use overload and snow load. This frame is not receiving wind load at all.

Permanent loads (own weight):		
One way floor slab (Steel deck floor system + joists)	g_1	$2{,}5\ kN/m^2$
Floor and inner partitions	g_2	$1{,}5\ kN/m^2$
False ceiling	g_3	$0{,}5\ kN/m^2$
Flat roof (slope concrete, gravel…)	g_4	$1{,}5\ kN/m^2$
Façade (façade itself + wooden slats)	g_5	$8\ kN/ml$
Variable loads:		
Use overload	g_6	$2\ kN/m^2$
Snow	g_7	$0{,}5\ kN/m^2$

The total length of this beam is about 8,8 meters, and the floor piece they hold is 3,65 meters wide. The total floor is area is 31,12 m^2 but apart from this it also holds the party wall and the façade (3,65+3,65 = 7,3 m).



Total loads:

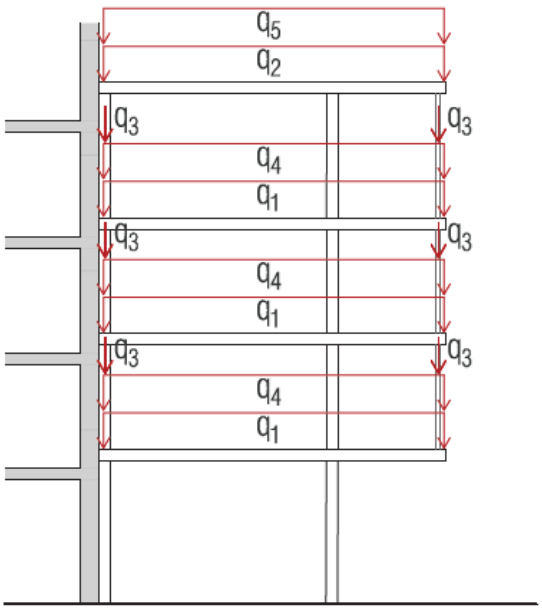
$$q_1: B.P\ (floors)\ = \frac{(g_1 + g_2 + g_3) \cdot 31{,}12}{8{,}8} = \frac{4{,}5 \cdot 31{,}12}{8{,}8} = 15{,}91\ kN/m$$

$$q_2: B.P\ (roof) = \frac{(g_1 + g_3 + g_4) \cdot 31{,}12}{8{,}8} = \frac{4{,}5 \cdot 31{,}12}{8{,}8} = 15{,}91\ kN/m$$

$$q_3: B.P.(façada) = g_5 \cdot 3{,}65 = 8 \cdot 3{,}65 = 29{,}2\ kN$$

$$q_4: U.O. = \frac{g_6 \cdot 31{,}12}{8{,}8} = \frac{2 \cdot 31{,}12}{8{,}8} = 7{,}07\ kN/m$$

$$q_5: snow\ = \frac{g_7 \cdot 31{,}12}{8{,}8} = \frac{0{,}5 \cdot 31{,}12}{8{,}8} = 1{,}77\ kN/m$$



Once we have quantify the loads affecting our frame we will combine them creating the load hypothesis. Through these hypothesis we will on the on hand verify the deflection (serviceability limit state), and on the other hand we will verify the resistance of the structure against collapse (ultimate limit state).

Serviceability limit state (deflection and collapse):

On this limit state we don´t have to increase the values but we have to apply some simultaneity factors:

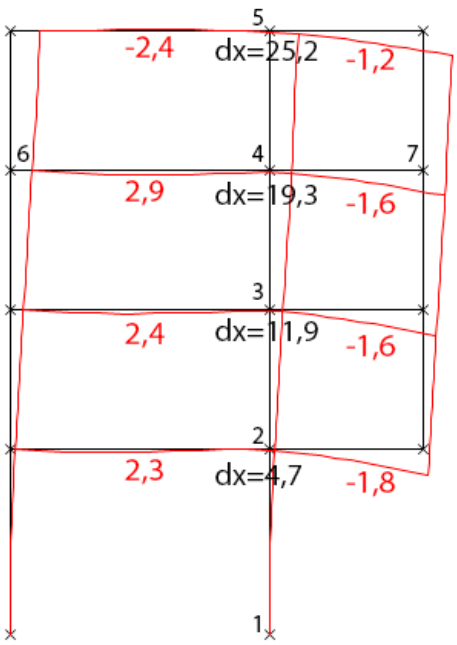
$$\sum_{j\geq 1} G_{k,j} + P + Q_{k,1} + \sum_{i>1} \Psi_{0,i} \cdot Q_{k,i}$$

Tabla 4.2 Coeficientes de simultaneidad (ψ)

	ψ_0	ψ_1	ψ_2
Sobrecarga superficial de uso (Categorías según DB-SE-AE)			
• Zonas residenciales (Categoría A)	0,7	0,5	0,3
• Zonas administrativas(Categoría B)	0,7	0,5	0,3
• Zonas destinadas al público (Categoría C)	0,7	0,7	0,6
• Zonas comerciales (Categoría D)	0,7	0,7	0,6
• Zonas de tráfico y de aparcamiento de vehículos ligeros con un peso total inferior a 30 kN (Categoría E)	0,7	0,7	0,6
• Cubiertas transitables (Categoría F)		(1)	
• Cubiertas accesibles únicamente para mantenimiento (Categoría G)	0	0	0
Nieve			
• para altitudes > 1000 m	0,7	0,5	0,2
• para altitudes ≤ 1000 m	0,5	0,2	0
Viento	0,6	0,5	0

	O.W.	U.O.	Snow
SLS – U.O.	1	1	1 x 0,5
SLS - Snow	1	1 x 0,7	1

After proposing the hypothesis we are going to introduce them in the WinEva computer program. As we have calculated before, the columns are a HEB 220 profile; the braces a 90.6 square shaped empty profile; the depth beam is a IPN 550 profile; and the rest beams are a IPN 320 profile. The worst of the two hypothesis is the use overload:



Beams maximum deflection:

$H_{64}: {}^{5600}/_{500} = 11{,}2\ mm > 2{,}9\ mm$ ✓

$H_{47}: {}^{3300}/_{500} = 6{,}6\ mm > 1{,}8\ mm$ ✓

Columns maximum collapse:

$H_T: {}^{13000}/_{250} = 52\ mm > 25{,}2\ mm$ ✓

$H_{12}: {}^{4000}/_{250} = 16\ mm > 4{,}7\ mm$ ✓

$H_{23}: {}^{3000}/_{250} = 12\ mm > 7{,}2\ mm$ ✓

$H_{34}: {}^{3000}/_{250} = 12\ mm > 7{,}4\ mm$ ✓

$H_{45}: {}^{3000}/_{250} = 12\ mm > 5{,}9\ mm$ ✓

Ultimate limit state (resistance):

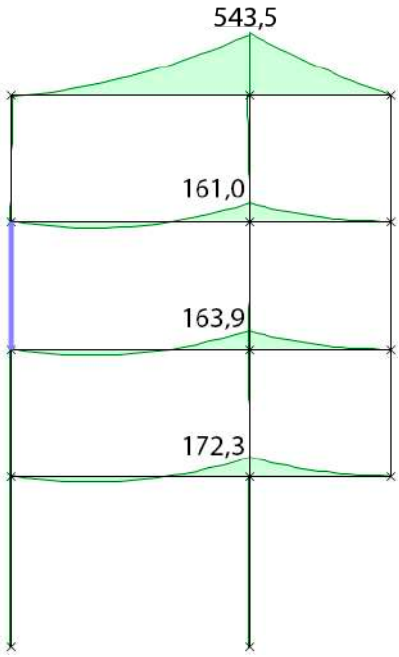
On the ultimate limit state, we verify the building structure resistance against the building structure collapse. This time we have to increase the loads (security factors) and also apply the simultaneity factors:

$$\sum_{j\geq 1} \gamma_{G,j} \cdot G_{k,j} + \gamma_P \cdot P + \gamma_{Q,1} \cdot Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \cdot \Psi_{0,i} \cdot Q_{k,i}$$

	O.W.	U.O.	Snow
ULS – U.O.	1 x 1,35	1 x 1,5	1,5 x 0,5
ULS - Snow	1 x 1,35	1,5 x 0,7	1 x 1,5

Among those two hypothesis the worst is the use overload. We will check the moment and shear strength diagram for the beams and the axial diagram for the columns and the braces.

Beam verification:



Flection moment:

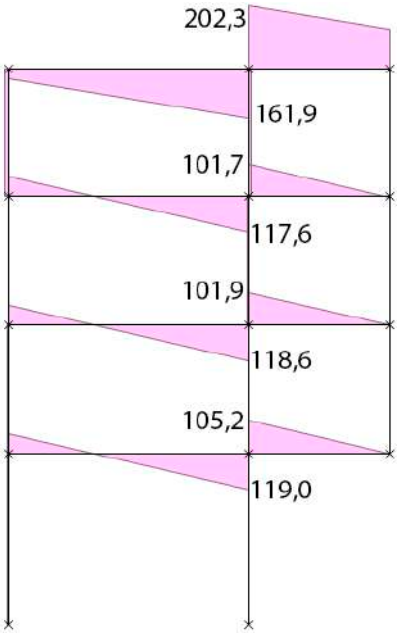
The greatest moment in the top depth beam is of 543,5 kN.m, and in the rest of the floor beams 172,3 kN.m:

$$\sigma_N = \frac{M}{W_x} \rightarrow W_x = \frac{543{,}5 \cdot 10^6}{261{,}9} = 2075000 = 2075\ cm^3$$

IPN 550 profile W_x=3610 cm^3, then, $\mathbf{2075 < 3610}$ ✓

$$\sigma_N = \frac{M}{W_x} \rightarrow W_x = \frac{172{,}3 \cdot 10^6}{261{,}9} = 640282 = 640{,}3\ cm^3$$

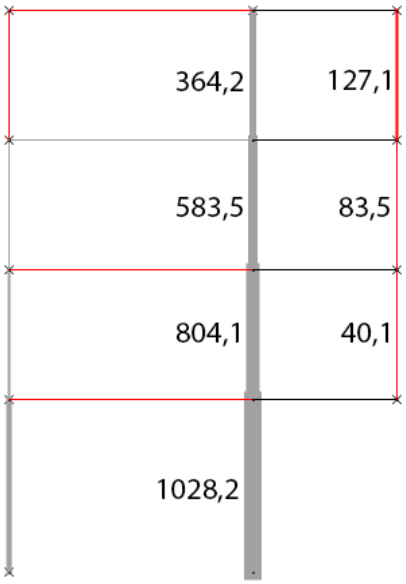
IPN 320 profile W_x=782 cm^3, then, $\mathbf{640{,}3 < 782}$ ✓



Cutting strength:

The cutting strength suffered by the beam is too small comparing to the flection moment, therefore we can consider it negligible.

Column verifications:



Axial strength:

The greatest load on the column is 1028,2 kN, and 127,1 kN in the braces:

$$A = \frac{N \cdot 1{,}2}{\delta_{CD}} = \frac{1028200 \cdot 1{,}2}{261{,}9} = 4711{,}11 = 47{,}11\ cm^2$$

HEB 220 profile has 91 cm^2, then $\mathbf{47,11} < 91$ ✓

$$A = \frac{N \cdot 1{,}2}{\delta_{CD}} = \frac{127100 \cdot 1{,}2}{261{,}9} = 582{,}35 = 5{,}82\ cm^2$$

90.6 empty profile has 18,9 cm^2, then $\mathbf{5,82} < 18{,}9$ ✓

This square shaped empty profile is going to bear lot less load than expected, so we could also change it to a smaller one to save material and money.

Second degree frame

This frame is going to be used to verify the resistance of the joists. The floor type we have chosen is a Steel deck floor system held by some metallic joists. This profiles are going to be 0,8 meters separated from each other so they will be holding a wideness of 0,8 meters of floor slab too. Being 18,5 the length of the building the total floor slab amount is of 14,44 m^2. The load combination will have the own weight of the floors, the use overload, the snow load and this time also the wind load.

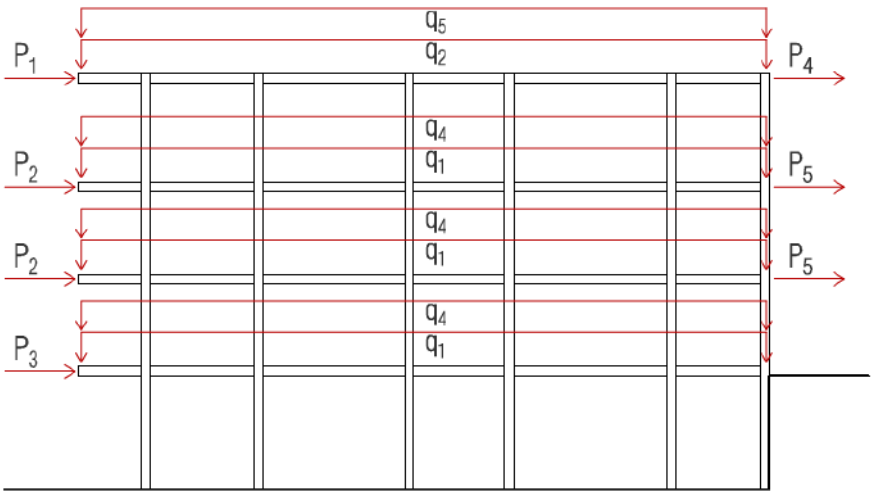
$$q_6: B.P\ (floors)\ = \frac{(g_1 + g_2 + g_3) \cdot 14{,}44}{18{,}5} = \frac{4{,}5 \cdot 14{,}44}{18{,}5} = 3{,}51\ kN/m$$

$$q_7: B.P\ (roof) = \frac{(g_1 + g_3 + g_4) \cdot 14{,}44}{18{,}5} = \frac{4{,}5 \cdot 14{,}44}{18{,}5} = 3{,}51\ kN/m$$

$$q_8: U.O. = \frac{g_6 \cdot 14{,}44}{18{,}5} = \frac{2 \cdot 14{,}44}{18{,}5} = 1{,}56\ kN/m$$

$$q_9: snow\ = \frac{g_7 \cdot 14{,}44}{18{,}5} = \frac{0{,}5 \cdot 14{,}44}{18{,}5} = 0{,}39\ kN/m$$

$$q_{10}: wind\ (P_1 = 19{,}5\ ;\ P_2 = 16{,}2\ ;\ P_3 = 21{,}6\ ;\ P_4 = 12{,}2\ ;\ P_5 = 10{,}2)$$

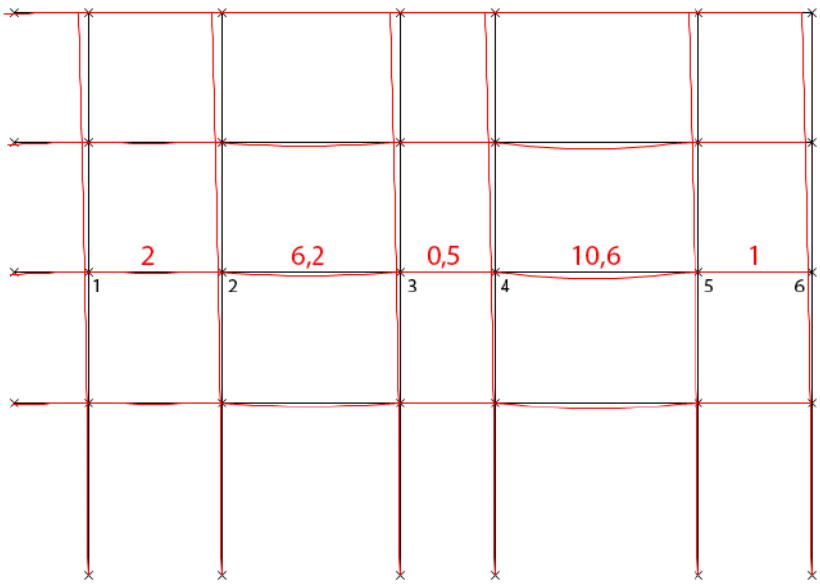


Serviceability limit state (defection):

	O.W.	U.O.	Snow	Wind
SLS – U.O.	1	1	1 x 0,5	1 x 0,6
SLS - Snow	1	1 x 0,7	1	1 x 0,6
SLS - Wind	1	1 x 0,7	1 x 0,5	1

As we have done the dimensioning of the columns according to the SLS horizontal deflection limit we don´t need to verify that again. Instead, we are going to check the deflection of the joists.

To verify that, the use overload hypothesis going to be the worst load combination (joist profile IPN 180):



$\delta_{12}: {}^{3100}/_{500} = 6{,}2\ mm > 2\ mm$ ✓

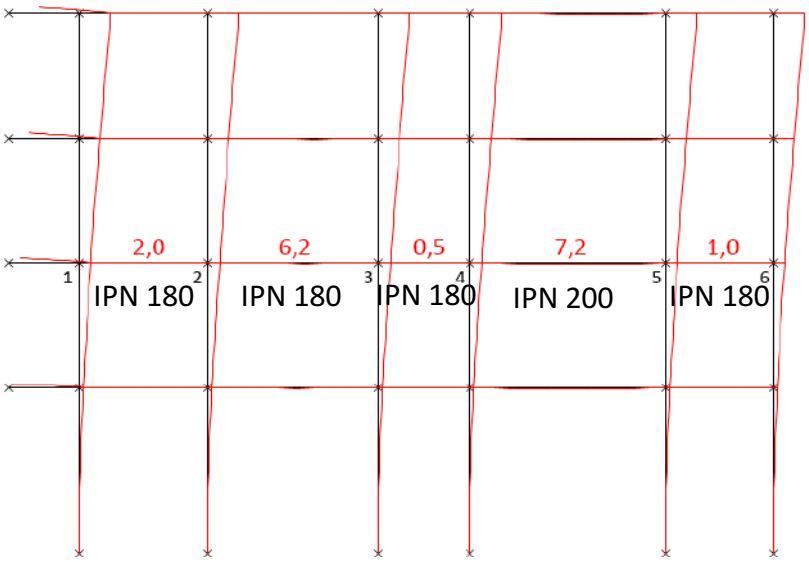
$\delta_{23}: {}^{4100}/_{500} = 8{,}2\ mm < 6{,}2\ mm$ ✓

$\delta_{34}: {}^{2200}/_{500} = 4{,}4\ mm > 0{,}5\ mm$ ✓

$\delta_{45}: {}^{4700}/_{500} = 9{,}4\ mm < 10{,}6\ mm$ ✗ → we need a bigger profile

$\delta_{56}: {}^{2600}/_{500} = 6{,}6\ mm > 1\ mm$ ✓

Instead of increasing the size of all the joists we are only going to increase the profile of the joists located on the 4-5 gap. These news joists will have IPN 200 profile.

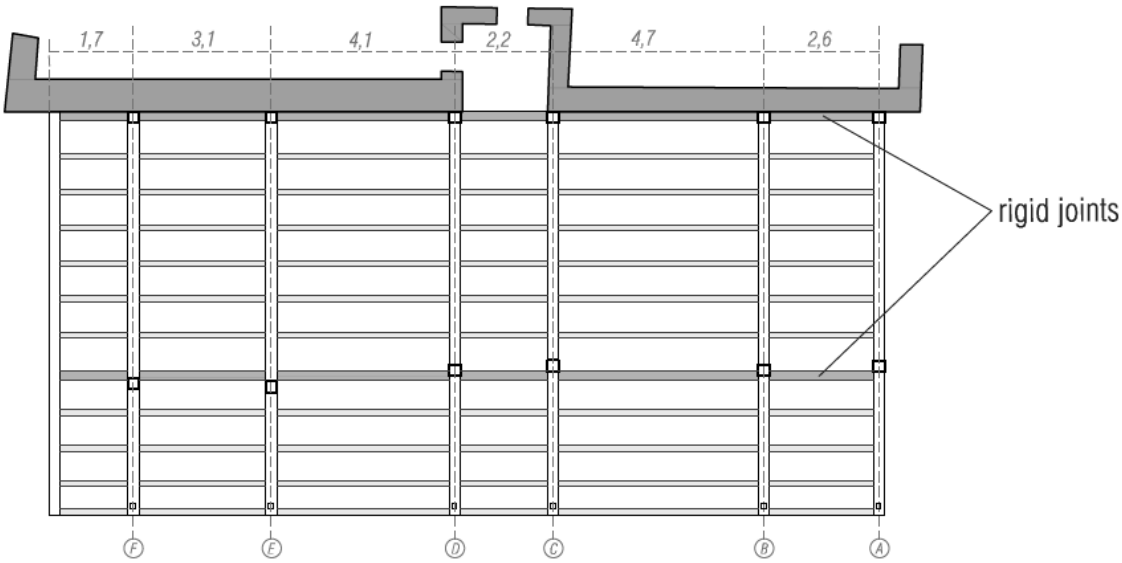


$H_{45}: {}^{4700}/_{500} = 9{,}4\ mm > 7{,}2\ mm$ ✓

<u>Ultimate limit state (resistance):</u>

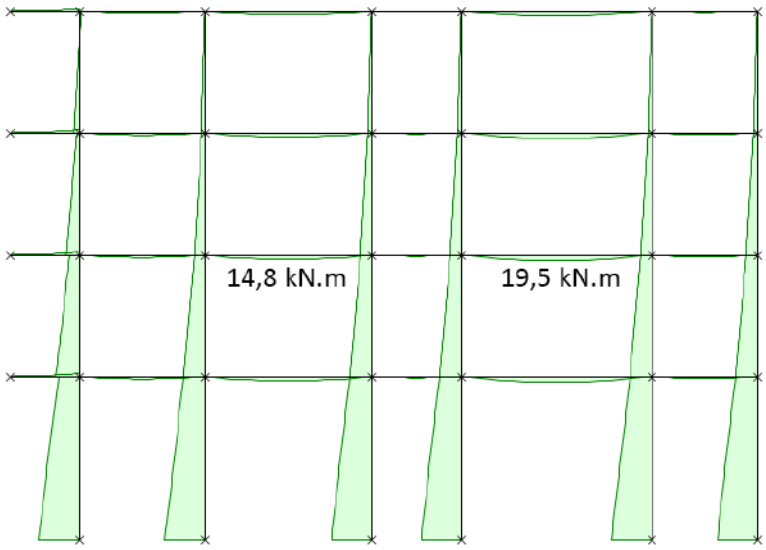
	O.W.	U.O.	Snow	Wind
ULS – E.O.	1 x 1,35	1 x 1,5	1,5 x 0,5	1,5 x 0,6
ULS - Snow	1 x 1,35	1,5 x 0,7	1 x 1,5	1,5 x 0,6
ULS - Wind	1 x 1,35	1,5 x 0,7	1,5 x 0,5	1 x 1,5

We will have to verify the resistance of joists working in two different ways: on one side the hinged beams, not transmitting the moments; and on the other side the joists with rigid joints, giving rigidity to the whole building frame. The moment diagram of this two joists is going to be absolutely different, so we will have to check both.



Hinged beams:

These beams only bear vertical loads, working under pure flection moment. The worst load combination is use overload (beams profiles, IPN 180 and IPN 200):



The greatest moment is 14,8 kN.m for IPN 180 profile, and 19,5 kN.m for IPN 200 profile:

$$\sigma_N = \frac{M}{W_x} \rightarrow W_x = \frac{14{,}8 \cdot 10^6}{261{,}9} = 56510 = 56{,}5\ cm^3$$

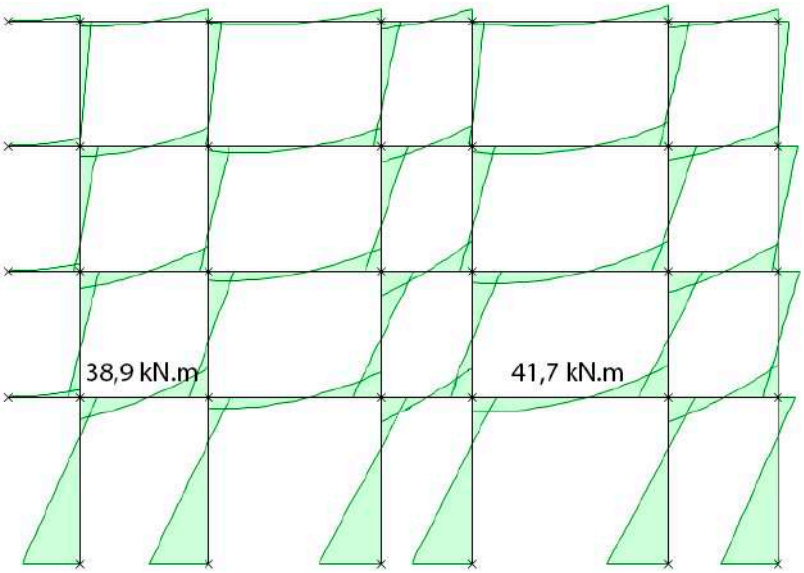
IPN 180 profile has W_x=161 cm^3, then, $\mathbf{56,5} < 161$ ✓

$$\sigma_N = \frac{M}{W_x} \rightarrow W_x = \frac{19{,}5 \cdot 10^6}{261{,}9} = 74455 = 74{,}5\ cm^3$$

IPN 200 profile has W_x=214 cm^3, then, $\mathbf{74,5} < 214$ ✓

Beams with rigid joints:

These beams will bear, apart from the previous vertical loads, the horizontal loads created by the wind. This time the worst hypothesis will be with the wind load (joist profiles, IPN 180 and IPN 200):



The greatest moment is 38,9 kN.m in IPN 180 profile, and 41,7 kN.m in IPN 200 profile:

$$\sigma_N = \frac{M}{W_x} \rightarrow W_x = \frac{38{,}9 \cdot 10^6}{261{,}9} = 148500 = 148{,}5\ cm^3$$

IPN 180 profile has W_x=161 cm^3, then, $\mathbf{148,5} < 161$ ✓

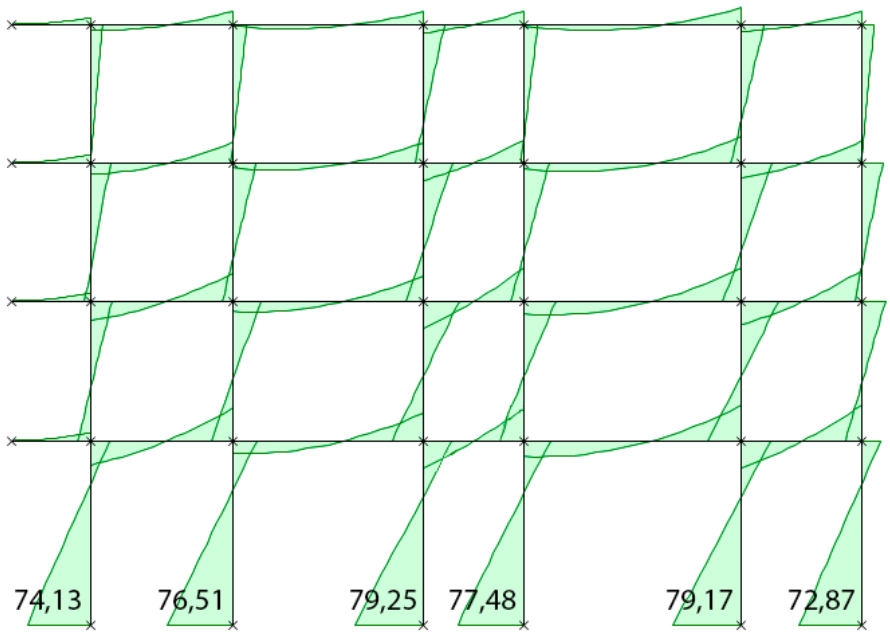
$$\sigma_N = \frac{M}{W_x} \rightarrow W_x = \frac{41{,}7 \cdot 10^6}{261{,}9} = 159200 = 159{,}2\ cm^3$$

IPN 200 profile has W_x=214 cm^3, then, $\mathbf{159,2} < 214$ ✓

As we can see there is no need of increasing the size of the joists with rigid joints, as with their current size can bear the moment.

Columns:

To verify the **moments in the columns**, we will do the verification with the wind load combination (column profile HEB 20):



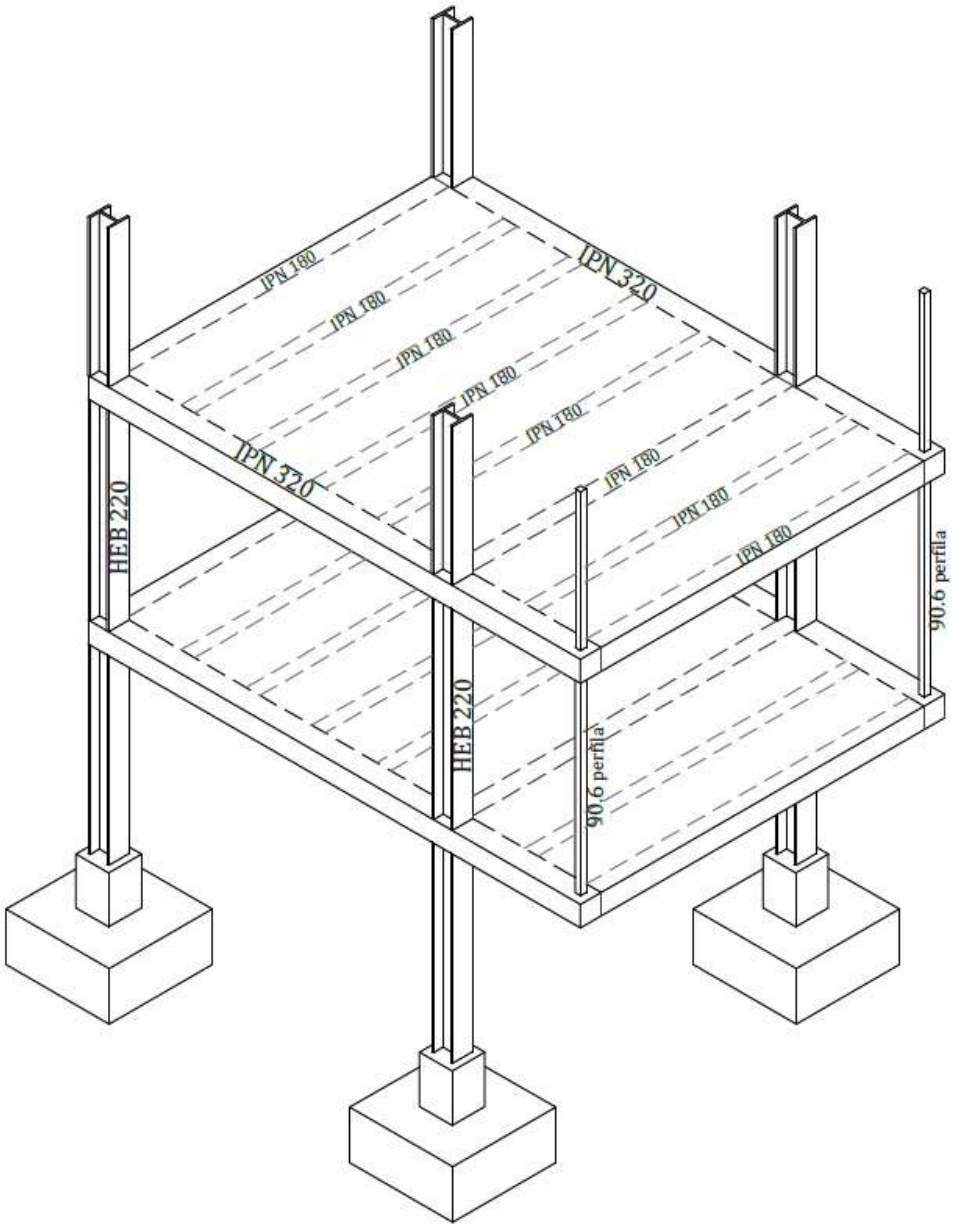
The greatest moment happens in the first column with 79,25 kN.m:

$$\sigma_N = \frac{M}{W_x} \rightarrow W_x = \frac{79{,}25 \cdot 10^6}{261{,}9} = 302600 = 302{,}6\ cm^3$$

HEB 220 profile has W_x=736 cm^3, then, $\mathbf{302{,}6} < 736$ ✓

SUMMARY

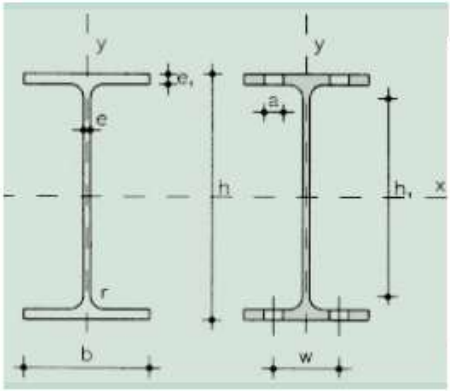
- Columns: **HEB 220**
- Braces: **90.6 perfil karratua**
- Beams:
  - Top depth beam: **IPN 550**
  - Rest of the floors beams: **IPN 320**
- Joists:
  - Long span (4,7 m): **IPN 200**
  - Rest: **IPN 180**



ANNEX 4. SUMARY INFORMATION FOR EXAM EXERCISES

PROFILE CHARTS

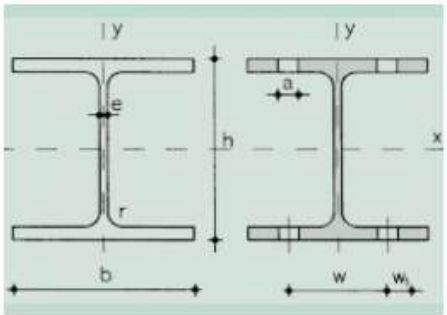
IPE profile



A = Section's area
S_x = Half section's static moment against X axis
I_x = Section's inertia moment against X axis
$W_x = 2I_x : h$. Section's resistant module against X axis
$i_x = \sqrt{I_x : A}$. Section's rotation radious against X axis
I_y = Section's inertia moment against Y axis
$W_y = 2I_y : b$. Section's resistant module against Y axis
$i_y = \sqrt{I_y : A}$. Section's rotation radious against Y axis
I_t = Section's torsion module
I_a = Section's warping module
u = Section's perimeter
a = Rivet's hole diameter
w = Distance between holes' axis
h_1 = Soul's flat part's height
p = Weight per meter

Profile	Dimensions							Section's terms										Weight
	h mm	b mm	a mm	e_1 mm	r_1 mm	h_1 mm	u mm	A cm²	S_x cm³	I_x cm⁴	W_x cm³	i_x cm	I_y cm⁴	W_y cm³	i_y cm	I_t cm⁴	I_a cm⁶	p kp/m
IPE 80	80	46	3,8	5,2	5	60	328	7,64	11,6	80,1	20,0	3,24	8,49	3,69	1,05	0,721	118	6,00
IPE 100	100	55	4,1	5,7	7	75	400	10,30	19,7	171,0	34,2	4,07	15,90	5,79	1,24	1,140	351	8,10
IPE 120	120	64	4,4	6,3	7	93	475	13,20	30,4	318,0	53,0	4,90	27,70	8,65	1,45	1,770	890	10,40
IPE 140	140	73	4,7	6,9	7	112	551	16,40	44,2	541,0	77,3	5,74	44,90	12,30	1,65	2,630	1.981	12,90
IPE 160	160	82	5,0	7,4	9	127	623	20,10	61,9	869,0	109,0	6,58	68,30	16,70	1,84	3,640	3.959	15,80
IPE 180	180	91	5,3	8,0	9	146	698	23,90	83,2	1.320,0	146,0	7,42	101,00	22,20	2,05	5,060	7.431	18,80
IPE 200	200	100	5,6	8,5	12	159	788	28,50	110,0	1.940,0	194,0	8,26	142,00	28,50	2,24	6,670	12.990	22,40
IPE 220	220	110	5,9	9,2	12	178	848	33,40	143	2.770	252	9,11	205	37,3	2,48	9,15	22.670	26,20
IPE 240	240	120	6,2	9,8	15	190	922	39,10	183	3.890	324	9,97	284	47,3	2,69	12,00	37.390	30,70
IPE 270	270	135	6,6	10,2	15	220	1.040	45,90	242	5.790	429	11,20	420	62,2	3,02	15,40	70.580	36,10
IPE 300	300	150	7,1	10,7	15	249	1.160	53,80	314	8.360	557	12,50	604	80,5	3,35	20,10	125.900	42,20
IPE 330	330	160	7,5	11,5	18	271	1.250	62,60	402	11.770	713	13,70	788	98,5	3,55	26,50	199.100	49,10
IPE 360	360	170	8,0	12,7	18	299	1.350	72,70	510	16.270	904	15,00	1.040	123,0	3,79	37,30	313.600	57,10
IPE 400	400	180	8,6	13,5	21	331	1.470	84,50	654	23.130	1.160	16,50	1.320	146,0	3,95	48,30	490.000	66,30
IPE 450	450	190	9,4	14,6	21	379	1.610	98,80	851	33.740	1.500	18,50	1.680	176,0	4,12	65,90	791.000	77,60
IPE 500	500	200	10,2	16,0	21	426	1.740	116,00	1.100	48.200	1.930	20,40	2.140	214,0	4,31	91,80	1.249.000	90,70
IPE 550	550	210	11,1	17,2	24	468	1.880	134,00	1.390	67.120	2.440	22,30	2.670	254,0	4,45	122,00	1.884.000	106,00
IPE 600	600	220	12,0	19,0	24	514	2.010	155,00	1.760	92.080	3.070	24,30	3.390	308,0	4,66	172,00	2.846.000	122,0

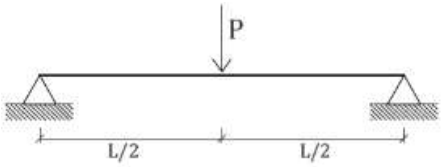
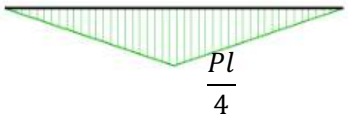
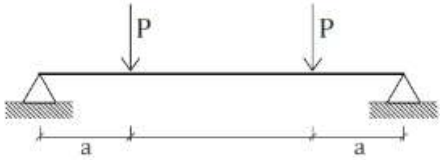
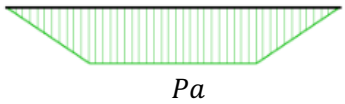
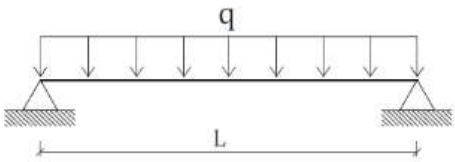
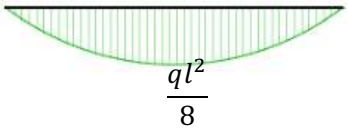
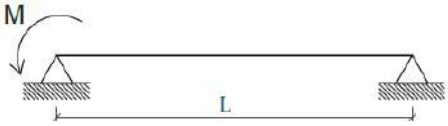
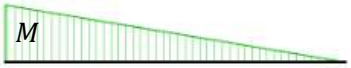
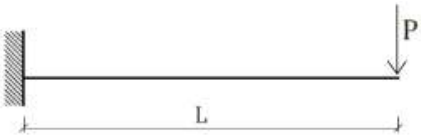
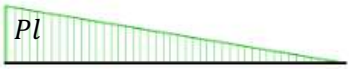
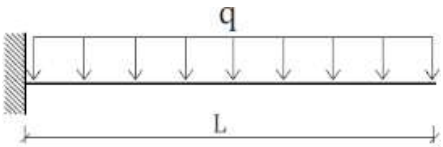
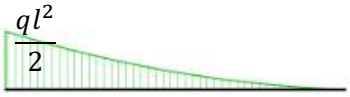
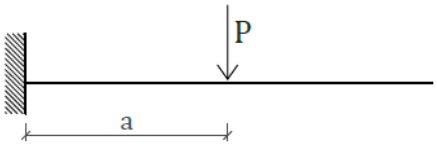
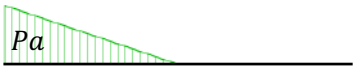
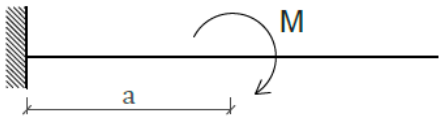
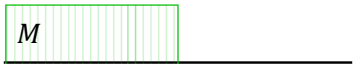
HEB profile



A = Section's area
S_x = Half section's static moment against X axis
I_x = Section's inertia moment against X axis
$W_x = 2I_x : h$. Section's resistant module against X axis
$i_x = \sqrt{I_x : A}$. Section's rotation radious against X axis
I_y = Section's inertia moment against Y axis
$W_y = 2I_y : b$. Section's resistant module against Y axis
$i_y = \sqrt{I_y : A}$. Section's rotation radious against Y axis
I_t = Section's torsion module
I_a = Section's warping module
u = Section's perimeter
a = Rivet's hole diameter
w = Distance between holes' axis
h_1 = Soul's flat part's height
p = Weight per meter

Profile	Dimensions							Section's terms										Weight
	h mm	b mm	e mm	e_1 mm	r_1 mm	h_1 mm	u mm	A cm^2	S_x cm^3	I_x cm^4	W_x cm^3	i_x cm	I_y cm^4	W_y cm^3	i_y cm	I_t cm^4	I_a cm^6	p kp/m
HEB 100	100	100	6,0	10,0	12	56	567	26,0	52,1	450	90	4,16	167	33	2,53	9,34	3.375	20,4
HEB 120	120	120	6,5	11,0	12	74	686	34,0	82,6	864	144	5,04	318	53	3,06	14,90	9.410	26,7
HEB 140	140	140	7,0	12,0	12	92	805	43,0	123,0	1.509	216	5,93	550	79	3,58	22,50	22.480	33,7
HEB 160	160	160	8,0	13,0	15	104	918	54,3	177,0	2.492	311	6,78	889	111	4,05	33,20	47.940	42,6
HEB 180	180	180	8,5	14,0	15	122	1.040	65,3	241,0	3.831	426	7,66	1.363	151	4,57	46,50	93.750	51,2
HEB 200	200	200	9,0	15,0	18	134	1.150	78,1	321,0	5.696	570	8,54	2.003	200	5,07	63,40	171.100	61,3
HEB 220	220	220	9,5	16,0	18	152	1.270	91,0	414,0	8.091	736	9,43	2.843	258	5,59	84,40	295.400	71,5
HEB 240	240	240	10,0	17,0	21	164	1.380	106,0	527,0	11.259	938	10,30	3.923	327	6,08	110,00	486.900	83,2
HEB 260	260	260	10,0	17,5	24	177	1.500	118,4	641,0	14.919	1.150	11,20	5.135	395	6,58	130,00	753.700	93,0
HEB 280	280	280	10,5	18,0	24	196	1.620	131,4	767,0	19.270	1.380	12,10	6.595	471	7,09	153,00	1.130.000	103,0
HEB 300	300	300	11,0	19,0	27	208	1.730	149,1	934,0	25.166	1.680	13,00	8.563	571	7,58	192,00	1.688.000	117,0
HEB 320	320	300	11,5	20,5	27	225	1.770	161,3	1.070,0	30.823	1.930	13,80	9.239	616	7,57	241,00	2.069.000	127,0
HEB 340	340	300	12,0	21,5	27	243	1.810	170,9	1.200,0	36.656	2.160	14,60	9.690	646	7,53	278,00	2.454.000	134,0
HEB 360	300	300	12,5	22,5	27	261	1.850	180,6	1.340,0	43.193	2.400	15,50	10.140	676	7,49	320,00	2.883.000	142,0
HEB 400	400	300	13,5	24,0	27	298	1.930	197,8	1.620,0	57.680	2.880	17,10	10.819	721	7,40	394,00	3.817.000	155,0
HEB 450	450	300	14,0	26,0	27	344	2.030	218,0	1.990,0	79.887	3.550	19,10	11.721	781	7,33	500,00	5.258.000	171,0
HEB 500	500	300	14,5	28,0	27	390	2.120	238,6	2.410,0	107.176	4.290	21,20	12.624	842	7,27	625,00	7.018.000	187,0
HEB 550	550	300	15,0	29,0	27	438	2.220	254,1	2.800,0	136.691	4.970	23,20	13.077	872	7,17	701,00	8.856.000	199,0
HEB 600	600	300	15,5	30,0	27	486	2.320	270,0	3.210,0	171.041	5.700	25,20	13.530	902	7,08	783,00	10.965.000	212,0

BEAM COMPENDIUM. MAXIMUM DEFLECTION

Load and support type	Bending moment diagram	Middle of the span
		$\delta = \frac{P \cdot l^3}{48EI}$
		$\delta = \frac{P \cdot a}{24EI}(3l^2 - 4a^2)$
		$\delta = \frac{5}{384} \cdot \frac{q \cdot l^4}{EI}$
		$\delta = \frac{M \cdot l^2}{16EI}$
Load and support type	**Bending moment diagram**	**End of the cantilever**
		$\delta = \frac{P \cdot l^3}{3 \cdot EI}$
		$\delta = \frac{q \cdot l^4}{8 \cdot EI}$
		$\delta = P \cdot a^2 \frac{3l - a}{6 \cdot EI}$
		$\delta = \frac{M \cdot a}{2 \cdot EI}(l - a)$

VERIFICATIONS

SECTION'S RESISTANCE

Normal stress:

$$\frac{N_{ED}}{A} + \frac{M_{y,ED}}{W_y} + \frac{M_{z,ED}}{W_z} \leq f_{yD}$$

$$\frac{N_{ED}}{A} + \frac{M_{y,ED}z}{I_y} + \frac{M_{z,ED}y}{I_z} \leq f_{yD}$$

Tangential stress:

$$\frac{V_{ED} \cdot S_y}{I_y \cdot b} \leq \frac{f_{yD}}{\sqrt{3}}$$

Stress combination:

$$\sqrt{\sigma^2 + 3\tau^2} \leq f_{yD}$$

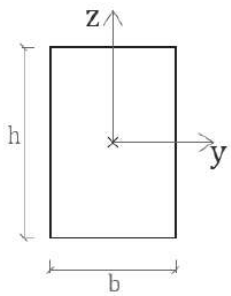
Centre of gravity:

$$Z_{G.C.} = \frac{\sum_i^N Z_{G.C._i} \cdot A_i}{A_i}$$

Steiner's theorem:

$$I_y = \sum_i^N \left(I_{yi} + A_i \cdot {d_i}^2\right)$$

Moments of inertia of a rectangle about the center of gravity:



$$I_y = \frac{bh^3}{12} \; ; \; I_z = \frac{hb^3}{12}$$

Connector's calculation

$$V_{X,ED} \leq n \cdot V_{kon}$$

$$V_{X,ED} = \frac{S_y{}^{z_0}}{I_y} \Delta_v \; ; V_{kon} = A_{kon} \cdot \frac{f_{yD}}{\sqrt{3}}$$

BUCKLING DUE TO AXIAL STRENGTH

$$\frac{N_{ED}}{\chi \cdot A} < f_{yD}$$

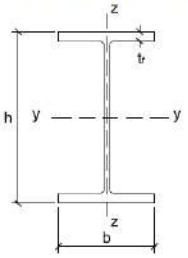
$$\chi : Buckling\ reduction\ factor \rightarrow \text{ Chart 6.3}$$

Tabla 6.3 Valores del coeficiente de pandeo (χ)

Esbeltez reducida	Curva de pandeo a_0	a	b	c	d
Coeficiente (α) de imperfección	0,13	0,21	0,34	0,49	0,76
≤ 0,20	1,00	1,00	1,00	1,00	1,00
0,30	0,99	0,98	0,96	0,95	0,92
0,40	0,97	0,95	0,93	0,90	0,85
0,50	0,95	0,92	0,88	0,84	0,78
0,60	0,93	0,89	0,84	0,79	0,71
0,70	0,90	0,85	0,78	0,72	0,64
0,80	0,85	0,80	0,72	0,66	0,58
0,90	0,80	0,73	0,66	0,60	0,52
1,00	0,73	0,67	0,60	0,54	0,47
1,10	0,65	0,60	0,54	0,48	0,42
1,20	0,57	0,53	0,48	0,43	0,38
1,30	0,51	0,47	0,43	0,39	0,34
1,40	0,45	0,42	0,38	0,35	0,31
1,50	0,40	0,37	0,34	0,31	0,28
1,60	0,35	0,32	0,31	0,28	0,25
1,80	0,28	0,27	0,25	0,23	0,21
2,00 (1)	0,23	0,22	0,21	0,20	0,18
2,20 (1)	0,19	0,19	0,18	0,17	0,15
2,40 (1)	0,16	0,16	0,15	0,14	0,13
2,70 (2)	0,13	0,13	0,12	0,12	0,11
3,00 (2)	0,11	0,10	0,10	0,10	0,09

(1) esbeltez intolerable en los elementos principales
(2) esbeltez intolerable incluso en elementos de arriostramiento

Tabla 6.2 Curva de pandeo en función de la sección transversal

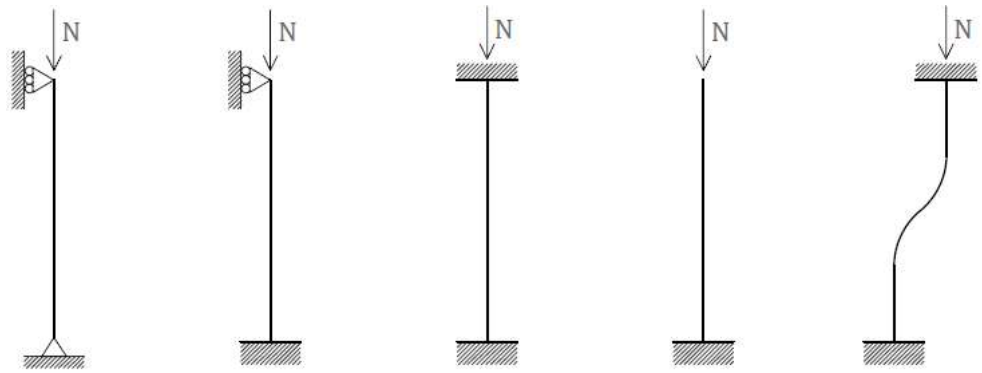
Tipo de sección		Tipo de acero	S235 a S355		S450	
		Eje de pandeo (1)	y	z	y	z
Perfiles laminados en I 	h/b > 1,2	t ≤ 40 mm	a	b	a_0	a_0
	40 mm < t ≤ 100 mm		b	c	a	a
	h/b ≤ 1,2	t ≤ 100 mm	b	c	a	a
	t > 100 mm		d	d	c	c

Reduced slenderness:

$$\bar{\lambda} = \sqrt{\frac{A \cdot f_y}{N_{CR}}}$$

Critical normal strength of Euler

$$N_{CR} = \frac{\pi^2}{l^2} \cdot EI \quad \rightarrow \quad l_k = l \cdot \beta \ \ (buckling\ length)$$



$\beta = 1$	$\beta = 0{,}7$	$\beta = 0{,}5$	$\beta = 2$	$\beta = 1$
Double-hinged	Hinged - fixed	Double - fixed	Fixed end	double-fixed with a diplacement

LATERAL TORSIONAL BUCKLING

1.) $M_{CR} = \sqrt{{M_{LTv}}^2 + {M_{LTw}}^2}$

$M_{LTv} = C_1 \cdot \frac{M}{l_C} \sqrt{I_z \cdot G \cdot I_t \cdot E}$ → In the next page

$M_{LTv} = C_1 \cdot W_{EC,y} \cdot \frac{M^2 \cdot E}{{l_C}^2} \cdot {i_{f,z}}^2$ → In the next page

2.) $\overline{\lambda}_{LT} = \sqrt{\frac{W_{EL,y} \cdot f_y}{M_{CR}}}$

$\overline{\lambda}_{LT} : lateral\ torsional\ reduced\ slenderness$

When lateral torsional reduced slenderness is smaller than 0,4, CTE regulation accepts $\chi_{LT} = 1$

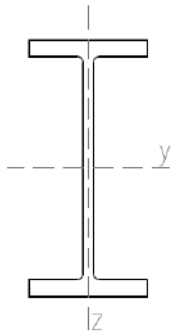
3.) $\left.\begin{array}{l} \overline{\lambda}_{LT} \\ Buckling\ curve\ (6.6\ chart) \end{array}\right\}$ χ_{LT} lateral buckling reduction factor. (6.3 chart)

Tabla 6.6 Factor de imperfección α_{LT}

Elemento	Límites	Curva de pandeo	α_{LT}
Perfil laminado con sección en doble T	h/b ≤ 2	a	0,21
	h/b > 2	b	0,34

4.) $\frac{M_{ED}}{\chi_{LT} \cdot W_{EL,y}} \leq f_{yD}$

CRITICAL MOMENT'S CHART



IPE profile

$\bar{\lambda}_{LT}$ calculation

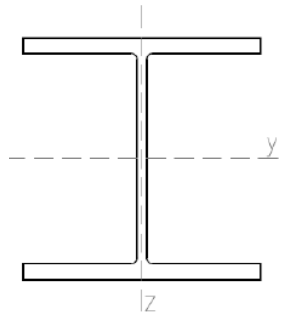
$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} \ , \qquad where \ \ M_{cr} = \sqrt{M_{LT,v}^2 + M_{LT,w}^2}$$

$$b_{LT,v} = \pi \cdot \sqrt{G \cdot I_t \cdot E \cdot I_z} \ , \qquad to\ get \ \ M_{LT,v} = b_{LT,v} \cdot \frac{C_1}{L_c}$$

$$b_{LT,w} = W_{el,y} \cdot \pi^2 \cdot E \cdot {i_{f,z}}^2 \ , \qquad to\ get \ \ M_{LT,w} = b_{LT,w} \cdot \frac{C_1}{{L_c}^2}$$

$i_{f,z}$ is compressed web's rotation radius regarding to z + nearest web's 1/3

PROFILE	$i_{f,z}$ mm	$b_{LT,v}$ $\cdot 10^6\ N \cdot mm^2$	$b_{LT,w}$ $\cdot 10^9\ N \cdot mm^3$	$W_{el,y} \cdot f_y$ $N \cdot mm$	$W_{pl,y} \cdot f_y$ $N \cdot mm$
IPE 80	11.4	9 994	5 387	5 500 000	6 380 000
IPE 100	13.5	17 897	12 918	9 405 000	10 835 000
IPE 120	15.7	28 445	27 076	14 575 000	16 720 000
IPE 140	17.9	42 974	51 333	21 257 500	24 310 000
IPE 160	20.0	64 248	90 366	29 975 000	34 100 000
IPE 180	22.2	90 121	149 134	40 150 000	45 650 000
IPE 200	24.4	128 995	239 386	53 350 000	60 500 000
IPE 220	26.9	176 678	377 941	69 300 000	78 650 000
IPE 240	29.4	247 810	580 441	89 100 000	100 650 000
IPE 270	33.0	335 251	968 287	117 975 000	133 100 000
IPE 300	36.5	451 683	1 538 012	153 175 000	172 700 000
IPE 330	38.8	610 245	2 224 702	196 075 000	221 100 000
IPE 360	41.3	807 215	3 195 858	248 600 000	280 500 000
IPE 400	43.3	1 063 933	4 507 677	319 000 000	360 250 000
IPE 450	45.2	1 373 321	6 351 658	412 500 000	467 500 000
IPE 500	47.2	1 791 060	8 911 695	530 750 000	605 000 000
IPE 550	49.1	2 349 973	12 191 912	671 000 000	764 500 000
IPE 600	51.3	3 068 100	16 745 269	844 250 000	968 000 000



HEB profile

$\bar{\lambda}_{LT}$ calculation

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} \ , \qquad where \ \ M_{cr} = \sqrt{M_{LT,v}^2 + M_{LT,w}^2}$$

$$b_{LT,v} = \pi \cdot \sqrt{G \cdot I_t \cdot E \cdot I_z} \ , \qquad to\ get \ \ M_{LT,v} = b_{LT,v} \cdot \frac{C_1}{L_c}$$

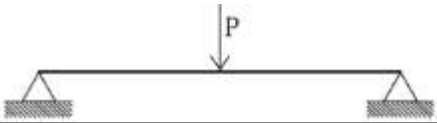
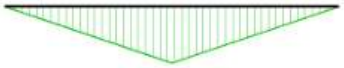
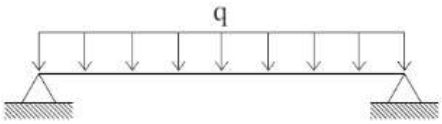
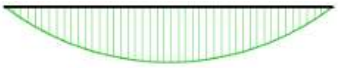
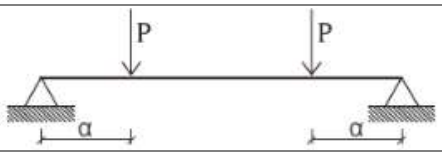
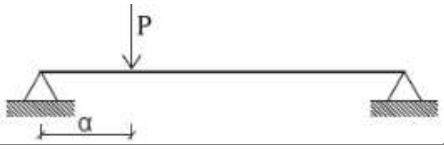
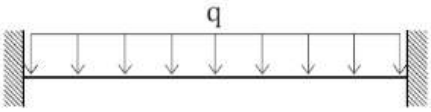
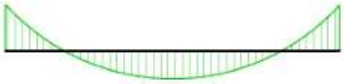
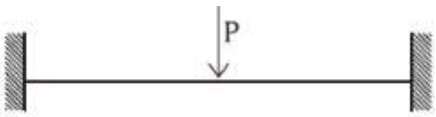
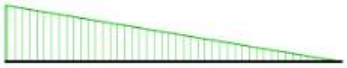
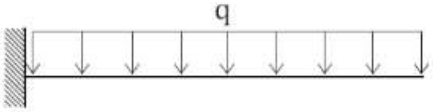
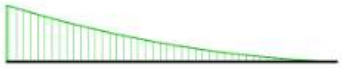
$$b_{LT,w} = W_{el,y} \cdot \pi^2 \cdot E \cdot {i_{f,z}}^2 \ , \qquad to\ get \ \ M_{LT,w} = b_{LT,w} \cdot \frac{C_1}{{L_c}^2}$$

$i_{f,z}$ is compressed web's rotation radius regarding to z + nearest web's 1/3

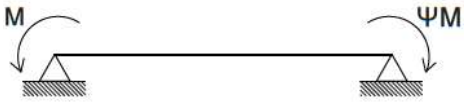
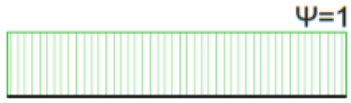
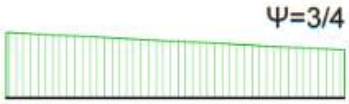
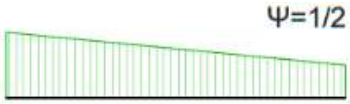
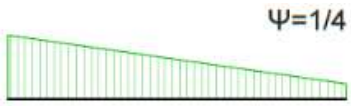
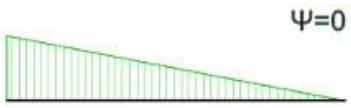
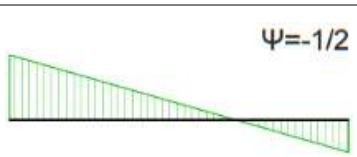
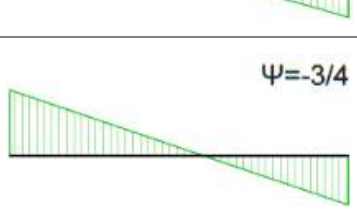
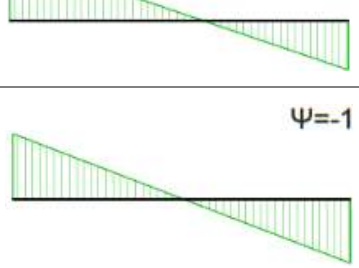
PROFILE	$i_{f,z}$ mm	$b_{LT,v}$ $\cdot 10^6\ N \cdot mm^2$	$b_{LT,w}$ $\cdot 10^9\ N \cdot mm^3$	$W_{el,y} \cdot f_y$ $N \cdot mm$	$W_{pl,y} \cdot f_y$ $N \cdot mm$
HEB 100	26.8	161 820	133 977	24 750 000	28 600 000
HEB 120	32.2	282 038	309 451	39 600 000	45 375 000
HEB 140	37.5	455 800	629 557	59 400 000	67 650 000
HEB 160	42.7	703 917	1 175 263	85 525 000	97 350 000
HEB 180	48.0	1 030 381	2 034 281	117 150 000	132 550 000
HEB 200	53.4	1 459 021	3 368 809	156 750 000	176 550 000
HEB 220	58.7	2 006 006	5 256 214	202 400 000	227 700 000
HEB 240	64.1	2 690 552	7 987 997	257 950 000	288 750 000
HEB 260	69.6	3 346 049	11 546 101	316 250 000	352 000 000
HEB 280	74.8	4 114 247	16 002 996	379 500 000	420 750 000
HEB 300	80.1	5 252 783	22 340 523	462 000 000	514 250 000
HEB 320	79.9	6 114 297	25 537 001	530 750 000	588 500 000
HEB 340	79.6	6 724 907	28 366 054	594 000 000	660 000 000
HEB 360	79.3	7 366 101	31 280 713	660 000 000	737 000 000
HEB 400	78.4	8 452 049	36 689 657	792 000 000	891 000 000
HEB 450	77.9	9 910 143	44 650 086	976 250 000	1 094 500 000
HEB 500	77.3	11 498 132	53 129 450	1 179 750 000	1 325 500 000
HEB 550	76.5	12 416 428	60 283 477	1 366 750 000	1 540 000 000
HEB 600	75.7	13 321 397	67 699 530	1 567 500 000	1 765 500 000

LATERAL TORSIONAL EQUIVALENT MOMENT FACTOR "C"

Table depending on the load

Load and support type	Bending moment diagram	Moment coeff.
		1.00
		1.75
		2.56
		1.35
		1.13
		$1 + \alpha^2$
		$1.35 + 1.95\,(0.5 - \alpha)^2$
		1.30
		1.70
		1.28
		2.04

CTE CHART

Support type	Bending moment diagram	Moment coeff.
		1.00
		1.14
		1.32
		1.56
		1.88
		2.28
		2.70
		2.93
		2.75

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